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A tabu search based solution approach to the competitive multiple allocation hub location problems

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Abstract

The hub location problems (HLP) constitute an important class of facility location problems that have attracted attention of the operations researchers in recent years. HLP is one of the strategic problems frequently encountered in designing logistics and transportation networks. This paper addresses the competitive multiple allocation HLP where the market is assumed to be a duopoly. It is assumed that an incumbent firm (the leader) is operating an existing hub network in a market and an entrant firm (the follower) tries to enter the market by configuring its own hub network trying to capture as much flow as possible from the leader. The customers choose one firm based on the service level (cost, time, distance, etc.) provided by these firms. We formulate the problem from the entrant firm's point of view and propose an efficient tabu search (TS) based solution algorithm to solve it. Computational experiments show the capability of the proposed solution algorithm to obtain the optimal solutions in short computational times.

Keywords:
Hub location, competitive models, mathematical formulation, tabu search.

1. Introduction

Hub networks play a major role in reducing cost and enhancing service level in many transportation, telecommunications and computer networks. These networks provide efficient service between many origins destination (O/D) pairs of nodes via a set of hubs that serve as switching and flow consolidation points. The hub location problem (HLP) deals with locating hub facilities and allocating non-hub nodes (spokes) to the installed hubs in order to route the traffic between O/D pairs [1].
Regarding the way the non-hub nodes are allocated to the hubs, there are two basic types of allocation in hub networks called single and multiple allocation. In a single allocation network, all the incoming and outgoing traffic to/from every demand node is routed through a single hub, whereas in a multiple allocation network each demand node can receive and send flow through more than one hub. An example of multiple allocation hub-and-spoke networks which is the underlying network topology in this paper is shown in Figure 1.

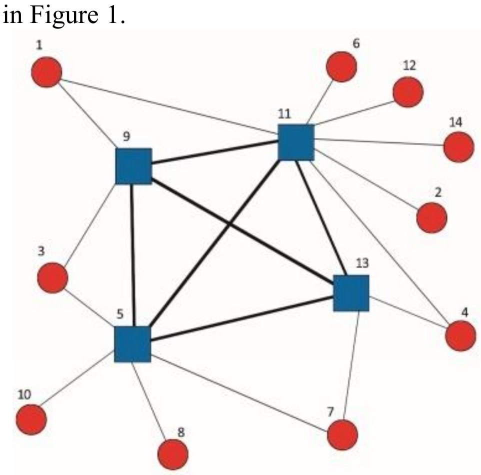

Figure 1- Examples of multiple allocation hub networks

Most of the studies in the literature of the HLP deal with situations in which the markets are monopolistic and try to model and solve the problems on behalf of a firm operating in such a market. However, in real world applications there may be competitors present in the market whose decisions would definitely affect the level of goal achievement for the other firms. In this paper, we address the competitive multiple allocation HLP where the market is assumed to be a duopoly. We assume that an incumbent firm (the leader) is operating an existing hub network in a market and an entrant firm (the follower) tries to enter the market by configuring its own hub network trying to capture as much

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flow as possible from the leader. The customers choose one firm based on the service level (cost, time, distance, etc.) provided by these firms. We consider the follower's problem which aims at optimally locating its hubs, based on the known decisions already made by leader. To this end, an MIP model is developed for the follower's problem and an efficient solution procedure based on the tabu search metaheuristic is proposed to solve the problem.

The remainder of this paper is organized as follows. The next section briefly reviews the relevant literature to the problem. In Section 3, we will present the mathematical formulation for the problem. The proposed tabu search based algorithm is presented in Section 4. Computational experiments and corresponding results using CAB and TR datasets are presented in section 5. Finally, some concluding remarks and directions for future works are given in section 6.

2. Literature review

HLP was first addressed by O'Kelly [2]. His paper presented the first mathematical formulation and solution method in the field of HLP. Later, O'Kelly [3] developed the first quadratic mathematical formulation of single allocation p-hub median problem. Campbell [4] proposed linear integer programming formulations for different versions of the HLP such as p-hub median problem, the uncapacitated hub location problem, p-hub center problem, and hub covering problem. For more details on HLP and recent advances in this field, the interested readers are referred to surveys [1], [5], and [6].

