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Applied sequential sampling plan in the truncated life test

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Abstract

The lifetime of a product is expected to be high, thus it is usual to terminate the life test at a prefixed time. This type of test for evaluating the lifetime of a product is called truncated life test. It is assumed the lifetime of a product as its quality characteristic and the sequential sampling plan is applied in the context of truncated life test. In the sequential sampling the samples are selected from the lot stage to stage. In each stage the total number of inspected item and the total number of defective items to that stage is specified and based on them it is decided whether to continue sampling, cease it or take another sample. Finally, by illustrating an example the application of the sequential sampling is indicated for the truncated life test.

Key words:

truncated life test; sequential sampling; lifetime

1. Introduction

Statistical method for quality control can be classified as: statistical process control, design of experiment and acceptance sampling. A typical application of acceptance sampling is as follows: a company receives a shipment from the supplier. Some pre-specified quality characteristics are inspected through a random sample from the shipment. Based on the information obtained from the sampling the decision is made about lot disposition. Usually this decision is to accept or reject the lot. This decision sometimes is called Lot sentencing. Generally, there are three approaches for lot sentencing: (1) accept with no inspection (2) 100% inspection and (3) acceptance sampling. Acceptance sampling is widely used when testing is destructive, the cost of thorough investigation is very high and/or it takes too much time [1].

Consider a product that its quality is represented by its mean lifetime, μ. A lot is considered good if its mean lifetime not less than the specified values as μ_0 , in the other word a lot is good if $\mu \geq \mu_0$. A lot is considered bad if $\mu < \mu_0$. Both consumer and producer are subject to risk. The producer's risk is the probability of rejection a good lot while the consumer's risk is the probability of acceptance a bad lot.

On the other hand, a lifetime of a product is expected to be high, thus it is usual to terminate the life test at a prefixed time. This type of test for evaluating the lifetime of a product is called truncated life test. Generally, the truncated life tests are studies from two aspects: the considered distribution for the product lifetime and the used sampling plane for this type of test. For example Gui and Xu [2] proposed double acceptance sampling plane for truncated life time while the lifetime of product is assumed to has a half exponential power distribution. Gui and Aslam [3] applied single sampling plane for a product that has weighted exponential distribution. Aslam and Jun [4] studied the group sampling plan while they assumed that the lifetime has a Weibull distribution. Bhattacharya et al. [5] developed a model for optimum reliability acceptance sampling plane under cost constraint in the framework on hybrid censoring.

Al - Omari [6] applied single sampling plane for a truncated life test while the lifetime follows inverse Weibull distribution. Lio et al. [7] develop a sampling plan for the truncated life test when the lifetime follows Burr type XII percentiles. The minimum sample size to guarantee the specified consumer's risk is also presented. Aslam and Jun [8] applied the zero and one failure scheme of double sampling plan while the lifetime follows a generalized log-logistic distribution. Tsai and Wu [9] applied sampling plan in the contest of truncated life test while the lifetime is based on the generalized Rayleigh distribution and the mean life time is under consideration. In this paper we consider a product that its lifetime follows a Weibull distribution. It is proposed a sequential sampling plane for the truncated life test of this product.

2. The sequential sampling

Suppose that the lifetime of a product follows a Weibull distribution with the following cumulative density function (c.d.f):

$$F(t;\mathrm{v},\lambda) = 1-\exp\left\{-(\lambda\,\mathrm{t})^{v}\right\} \qquad (1)$$

Where λ and ν are the scale and shape parameter of Weibull distribution, respectively. The mean time of the Weibull distributed item is equal:

$$\mu = \frac{1}{\lambda}.\Gamma(1+\frac{1}{v}) \qquad (2)$$

Where $\Gamma(\bullet)$ is the value of gamma function. It is desired to design an accept sampling plane to assure that the mean lifetime of this product is at least μ_0. The lot is accepted if there is enough evidence that $\mu \geq \mu_0$ at the certain level of the confidence for the consumer's and producer's risks. For convenience's sake it is assumed that the termination time of the test ,t_0, for each item, is a multiple of the specified mean lifetime, μ_0. Thus $t = g\,\mu_0$ where g is a constant coefficient for the test termination time. For example, if we want to test whether the mean lifetime of the product is bigger than 2000 hours and set g=0.5 then the test of each item is terminated after passing 1000h from the start of the test. For the specified test termination time as t_0, the probability of failure for each item, P, or in the other word, the probability that the lifetime of the item is less than t_0 is equal to the c.d.f of Weibull distribution at t_0 as follows:

$$\begin{aligned} P &= 1-\exp\left\{-(\frac{t_0}{\lambda})^{v}\right\} \\ &= 1-\exp\left\{-(\frac{\mu}{\mu_0})^{-v}\,(g\,\Gamma(1+\frac{1}{v}))^{v}\right\} \end{aligned} \qquad (3)$$

The quality level of each item is denoted by r and is expressed based on the ratio of its mean time to the specified mean that it would be tested as: $r = \frac{\mu}{\mu_0}$. The producer's risk is the probability of rejection a good lot while the consumer's risk is the probability of acceptance a bad lot. The producer and consumer's risk are denoted as α and β, respectively. The producer wants the reject probability of the specified lot at the higher quality level, denoted by P_1, be smaller than α. On the other hand, the consumer wants the probability of accepting a specified lot at the lower quality level, denoted by P_2, be smaller than β. Let assume the ratio of $r = \frac{\mu}{\mu_0}$ at the higher quality level and lower quality level are r_1 and r_2 respectively ($r_1 > r_2$). For the specified values of shape parameter of Weibull distribution (ν), test termination coefficient (g) and quality level of each item (r), equation (3) determines the probability of item failure before t_0. Hence, P_1 and P_2 ,corresponding to r_1 and r_2 , can be obtained using equation (3).

