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Control-Limit Policy Optimization of Redundant Systems in Condition-Based Maintenance

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Abstract

Condition-based maintenance (CBM) modelled through Proportional Hazards Model (PHM) is a kind of maintenance strategy in which a system is inspected in intervals of time and the optimal replacement policy is determined based on an optimal threshold value called control limit. In this paper, a model is presented to determine a control limit for redundant systems as whole systems in which their components would be replaced under the control limit policy for the whole system. In redundant systems, during each inspection time, a system failure is occurred less single-unit systems. The proposed approach is demonstrated through an example of redundant systems.

Keywords:
Condition Based Maintenance, Proportional Hazards Model, Redundant System

Introduction

Condition-based maintenance (CBM) is a class of maintenance strategy in which the health of an asset would be continuously monitored or discretely inspected to prevent the destroying failures. Against the planned periodic maintenance and corrective maintenance, condition-based maintenance can predictively give an effective information to preventively do appropriate maintenance action prior to failure.

Over condition-based maintenance (CBM) process, the collected inspection data related to the condition of an equipment can provide some information about the future health of the equipment and the appropriate maintenance actions would be planned to prevent dangerous or destructive failures [1]. Actually, age of the equipment is under control through measuring the operating condition by various parameters such as temperature, state of the oil, vibration, noise, etc. These parameters are inspected as signs to motivate maintainer set some maintenance plans before occurring any serious failure.

The process of CBM constitutes of two main steps: first one is condition monitoring and the second one is maintenance decision making. The core of CBM is the condition monitoring step wherein the mentioned parameters are monitored either continuously using specific types of sensors associated to the technology, the nature and the structure of an equipment or in discrete using periodic or no-periodic inspections [13], consequently after observing the status of parameters, an optimal decision including replacement or repair would be made to reduce unnecessary maintenance leading to cost reduction and to prevent the destructive failures leading to a better safety and more availability of the equipment.

Both periodically and continuously condition monitoring need to a device through which real-time data regarding the condition of the equipment could be collected and assessment could start over. These devices can be portable indicators for discrete-time inspections and sensors for continuous monitoring.

Two main constraints were stated in [1] for continuous monitoring: being expensive due to special sensors and inaccurate information may be obtained due to failure or bad status of sensors, however it has to work continuously.

Discrepant models for CBM have been proposed [2-8] and many survey studies have been done on CBM [9, 10, 11, 12]. Proportional Hazard Model (PHM) was introduced by Cox [14] and used widely in various fields, it was also used to model the hazard rate in maintenance optimization using covariates as parameters monitoring. A CBM approach for single unit have been presented using PHM to determine an optimal control limit for hazard rate by which maintainer can replace the equipment economically [15, 16]. Ghasemi et al [17] using partially observed Markov decision process (POMDP) and PHM presented an optimal replacement policy in which equipment's state is unknown and solved the model using dynamic programming. A multi-objective model for CBM optimization based on PHM has been proposed by Tian et al. [18]. They offered a model in which reliability maximization and cost minimization are objectives and two constraints for cost and reliability are contemplated. The multi-objective model is transformed into single objective using physical programming approach to solve the problem more convenient in their study. Golmakani and Fattahipour [19] developed the model proposed by Makis et al [15] in which a new cost parameter called the cost of inspection is considered and inspection interval is a new variable in their model. A development created by Golmakani and Fattahipour is a scheme in which interval inspection is a variable with non-same interval times. Interval times directly depends on the age of the equipment [20]. A new consideration has been done on PHM that repair policy with its cost is a new parameter in the model and both control limit and repair policy are simultaneously determined in a optimization problem in [21]. A model has been proposed by Golmakani and Pouresmaeeli [22] to develop the CBM optimization model with PHM in which failure replacement cost depends on the equipment's degradation state and inspection interval is also considered as a variable with its cost. Lam and Banjevic [23] proposed a decision policy for CBM optimization model that schedules inspections according to the current health of the equipment and optimized myopically during the next inspection. They used PHM for hazard rate and a Markovian process for the system covariates.

All reported CBM optimization model based on PHM were applied for a single unit or equipment. Actually for other structures such as series, parallel, series-parallel, parallel-series, k-out-of-n and load sharing based on proportional hazard model has not worked seriously. However most studies does focus on individual component's age as a single item and individually model component by component.