The hub location problem with competition was first addressed by Marianov et al. [7]. Given a set of existing hubs for the leader, they formulated the follower's problem trying to maximize its market share. Their model allows partial captures by the follower depending on the provided service levels. Wagner [8] tackled a similar problem with a different capture paradigm where the follower gets nothing in case of equal service levels for the same problem. In another competitive HLP, an entrant airline transportation firm was assumed to enter a competitive market [9]. It was also assumed that the customers choose an airline depending on a combination of factors such as flying time and travel fare based on gravity like utility functions. Lin and Lee [10] studied a competition game on hub network design and determined a hub network for each of all carriers in the oligopolistic market based on the long-term Cournot-Nash equilibrium steady state. Lüer-Villagra and Marianov [11] addressed a competitive HLP where location and pricing decisions were made by an entrant firm entering to a market where some other firm had already been operating. Customer preferences were modeled using logit function resulting in a nonlinear model maximizing the profit of the entrant firm. Another research conducted by Sasaki et al. [12] deals with a competitive hub arc location problem under Stackelberg competition. In their problem, rather than locating hub facilities, hub arcs are located in the network. To maximize its own revenue, the leader and the follower locate p and r hub arcs, respectively. They modeled the problem as a bilevel program.

In a very recent study, Mahmutogullari and Kara [13] considered a competitive multiple allocation HLP based on Stackelberg competition where the market was assumed to be a duopoly. Two firms decide locations of their hubs and then customers choose one firm with respect to provided service levels. They named the follower's problem as $(r|X_p)$ hub-medianoid and the leader's problem as $(r|p)$ hub-centroid problem. Both problems were formulated as MIP models and exact solution algorithms based on enumeration are proposed for solving them.

Although integer programming optimization approaches are used to solve small hub problems, larger instances of the HLP are usually solved by heuristic or metaheuristic procedures. In fact, development of metaheuristic algorithms has helped many real world applications, in which optimal or near-optimal solutions can be obtained in less computational time.

So far, many researchers has addressed different variants of the HLP using metaheuristic algorithms. A tabu search (TS) heuristic is proposed for the for uncapacitated single allocation p-hub median problem (USApHMP) in [6]. Ernst and Krishnamoorthy [14] developed a simulated annealing (SA) heuristic for the same problem and showed that it is comparable, in both solution quality and computational time, to the TS heuristic in [15]. Silva and Cunha [16] proposed three variants of a simple and efficient multi-start TS heuristic as well as a two-stage integrated TS heuristic to solve the uncapacitated single allocation HLP.

In this paper, we consider the competitive hub location problem stated in [13] from the follower perspective. A mathematical programming formulations is proposed for the follower and efficient tabu search based solution heuristic is developed for solving the problem.

3. Mathematical Formulation

Assume that $G = (N , E)$ is a graph in which N is the set of nodes and E is the set of edges ($E \subseteq N \times N$). Let $H \subseteq N$ be a subset of nodes that are candidate for opening hubs. Also, for all $i, j \in N$, let w_{ij} and c_{ij} denote respectively the amount of flow originated at node i and destined to node j, and the transportation cost of a unit flow from node i to node j. Transportation costs on inter-hub connections are discounted by a constant factor α ($0 \leq \alpha \leq 1$) to reflect the scale economies on connections between hubs. It is assumed that the leader is operating its network of p hubs and the follower wants to enter the market. The number of hubs to be located by the follower is r. Customers are captured by the leader or follower based on the respective provided service levels. Service level is defined as the cost of routing a unit flow on the route from its origin to its destination. A customer prefers the follower if the service level provided by the follower is strictly better than that of the leader, otherwise the demand is captured by the leader. In case of equal service levels, ties are broken in favor of the leader as the customer has no incentive to change the current position.

Let us assume that the leader has already opened its hubs at

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a subset of nodes $X_p = \{x_1, x_2,\ldots, x_p\}$, $X_p \subseteq H$ and is serving the market with these hubs. For every node pair i and j, the service level provided by the leader, denoted by β_{ij} , can easily be calculated as [13]:

$$\beta_{ij} = \min_{k,m\in X_p} \{ c_{ik} + \alpha c_{km} + c_{mj} \} \quad \forall i,j \in N. \tag{1}$$

Assume now that the follower enters the market and establishes its hubs on a subset of nodes $Y_p = \{y_1, y_2,\ldots, y_r\}$, $Y_r \subseteq H$. In a similar manner, the follower's service levels, denoted by γ_{ij} , for all node pairs i and j can be calculated as [13]:

$$\gamma_{ij} = \min_{k,m\in Y_r} \{ c_{ik} + \alpha c_{km} + c_{mj} \} \quad \forall i,j \in N. \tag{2}$$