In the sequential sampling the samples are selected from the lot stage to stage. In each stage the total number of inspected item and the total number of defective items to that stage is specified and based on them it is decided whether to continue sampling, cease it or take another sample. If the sample size at each stage be greater than 1, the sampling plane is called group sequential sampling and if in each stage sample size is 1, the sampling plan is called item-to-item sequential sampling. In this paper item – to - item sequential sampling is applied and in summary we call it sequential sampling. The procedure of sequential sampling is illustrated in figure1. In each stage the cumulative observed number of inspected item and defective item are plotted on the chart as one point. If this point falls on or below the acceptance line the lot is accepted. If this point falls on or above the rejection line the lot is rejected and if this points falls between these two lines another sample must be taken. This process must be continued till the plotted point does not fall between two boundaries line. Hence the stages of sampling are terminated as soon as the plotted points does not fall between these two boundaries lines.

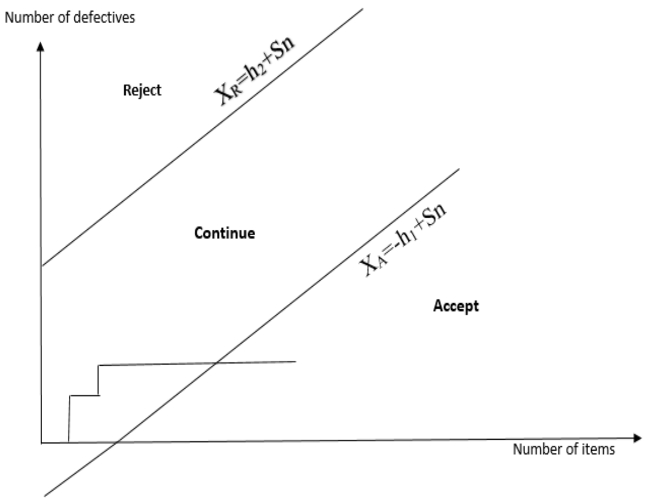

Figure 1. The procedure of sequential sampling

The equations of two boundary lines for the given values of α, P_2, β and P_1 are as follows:

$$X_A = -h_1 + sn$$
$$X_R = h_2 + sn \qquad (4)$$

While h_1, h_2 and s are computed as follows:

$$h_1 = (\log \frac{1-\alpha}{\beta}) / \mathrm{k} \qquad (5)$$

$$h_2 = (\log \frac{1-\beta}{\alpha}) / \mathrm{k} \qquad (6)$$

$$s = \log\left[(1-P_1)(1-P_2)\right] / k \qquad (7)$$

And k is obtained using the following equation

$$k = \log \frac{P_2(1-P_1)}{P_1(1-P_2)} \qquad (8)$$

3. Operating characteristic (OC) and average sample number (ASN)

Operating characteristic curve displays the probability of lot acceptance in each quality level. The lifetime of the product is assumed as its quality characteristic and it is expressed as the ratio of mean lifetime to the specified mean that it would be test ($r = \frac{\mu}{\mu_0}$). Increasing the value of r, increases the probability of lot acceptance. Hence, in the OC function the probability of lot acceptance should be determined respect to r. On the other hand, ASN provides another criteria about the sampling scheme and quality and reliability engineers are interested about it for a given sampling plane. In this section a procedure is presented for computing OC and ASN for the proposed sequential sampling plane.

The following formula is held for computing ASN in the sequential sampling:

$$ASN = \frac{P_a \log(\frac{\beta}{1-\alpha}) + (1-\mathrm{P_a})\log\frac{1-\beta}{\alpha}}{P\log\frac{P_2}{P_1} + (1-P)\log\frac{1-P_2}{1-P_1}} \qquad (9)$$

P and P_a are the proportion of nonconforming product and the probability of acceptance, respectively and can be obtained based on these two parametric equations:

$$P = \frac{1-(\frac{1-P_2}{1-P_1})}{(\frac{P_2}{P_1})^{\theta} - (\frac{1-P_2}{1-P_1})^{\theta}} \qquad (10)$$

$$P_a = \frac{(\frac{1-\beta}{\alpha})^{\theta} - 1}{(\frac{1-\beta}{\alpha})^{\theta} - (\frac{\beta}{1-\alpha})^{\theta}} \quad (11)$$

Hence, our procedure for computing ASN and P_a are as follows:

Procedure1.