We propose a method that determines a same control limit policy for components in a redundant system. In our model, the replacement policy is based on an optimal control limit for all component structured in parallel. Proportional hazards model is used to model the hazard rate of the whole system, not single component by single component and a combination of states for all components is considered through Markovian process.

This paper presents the classical PHM for single unit in section 2. Section 3 is assigned to describe the PHM for redundant systems and section 4 present some numerical examples taken from other studies in the literature are used for redundant systems and a comparison will be discussed. Section 5 consists of conclusion and future extensions.

PHM for Single Unit

In this model T is time to failure of the system and $Z(t)$ is stochastic covariate at time t that is a diagnostic process denoting the effect of the operating environment on the system (e.g, it can be the vibration level of the equipment based on vibration analysis). Discrete time intervals are determined such as $\Delta, 2\Delta, 3\Delta, \ldots$ to perform inspections in which stochastic covariate is estimated by the beginning of the interval, i.e., $k\Delta \le t < (k+1)\Delta$ interval is estimated by $Z(k\Delta)$. Hazard rate function in the model is shown by the product of a baseline failure rate, e.g., for Weibull distribution, $h_0(t) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}$ and a positive function to show the effect of covariate values denoting $\psi(Z(t)) = \exp\{\sum_{i=1}^{n}\gamma_i z_i(t)\}$, where γ_i is constant value and $z_i(t)$ is observed value of covariate at time t for *i*-th parameter.

Based on Makis and Jardine [15] for Weibull hazard function and the stochastic covariate, the hazard function of the system is given by:

$$h(t, Z(t)) = \frac{\beta}{\eta}(\frac{t}{\eta})^{\beta-1}\exp\{\gamma Z(t)\} \qquad (1)$$

$$t = 0, \Delta, 2\Delta, \ldots, \Delta \ge 0$$

where the parameters of hazard function .,i.e., β, η and γ are estimated using maximum likelihood method in the presence of system's failure histories [16]. The method maximum likelihood is also used to estimate the transition probabilities of the covariate process. The result is a $m \times m$ matrix, say $p(k)$, where the elements of $p(k)$ includes $p_{ij}(k)$ representing the conditional probability of the process $\{Z(k\Delta)\}$, i.e., the probability that in the next inspection , the state of the system is j given that the current state is i and failure may happen after and m denotes the number of covariate condition, e.g., the level of vibration, it constitutes a set $S = \{0,1,\ldots,m\}$ that $Z(k\Delta) \in S$.

The replacement policy based on PHM is to perform a preventive maintenance if the failure hazard function constructed based on PHM is larger than the constant predetermined value denoted by d as a control limit and the time when the failure risk reaches the control limit is denoted by T_d. The average cost per unit time consists of two components; the total expected cost, i.e., C the cost of preventive maintenance (replacement) with probability that the failure does not happen before time T_d and $C+K$ the cost of replacement and failure the probability that the failure does happen before time T_d denoted by $Q(d)$, and the expected time interval, i.e., the expected time interval of minimum time to failure and T_d denoted by $W(d)$. This function is given by:

$$\varphi(d) = \frac{C(1-Q(d)) + (C+K)(Q(d))}{W(d)} \qquad (2)$$

Where $Q(d) = \Pr\{T \le T_d\}$ and $W(d) = E(\min\{T, T_d\})$.

The aim of the model is to find the optimal control limit,

d^* , that minimize the objective function, φ . Based on iteration procedure developed by Makis and Jardine (1992), d^* can be obtained through $d^{(n)} = \varphi(d^{(n-1)}), n = 1,2,...$ where $d^{(0)}$ as an initial value can be set an arbitrary value. After some iteration, a unique value for d is found called d^* that is satisfied in equation (3) as below.

$$\varphi(d^*) = d^*$$

PHM for Redundant Systems

Condition-based maintenance optimization of redundant systems with repairable components through proportional hazards model is carried out in this study. A redundant system as whole system can have a failure rate function in which the condition of each component can influence the failure rate function of the whole system. Two approaches can be used to model CBM optimization of a redundant system: the first assumption would be to apply PHM for all components separately as mentioned the previous section and the second approach, against the firs approach, to consider all components in parallel as a whole system and to apply PHM for the whole system that is the main contribution of our study.