For all $i, j \in N$, the follower captures the flow w_{ij} if $\gamma_{ij} < \beta_{ij}$. Therefore, total flow captured by the follower can be expressed by a mapping $f : P_p(H) \times P_r(H) \rightarrow [0,W]$ such that:

$$f(X_p,Y_r) = \sum_{i,j\in N:\gamma_{ij}<\beta_{ij}} w_{ij} \tag{3}$$

where $P_p(H)$ is the set of all subsets of cardinality p from H and W is the sum of flows over the network:

$$W = \sum_{i,j\in N} w_{ij}. \tag{4}$$

Given the leader's hubs located on X_p, the follower's problem is to locate a set of r hubs that maximizes the captured demand by him/her. To model this problem, assume that q_{ijkm} is a binary covering parameter that takes the value of 1 if the flow between nodes i and j is captured by the follower and 0, otherwise. In other words, with β_{ij} defined by (1) for a fixed X_p:

$$q_{ijkm} = \begin{cases} 1, & \text{if } c_{ik} + \alpha c_{km} + c_{mj} < \beta_{ij} \\ 0, & \text{otherwise} \end{cases} \quad \forall i,j \in N, \forall k,m \in H \tag{5}$$

Let the variable z_{ijkm} denote the fraction of flow w_{ij} that is sent from node i to node j using the link between the hubs k and m by the follower. Let also the binary variable $y_k \in \{0, 1\}$ be 1 if node k is selected by the follower as a hub and 0, otherwise. The problem consists of the selection of nodes which will act as the follower's hubs and determining how the non-hub nodes will be allocated to the hub nodes and the flows will be routed in the network so that total captured flow by the follower is maximized. The MIP model for the follower's problem can now be written as:

$$\max \sum_{i\in N}\sum_{j\in N}\sum_{k\in H}\sum_{m\in H} w_{ij} q_{ijkm} z_{ijkm} \tag{6}$$

s.t.

$$\sum_{k\in H} y_k = r \tag{7}$$

$$\sum_{k\in N}\sum_{m\in H} z_{ijkm} = 1 \quad \forall i,j \in N \tag{8}$$

$$\sum_{m\in H} z_{ijkm} + \sum_{m\in H|m\neq k} z_{ijmk} \leq y_k \quad \forall i,j \in N,\ k \in H \tag{9}$$

$$z_{ijkm} \geq 0 \quad \forall i,j \in N,\ k,\ m \in H \tag{10}$$

$$y_k \in \{0,1\} \quad \forall k \in H \tag{11}$$

The objective function (6) maximizes the total flow captured by the follower. Constraint (7) determines the number of hubs to be located by the follower. Constraints (8) assure that the whole flow associated with each O/D pair is routed via some hub pair. Constraints (9) state that the flows can only be routed via nodes that have been designated as hubs. (10) and (11) are positive and binary constraints, respectively.

4. Metaheuristic Solution Algorithm

The tabu search algorithm proposed by Glover [17], is a local search algorithm which has effectively tackled a variety of hard real-world optimization problems. This procedure starts with an initial solution and uses a tabu list to control moves in the neighborhood so that trapping in local optima and re-visiting the same solution will not occur. From the current solution, all the non-tabu moves are explored and the best one is selected. This move which might lead to an either better or worse solution than the current solution, is recorded in the tabu list. The future move is among those not listed in the tabu list, unless it fulfills aspiration level. TS would be terminated whenever a termination criterion such as a maximum number of iterations, a fixed number of iterations with no improvement in the best solution, etc. is met.

We use a one-dimensional array to represent the solutions in the multiple allocation HLP. This array of size p includes the numbers associated with the nodes that are selected as hubs. The sorting of numbers within the arrays is not important. Note that having known the selected hubs, for each O/D pair i - j, one can easily determine the paths for routing the associated flow w_{ij} by solving a shortest path problem. The initial solution is generated randomly by selecting p out of n nodes as hub nodes.

We define and use two operators for generating neighboring solutions in our algorithm. The first operator is used to alter one of the hubs in the solutions. First, we randomly select a hub node and a non-hub node. Then if the selected non hub node is not in the tabu list, the selected hub node becomes a non-hub node and the selected non-hub node becomes a hub. In contrast, if the selected non hub node exist in the tabu list, TS algorithm should use the second operator. This operator is used if the randomly selected non-hub is in the tabu list but improve the best solution.