1- Based on the equation 3, compute the value of P for the given value of r
2- By replacing the value of P in equation 10, compute the corresponding value of θ.
3- By inserting the value of θ in equation 11, compute P_a
4- Insert the values of P and P_a in equation 9, yields the corresponding value of ASN

The four steps of procedure1 is coded in MATLAB program and in step 2 it is used "vpasolve" function for computing θ.

4. An illustrative example

Suppose that the lifetime of a product follows a Weibull distribution with the shape parameter 2 value. It is desired to design a sequential sampling plane to assure that the mean lifetime of this product is greater than 1000 hours, while the experiment must be terminated after 1000h. The consumer wants the risk of accepting the lot that its true mean is 1000 be at most 0.25 and the producer wants the risk of rejection a lot that its true mean is 2000h be at most 0.05. Based on this information we have g=1, r_1=1, r_2=2, v=2, α=0.05, β=0.25. The first step for designing sequential sampling for this experiment is computing P_1 and P_2. Based on the equation (3) the values of P_1 and P_2 are 0.5441 and 0.1783. Based on the equations (8), (7), (6) and (5) the value of k, s, h_1 and h_2 are equal to: -1.7048, 0.5759, -0.7831, -1.5885. Inserting the values of k, s, h_1 and h_2 in equation (4) yields the following equation for acceptance and rejection lines:

$$\begin{aligned} X_A &= 0.7831 + 0.5759n \\ X_R &= -1.5885 + 0.5759n \end{aligned} \quad (12)$$

The result of the sequential sampling is presented in the table1. For example, consider the computation of rejection and acceptance number for n=35. Inserting n=35 in equation (12) yields:

$$\begin{aligned} X_A &= 0.7831 + 0.5759 \times 35 = 1.1629 \\ X_R &= -1.5885 + 0.5759n = 3.5345 \end{aligned}$$

Acceptance and rejection number must be integer, hence the value of X_A is rounded downward and the value of X_R is rounded upward. Finally, the rejection and acceptance numbers for n=35 are 0 and 4 respectively. Based on this result for n= 35 if the number of the defects observed is 2 or 3 then the sampling process must be continued. If the number of defects are 0 or 1 then accept the lot and if number of defects are greater than or equal 4, the lot should be rejected.

Table1. The result of sequential sampling for the illustrative example

n	AC	RC	n	AC	RC	n	AC	RC
1	a	b	17	0	3	33	1	4
2	a	2	18	0	3	34	1	4
3	a	2	19	0	3	35	1	4
4	a	2	20	0	3	36	1	4
5	a	2	21	0	3	37	1	4
6	a	2	22	0	3	38	1	4
7	a	2	23	0	3	39	1	4
8	a	3	24	0	3	40	1	4
9	a	3	25	0	3	41	1	4
10	a	3	26	0	4	42	1	4

11	a	3	27	0	4	43	1	4
12	a	3	28	0	4	44	1	5
13	a	3	29	0	4	45	1	5
14	a	3	30	0	4	46	1	5
15	0	3	31	0	4	47	1	5
16	0	3	32	0	4	48	1	5

In table 1 the means of the used notations are as follows: n:number of item inspected; AC: accepted number; RC: rejection number; a: means accept not possible; b: means reject not possible. Figure 2 shows the OC functions for the v=1 and v=4 in this example. As, it is expected, increasing r leads to increase in the OC function and increasing v form 1 to 4 has not significant effect on the values of the OC function.

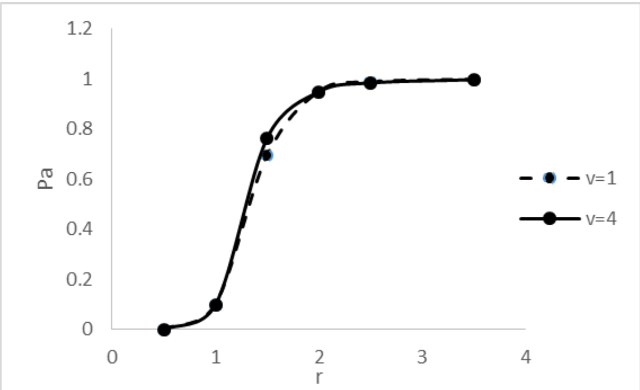

Figure 2: the OC function for the different values of the shape parameter of Weibull distribution

5. Conclusion

We consider the lifetime of a product as it is quality characteristic while this lifetime follows a Weibull distribution. Because the lifetime of the product usually is high, truncated life test is applied for evaluating and decision about the true mean lifetime. In this paper sequential sampling plane is proposed for decision about acceptance or rejection a lot based on the truncated life test. It is assumed that the termination time of the test ,t_0, for each item, is a coefficient of the specified mean lifetime , μ_0 . In the sequential sampling the samples are selected from the lot stage to stage. In each stage the total number of inspected item and the total number of defective items to that stage is specified and based on them it is decided whether to continue sampling, cease it or take another sample. Finally by illustrating an example the application of the sequential sampling is indicated for the truncated life test.

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