Assuming a redundant system with identical n components as depicted in Fig.1, the hazard rate of the system can be given as below:

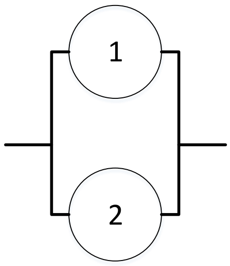

Figure 1 - A redundant system with two components.

The hazard rate function based on PHM can be given as below:

$$h(t,z) = \lim_{\Delta t \to 0} \frac{P(t < T \le t + \Delta t | z) P_z(z) / \Delta t}{P(t < T | z) P_z(z)} \quad (3)$$

$$P(t < T | Z_s, 0 \le s \le t) = \exp(-\int_0^t h_0(s) \psi(Z_s) ds) \quad (4)$$

The hazard rate function of a redundant systems with identical components can be given by:

$$\begin{aligned} & P_{Sys}(t < T | Z_{1,s}, Z_{2,s}, ..., Z_{n,s}, 0 \le s \le t) \\ & = 1 - \prod_{l=1}^{n} (1 - P_l(t > T | Z_{l,s}, 0 \le s \le t)) \quad (5) \\ & = 1 - \prod_{l=1}^{n} (1 - \exp(-\int_0^t h_{0,l}(s) \psi(Z_{l,s}) ds)) \end{aligned}$$

$$\begin{aligned} & \lim_{\Delta t \to 0} \frac{P_{Sys}(t < T | Z_{1,s}, Z_{2,s}, ..., Z_{n,s}, 0 \le s \le t)}{\Delta t} \\ & = -\frac{d(P_{Sys}(t < T | Z_{1,s}, Z_{2,s}, ..., Z_{n,s}, 0 \le s \le t))}{dT} \end{aligned} \quad (6)$$

$$\begin{aligned} & h(t, (Z_{1,s}, Z_{2,s}, ..., Z_{n,s})) \\ & = \frac{-\dfrac{d(P_{Sys}(t < T | Z_{1,s}, Z_{2,s}, ..., Z_{n,s}, 0 \le s \le t))}{dT}}{1 - \prod_{l=1}^{n} (1 - \exp(-\int_0^t h_{0,l}(s) \psi(Z_{l,s}) ds))} \\ & = -d \ln(1 - \prod_{l=1}^{n} (1 - \exp(-\int_0^t h_{0,l}(s) \psi(Z_{l,s}) ds))) / dt \end{aligned}$$

(7)

Where $Z_{l,s}, l = 1,2,...,n$ and $s \in S = \{0,1,...,m\}$ implies that the state of the component l is s , $h_{0,l}(t)$ implies the basic hazard rate function of component l and $h(t, (Z_{1,s}, Z_{2,s}, ..., Z_{n,s}))$ implies the hazard function based on PHM of the whole system in which state of the component l is s.

The replacement policy based on PHM for redundant systems with identical components is to find a same control limit for all components if the hazard rate function of each components constructed based on PHM reaches the control limit value denoted by d at each inspection time. The time when the failure risk of the system reaches the control limit is denoted by T_d . The average cost per unit time consists of total expected cost, i.e., nC the cost of preventive maintenance of all component, replacement cost, with the probability that the failure of all components does not occur before time T_d and $nC + K$, the cost of replacement and the failure with the probability that all components fail before time T_d which is denoted by $Q(d)$. The expected time interval is the expected minimum time between time to failure of all components and T_d which is denoted by $W(d)$. This function can be given as below:

$$\varphi(d)=\frac{nC+KQ(d)}{W(d)} \quad (8)$$

Where $\varphi(d)=\Pr\{T\le T_d\}$ and $W(d)=E(Min\{T,T_d\})$.

The optimal control limit, d^*, can be determined for redundant components using iteration procedure for single system developed by Makis and Jardine (1992) [15].

To find a unique solution in iteration procedure, $W(d)$ and $Q(d)$ have to be computed in each iteration. Thus an algorithm based on recursive computational procedure proposed by Makis and Jardine (1992) is developed to compute for the redundant systems step by step.

