

## Reliable Location-Allocation Problem with Correlated Geographical Failure Probability

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### Abstract

*In this study, we have presented a mixed integer programming model for the Reliable location-allocation problem. The model determines the located distribution centers and allocated customers to minimize total network cost. Furthermore, we used to improve the reliability of the network using correlated geographical failure probability. The correlated failure probability of the arcs between the distribution centers is estimated using a spatial statistic model. Finally, we solve the model using CPLEX solver version 12.5. A distribution company in the North-Khorasan province is used as an example for this model. The numerical results have shown improving the failure probability can reduce the total cost of the supply chain network.*

### Keywords:

Supply Chain, Reliability, Failure probability, Location-Allocation.

### Introduction

A supply chain is a set of activities, corresponding flow and production of goods from raw material to end users. A supply chain contains of different levels such as supplier, manufacture, distributor, etc. To be successful in supply chain, it needs coordinated actions. However, many factors are effective in a successful supply chain. One of a key factor is reliability.

Reliability is applied for explanation degree of accurate operation of a part or generally a set of factors in a time period. In fact, reliability is success probability of an operation without disrepair at specified tasks under given conditions such as temperature, humidity, etc.

The issue of reliability value has always been a critical challenge for researches. To measure system reliability, beginning system is decomposed to components. Therefore,

calculating the reliability of the supply chain is in terms of reliability achieved from different levels of chain. As mentioned above, a supply chain can have several different levels. However, we consider the supply chain with two levels such as producer and customer or distributor and retailer. The problem of locating reliable bi-level supply chain called as Reliable Location-Allocation Problem (RLAP). Location-allocation determines the best location for one or more facilities that will service from a given set of points and then assigning those points to the facilities. Different studies exist in the literature as follows. Most research on reliability in the supply chain has been carried out in issues that reliability calculation with regard to the concept of reliability theory. Similarly, Chen et al. [1] used tree error analysis to determine the key factors of supply chain failure, and employed the Monte Carlo simulation method to calculate the reliability. Andronov and Jurkina [2] are considered a supply chain with random environment. Moreover, the possibility of failure in the supply chain has been estimated by using continuous time Markov chains. Gillespie [3] proposed an Application program maintenance and reliable for logistics and supply chain. A general method has offered to optimize the supply chain with reliable data. Gołda [4] proposed a reliable method for the control and management of supply chain logistics processes. As another example, Soni and Kodali [5] considered the lack of standard constructs in frameworks of lean, agile and lean-agile supply chain. The goal is achieved by evaluating reliability of lean, agile and lean-agile supply chain in an Indian manufacturing industry. Also, Bauer et al. [6] embedded active and passive components in an interconnected substrate, which improved performance by cutting interconnected parasitic, achievements reliability through eliminating wire-bonds and solder bumps.

A broader perspective of this issue is studied by Artsiomchyk and Zhivitskaya [7] who offer models to improve supply chain operation by designing robust and reliable system. System reliability is estimates the analysis

reliability of system components. Furthermore, Ogunwolu et al. [8] suggested a multi-criteria model of a three-stage supply chain network. This model is very sensitive to changes in demand and reliability. The purpose of the model is to minimize the total cost of the supply chain. Miguel et al. [9] purpose of this study, the ability of a logistics service supply chain coordination with regard to their reliability. The expected profit was made under uncertainty in demand. The optimal order quantity logistics capability and the expected profit can be calculated when the system is supply chain logistics services coordination. Moreover, Ghayebloo et al. [10] a bi-objective mixed integer programming model is developed and a logistics network for forward / reverse that three ranks in the forward direction (suppliers, assembly centers and customer locations) and two in reverse (disassembly and recycling center) said, In this paper reliability for archers for each part of the supply chain is considered centers.