5. Numerical Experiments

In this section we present the results obtained from our computational experiments. Extensive experimentations are conducted in order to test the efficiency of the proposed mathematical models as well as the TS algorithms. To this end, we use two famous data sets from the literature of HLP: CAB and TR data sets. The CAB data set introduced by O'Kelly [3] is based on the airline passenger interactions between 25 US cities in 1970 evaluated by the Civil Aeronautics Board (CAB). This data set has been used by most of the hub location researchers in the literature in

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which all the 25 nodes are candidates for being hubs (|H| = 25). The second data set that is used in our computational experiments is the TR data set [18] which is based on the cargo flows between 81 cities of Turkey where only 22 of these cities are candidate nodes for locating hubs (|H| = 22). The proposed TS algorithms are implemented in Microsoft Visual C# 2013 (version 5.0). Also, the mathematical model for the follower problem is solved independently using CPLEX version 12.6. All the experiments have been run on a computer with Intel(R) Core(TM) i7-4500U CPU of 1.8 GHz and 4 GB of RAM, using the Microsoft Windows 8 operating system.

In an initial set of experiments, different combinations of tabu list size were tested on a large number of test instances and the best values are 10 and 7 for CAB and TR data set, respectively, which lead to high-quality solutions in short CPU times. We have also set the termination criterion for our algorithm as 15 iterations with no improvement in the quality of best solution. We have generated and used 32 instances from the CAB data set and the 75 instances from the TR data set. The number of hubs to be opened by the leader (p) and the follower (r) for CAB data set is taken as 2, 3, 4, or 5, whereas in TR data set, p and r are chosen as 6, 8, 10, 12, or 14. The parameter α is considered at two levels: 0.6, and 0.8 for CAB data set and at three levels: 0.6, 0.8, and 0.9 for TR data set as in [13].

Table 1 and 2 show the results obtained by solving the problem using the proposed TS algorithm as well as CPLEX based on the proposed mathematical models with the CAB data set for different discount factors (α). Since the distance matrix in both the CAB and TR data sets are symmetric, it can be concluded that if the flow w_{ij} from node $i \in N$ to node $j \in N$ is captured by the follower, the flow from node j to node i is also captured by the follower. Therefore, to reduce the size of our model, the constraints (7) - (9) are imposed for only $i < j$ and the objective (5) is modified as:

$$\sum_i \sum_{j|j>i} \sum_k \sum_m (w_{ij} + w_{ji}) q_{ijkm} z_{ijkm}$$

in our computational studies.

To set the location of hubs in the incumbent leader's network, we assume that the leader has already located its hubs based on the uncapacitated multiple allocation p-hub median problem (UMApHMP). Therefore, before solving the follower's problem, we solve the leader's problem based on the UMApHMP and obtain the optimal location for its hubs which are subsequently used in our numerical experimentations.

The columns entitled p and r denote the number of hubs which are opened by the leader and the follower, respectively. The next two columns show the follower's capture as the optimal objective function value that has been obtained by CPLEX and the CPU time, in seconds, needed to reach that solution. The columns under the label "TS" give the best objective function obtained through solving the instances with the TS algorithm and the average CPU time for three runs of the TS algorithm. Finally, the column entitled as "GAP (%)" show the gap percentage between the objective function values obtained by CPLEX and the proposed TS algorithm.

Table 1 - Results for the CAB data set with α = 0.6

p	***r***	**CPLEX**		**TS**		**Gap (%)**
		Follower's capture	**CPU (s)**	**Follower's capture**	**CPU (s)**	
2	2	65.62%	14.74	65.62%	0.02	0.00
	3	78.25%	17.56	78.25%	0.06	0.00
	4	87.08%	17.83	87.08%	0.07	0.00
	5	92.38%	16.05	92.38%	0.10	0.00
3	2	30.49%	26.60	30.49%	0.06	0.00
	3	45.13%	24.56	45.13%	0.08	0.00
	4	53.69%	22.13	53.69%	0.12	0.00
	5	62.02%	23.93	62.02%	0.15	0.00
4	2	18.89%	27.72	18.89%	0.06	0.00
	3	28.39%	30.68	28.39%	0.09	0.00
	4	37.73%	33.16	37.73%	0.20	0.00
	5	46.18%	25.76	46.18%	0.22	0.00
5	2	18.64%	27.97	18.64%	0.08	0.00
	3	28.14%	23.62	28.14%	0.09	0.00
	4	35.04%	19.22	35.04%	0.19	0.00
	5	42.32%	22.77	42.32%	0.16	0.00
Avg.		48.12%	23.39	48.12%	0.10	0.00