Suppose that $t_{i_1,i_2,\dots,i_n}$, $i_l\in S$ is defined for a given $d>0$ as:

$$t_{i_1,i_2,\dots,i_n}=\inf\{t\ge 0 \mid K\times h(t,(i_1,i_2,\dots,i_n))\ge d\} \quad (9)$$

and $K_{(i_1,i_2,\dots,i_n)}$, $i_l\in S$ is an integer value that is defined as:

$$(K_{(i_1,i_2,\dots,i_n)}-1)\Delta\le t_{i_1,i_2,\dots,i_n}<K_{(i_1,i_2,\dots,i_n)}\Delta \quad (10)$$

Also, let $Q(j,(i_1,i_2,\dots,i_n))$ be the probability of replacement due to failure and $W(j,(i_1,i_2,\dots,i_n))$ be the expected time until replacement given that the age of the system is $j\Delta$ and the current state of component l is i_l, in other words, $Z(j\Delta)=(i_1,i_2,\dots,i_n)$.

Suppose that $Z(0)=(0,0,\dots,0)$, the following equations can be given as follows.

$$W(d)=E(Min\{T,T_d\})=W(0,(0,0,\dots,0)) \quad (11)$$

$$Q(d)=P(T\le T_d)=Q(0,(0,0,\dots,0)) \quad (12)$$

The following backward recursion equations can be applied to compute $W(0,(0,0,\dots,0))$ and $Q(0,(0,0,\dots,0))$.

$$\begin{cases}W(j,(i_1,i_2,\dots,i_n))\\ Q(j,(i_1,i_2,\dots,i_n))\end{cases} ,\ j>K_{(i_1,i_2,\dots,i_n)}-1 \quad (13)$$

$$W(j,(i_1,i_2,\dots,i_n))=\int_0^{t_{i_1,i_2,\dots,i_n}-(K_{(i_1,i_2,\dots,i_n)}-1)\Delta} R(K_i-1,i,s)ds$$

$$Q(j,(i_1,i_2,\dots,i_n))$$
$$=1-R(K_{(i_1,i_2,\dots,i_n)}-1,(i_1,i_2,\dots,i_n),t_{i_1,i_2,\dots,i_n}-(K_{(i_1,i_2,\dots,i_n)}-1)\Delta$$

$$j=K_{(i_1,i_2,\dots,i_n)}-1 \quad (14)$$

$$W(j,(i_1,i_2,\dots,i_n))=\int_0^{\Delta}R(j,(i_1,i_2,\dots,i_n),s)ds$$
$$+R(j,(i_1,i_2,\dots,i_n),\Delta)\sum_{r=(i_1,i_2,\dots,i_n)}^{(m,m,\dots,m)}W(j+1,r)P_{(i_1,i_2,\dots,i_n)r}(j) \quad (15)$$

$$Q(j,(i_1,i_2,\dots,i_n))=1-R(j,(i_1,i_2,\dots,i_n),\Delta)$$
$$+R(j,(i_1,i_2,\dots,i_n),\Delta)\sum_{r=(i_1,i_2,\dots,i_n)}^{(m,m,\dots,m)}Q(j+1,r)P_{(i_1,i_2,\dots,i_n)r}(j)$$
$$j<K_{(i_1,i_2,\dots,i_n)}-1 \quad (16)$$

Where the conditional reliability function, $R(j,(i_1,i_2,\dots,i_n),t)$, is obtained through equation below:

$$R(j,(i_1,i_2,\dots,i_n),t)=1-$$
$$\prod_{l=1}^{n}(1-\exp(-\int_0^t\frac{\beta_l}{\eta_l}(\frac{u}{\eta_l})^{\beta_l-1}\exp\{\gamma_l i_l\}du)),t\in[0,\Delta) \quad (17)$$

Where $R(j,(i_1,i_2,\dots,i_n),t)$ represents conditional reliability for next time t given that the state of the system is $(i_1,i_2,\dots,i_n)$ and inspection value is j.

Numerical Instance

Assume that a redundant system would be studied to determine an optimal control limit. The baseline distribution of each component is a Weibull distribution with the following parameters; $\alpha=1$, $\beta=2$ and let $\psi(z)=e^{-0.5z},K=2,C=5,\Delta=1$. We assume that the status of each component belongs to $S=\{0,1\}$ with the transition probability as follows.

$$P=\begin{pmatrix}0.4 & 0.6\\ 0 & 1\end{pmatrix}$$

The optimal control limit and the optimal expected average cost for a system with single component calculated by Makis and Jardine (1992) [15] are $d^*=8.15$ and $\varphi(d^*)=8.15$.

A system would be studied to compare with condition in which control limit policy is determined for each component separately. Redundant systems with two identical components are given as follows.

Assume that there exists a redundant system with two identical components shown in Fig.1 that their components follow above-mentioned parameters. The status of two components is simultaneously essential in every inspection. Four possible status can be occurred as shown through the transition diagram in Fig.2.

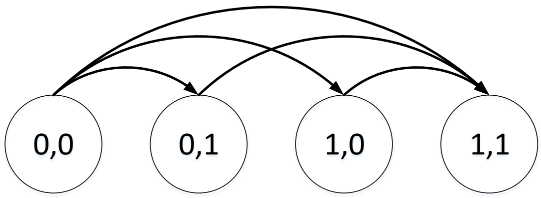

Figure 2- Transition diagram for a redundant system with two components

In Fig.2, each status denotes the status of two components, e.g., (1, 0) means the status of component 1 is 1 and the status of component 2 is 0. The transition matrix for the whole system is given as below.

$$P = \begin{pmatrix} 0.4 & 0.25 & 0.25 & 0.1 \\ 0 & 0.5 & 0 & 0.5 \\ 0 & 0 & 0.5 & 0.5 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

Where, the elements of matrix P denote the probability of transition between status of two component, e.g., $P_{11} = 0.4$ means that the status of component 1 is 0 and the status of component 2 is also 0 and the probability of transition again to this status is 0.4

Using the recursive method based on equations (17) to (24), the optimal control limit is determined. The calculation of each step is presented in Table 1.

Table 1 - Determination of control limit policy for the system with two redundant components.

Iteration	1	2	3
d	5	9.49	10.23
$t_{0,0}$	1.364	2.376	2.561
$t_{0,1}$	1.166	2.329	2.534
$t_{1,0}$	1.166	2.329	2.534
$t_{1,1}$	0.895	1.460	1.566
$K_{0,0}$	2	3	3
$K_{0,1}$	2	3	3
$K_{1,0}$	2	3	3
$K_{1,1}$	1	2	2
$W(0,(0,0))$	1.1634	1.1648	1.1679
$Q(0,(0,0))$	0.5206	0.9625	0.9743
$\varphi(d)$	9.49	10.23	10.23

As shown in Table 2 and Table 3 corresponding to a redundant system with two components and three components, respectively, the control limit for the system with two components is $d^* = 10.23$.

A useful comparison would be considered between two scenarios. Scenario1: it is assumed that a control limit is computed for each component in a redundant system by the control limit policy proposed by Makis and Jardine [15], then total expected cost $\varphi(d)$ for the whole system must be the summation of total expected cost of each component. Scenario2: it is to compute a control limit for the whole system, not for each component of the system, by the proposed method in this study. A comparison is shown in Table 2.

Table 2 - A comparison between two scenarios.

	Scenario1	Scenario2
A redundant systems with two identical components	16.3	10.23

As shown in Tale 2, the total expected cost of the proposed method in this study is less than the classical method.

Conclusion

In this paper, we extended the CBM model and the recursive method proposed in Makis and Jardine (1992) [15] for redundant systems. The objective of the system is to minimize the total average cost per time in order to find an optimal control limit for the redundant system, not for each component. Every fixed interval is 1 in this study, the failure rate of the whole system constructed by PHM is supposed to be computed and compared with the optimal control limit, if it exceeds, all components must be replaced with new one or if it does not exceed , the system must be allowed to continue. During interval time if all components fail, all components must be replaced with new components. It is notable that the control limit is determined for the whole system and it does not need to compute for each component. A comparison was also done between two scenarios in which against the proposed scenario in this study, in every interval time, the control limit is individually computed for each component based on the proposed method by Makis and Jardine (1992), and the result shows that the method proposed in this paper has less total expected cost per time than that scenario.

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