Table 2 - Results for the CAB data set with α = 0.8

p	***r***	**CPLEX**		**TS**		**Gap (%)**
		Follower's capture	**CPU (s)**	**Follower's capture**	**CPU (s)**	
2	2	65.84%	8.82	65.84%	0.03	0.00
	3	74.19%	22.03	74.19%	0.05	0.00
	4	80.69%	25.10	80.69%	0.10	0.00
	5	87.14%	18.35	87.14%	0.11	0.00
3	2	29.18%	24.99	29.18%	0.05	0.00
	3	42.92%	22.58	42.92%	0.06	0.00
	4	52.83%	20.57	52.83%	0.11	0.00
	5	60.14%	22.47	60.14%	0.13	0.00
4	2	21.06%	24.48	21.06%	0.05	0.00
	3	32.69%	21.92	32.69%	0.06	0.00
	4	42.10%	23.47	42.10%	0.13	0.00
	5	48.60%	25.25	48.60%	0.14	0.00
5	2	18.19%	23.49	18.19%	0.08	0.00
	3	29.12%	20.11	29.12%	0.14	0.00
	4	36.93%	24.00	36.93%	0.11	0.00
	5	44.32%	25.48	44.32%	0.13	0.00
Avg.		47.87%	22.07	47.87%	0.09	0.00

Observe that the proposed TS algorithm solves all instances to optimality within fraction of a second. This is an indication of the efficiency of the proposed TS algorithm. Also, the solution times for CPLEX using the proposed mathematical models are also acceptable for the CAB data set. Note that since the leader decides on the location of its hubs based on cost minimization criterion as in UMApHMP which does not take into account the upcoming competition, the follower can capture a considerable share of market upon entrance to market. For instance, when the follower locates the same number of hubs as the leader's,

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i.e., $p = r$, its captured market share is larger than that of the leader. For the cases where $p \leq r$, the lost market share by the leader gets even larger. However, as p increases (p = 4 or 5), the follower's capture is not as much as that of the leader.

Tables 3, 4, and 5 show the results obtained by solving the problem with TR data set for different values of α. Here also it is assumed that the leader has already selected its p hubs based on UMApHMP. To evaluate the performance of the proposed TS algorithm on the TR data set, for all instances have been solved to optimality by Mahmutogullari and Kara [13], for these instances the results obtained by the TS are compared to their corresponding optimal values which are reported under the column labeled as “M&K”.

Table 3- Results for the TR data set with α = 0.6

p	*r*	M&K	TS		Gap (%)
		Follower's capture	**Follower's capture**	**CPU (s)**	
6	6	39.31%	39.31%	2.43	0.00
	8	49.19%	49.19%	3.59	0.00
	10	56.94%	56.94%	3.36	0.00
	12	64.02%	64.02%	4.76	0.00
	14	68.91%	68.91%	4.50	0.00
8	6	28.58%	28.58%	3.27	0.00
	8	37.09%	37.09%	5.17	0.00
	10	44.37%	44.37%	4.33	0.00
	12	51.77%	51.77%	3.12	0.00
	14	57.97%	57.97%	2.51	0.00
10	6	19.91%	19.91%	3.87	0.00
	8	27.13%	27.13%	2.57	0.00
	10	34.10%	34.10%	3.49	0.00
	12	40.48%	40.48%	3.14	0.00
	14	45.73%	45.73%	4.15	0.00
12	6	15.83%	15.83%	3.92	0.00
	8	21.79%	21.79%	4.39	0.00
	10	27.06%	27.06%	5.44	0.00
	12	31.37%	31.37%	5.32	0.00
	14	35.48%	35.48%	3.42	0.00
14	6	13.04%	13.04%	7.79	0.00
	8	17.87%	17.87%	4.89	0.00
	10	22.25%	22.25%	6.84	0.00
	12	26.00%	26.00%	4.60	0.00
	14	28.42%	28.42%	5.07	0.00
Avg.		36.18%	36.18%	4.23	0.00

Table 4 - Results for the TR data set with α = 0.8

p	*r*	M&K	TS		Gap (%)
		Follower's capture	**Follower's capture**	**CPU (s)**	
6	6	37.97%	37.97%	3.16	0.00
	8	48.24%	48.24%	3.08	0.00
	10	55.70%	55.70%	2.59	0.00
	12	61.84%	61.84%	4.15	0.00
	14	67.97%	67.97%	4.31	0.00
8	6	29.37%	29.37%	3.41	0.00
	8	37.08%	37.08%	4.82	0.00
	10	44.35%	44.35%	4.30	0.00
	12	50.71%	50.71%	3.40	0.00
	14	56.33%	56.33%	3.17	0.00
10	6	20.12%	20.12%	3.87	0.00
	8	27.03%	27.03%	2.57	0.00
	10	33.84%	33.84%	3.49	0.00
	12	40.74%	40.74%	3.14	0.00
	14	46.84%	46.84%	4.15	0.00
12	6	16.93%	16.93%	3.92	0.00
	8	23.41%	23.41%	4.39	0.00
	10	28.62%	28.62%	5.44	0.00
	12	32.81%	32.81%	5.32	0.00
	14	35.85%	35.85%	3.42	0.00
14	6	13.02%	13.02%	7.79	0.00
	8	18.57%	18.57%	4.89	0.00
	10	22.52%	22.52%	6.84	0.00
	12	25.20%	25.20%	4.60	0.00
	14	27.40%	27.40%	5.07	0.00
Avg.		36.10%	36.10%	4.21	0.00

Table 5 - Results for the TR data set with α = 0.9

p	*r*	M&K	TS		Gap (%)
		Follower's capture	**Follower's capture**	**CPU (s)**	
6	6	40.86%	40.86%	3.89	0.00
	8	49.44%	49.44%	2.75	0.00
	10	56.06%	56.06%	3.15	0.00
	12	61.54%	61.54%	4.10	0.00
	14	66.45%	66.45%	5.10	0.00
8	6	31.11%	31.11%	3.41	0.00
	8	38.69%	38.69%	5.04	0.00
	10	44.83%	44.83%	4.87	0.00
	12	50.49%	50.49%	4.13	0.00
	14	55.77%	55.77%	2.98	0.00
10	6	20.74%	20.74%	3.75	0.00
	8	27.77%	27.77%	2.41	0.00
	10	33.86%	33.86%	4.19	0.00
	12	39.89%	39.89%	3.39	0.00
	14	44.90%	44.90%	4.72	0.00
12	6	18.45%	18.45%	4.05	0.00
	8	24.59%	24.59%	3.98	0.00
	10	29.08%	29.08%	6.15	0.00
	12	32.98%	32.98%	5.09	0.00
	14	36.18%	36.18%	3.86	0.00
14	6	13.66%	13.66%	6.74	0.00
	8	18.81%	18.81%	5.15	0.00
	10	22.50%	22.50%	6.85	0.00
	12	25.60%	25.60%	5.16	0.00
	14	28.18%	28.18%	4.75	0.00
Avg.		36.50%	36.50%	4.38	0.00

The results reported in the above tables reveal that the proposed TS algorithm is able to obtain the optimal solutions for all the instances in the TR data set. From a solution time perspective, it is shown that the TS solves the problem instances for the TR data set in quite short CPU

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times. For TR data set in case of equal number of hubs, that is p=r, the leader captures more than half of the market. The follower should open at least two more hubs to defeat the leader. Moreover, since the same discount factor applies for both firms, there is no important correlation between market shares and α value. Another important observation form the above tables is that as the number of hubs opened by the leader (*p*) increases, the follower fails to capture much of the market share even if $r > p$. One possible reason for this observation can be the fact that as *p* increases, the leader selects more of the critical locations for opening hubs and increases his/her provided service level. In addition, since the customers choose the leader's service for equal service levels offered by the leader and the follower, the leader's market share stays higher than that of the follower.

6. Conclusion

In this research the competitive multiple allocation HLP in a duopoly market is considered. It is assumed that an incumbent firm (the leader) is operating an existing hub network in a market and an entrant firm (the follower) tries to enter the market by configuring its own hub network trying to capture as much flow as possible from the leader. The customers choose one firm based on the service level (cost, time, distance, etc.) provided by these firms. Therefore, the follower aims at locating its hubs in such a way that the total captured flow (market share) is maximized. We proposed a mathematical formulation the follower's problem and an efficient tabu search (TS) based heuristics is proposed for solving the problem. Extensive computational experiments based on two data sets of the CAB and TR are conducted to analyze different properties of these problems and to evaluate the performance of the proposed TS algorithm as well as the mathematical model. In all the instances for the two data sets, the proposed TS algorithms obtained the optimal solutions. Furthermore, the computational results show the efficiency of the proposed algorithms in terms of CPU times. Further research can consider more than two players (decision makers) in competitive hub location problem which can make the problem fit better to more complex real life situations.

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