A great deal of previous research into reliability in the supply chain has focused on Calculated according to the theory of reliability while Can with regard to the concept of failure probability estimate reliability. Up to now, several studies have found in this field, as Kamalahmadi and Mellat-Parast [11] pointed out that the optimal allocation of demand across a range of suppliers in the supply chain that is exposed to supply risk and environmental risk. As another example, Shaochuan and Qingming [12] offered a supply chain model with reliability constraint based on the reliability theory. Moreover, Gao and Gao [13] provided a reliability evaluation model in order to ensure the normal operation of public equipment supply chain's. In addition, Pasandideh et al. [14] proposed a bi-objective optimization of three-level and multi-period for multi-product supply chain network including manufacturing plants, distribution centers with uncertain service, and customers. Their model minimized the total cost, while maximizing the average number of products sent to customers. Furthermore, Jia and Cui [15] of pointed out that Suppliers in the supply chain are not independent from one another and the relationship of dependence may be linear or non-linear correlation. Hatefi and Jolai also [16] suggested a robust and reliable model for closed-loop integrated logistics network design, which simultaneously used uncertain parameters and allowed disruption facilities. Later, Yildiz [17] presented a bi-objective mixed integer programming model for supply chain network design to minimize cost and maximize reliability. Furthermore, Snyder and Daskin [18] proposed a basic model that have been used in many recent models. In their model, the centers failure probability are considered independently. In addition, Poudel et al. [19] provided a pre-disaster planning model that seeks to strengthen the link given several facilities in biofuel supply chain system, while is limited the budget available.

In this study, we consider the failure correlated failure probability according to geographical characteristics. In fact, the spatial correlation is used between environment

variables based on the spatial distance between them. This means that the relationship between the amounts depends on the distance or proximity of points. Therefore, increasing geographical correlation of the distance between the observations less and more distance greater spatial correlation value is close to zero. To cope this problem, we offer a mixed integer programming model for RLAP. Then, we solve the model using CPLEX solver version 12.5.

The overall structure of this paper is as follows. Section 2 formulates the mathematical model; Section 3 discusses the method of predicting the failure probability of arcs; Section 4 presents numerical results; and finally, Section 5 provides conclusions and future research directions.

## Problem description and model formulation

This paper builds a reliable model that aids the design of RLAP by considering the correlated geographical failure probability. The general structure of the proposed supply chain network is illustrated in Fig. 1. This model seeks to increase reliability by strengthen the arcs between facilities. The method uses scenarios to represent uncertain events proposed with the objective of minimizing transport costs.

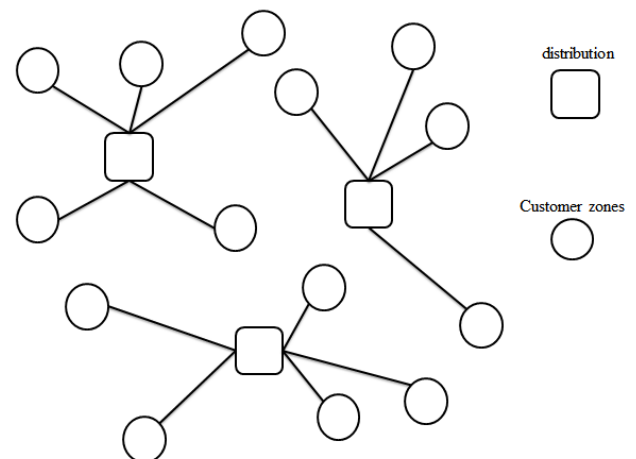


Figure 1 –bi-echelon supply chain network.

### Sets

- $I$ : Sets of customer zones;
- $J$ : Sets of candidate distribution centers;
- $S$ : Sets of scenario;

### Parameters

- $F_j$  : Fixed cost of opening a center at location  $j$ ;
- $h_i$  : Demand of customer zone  $i$ ;
- $q_{ijs}$  : Probability of failing arc  $(i, j)$  under Scenario  $S$ ;
- $d_{ij}$  : Unit transportation cost from a center  $j$  to customer  $i$ .

**Decision variables**

$$X_j = \begin{cases} 1 & \text{if distribution center } j \text{ is open} \\ 0 & \text{otherwise} \end{cases}$$

$y_{ijs}$  : The fractional demand of customer  $i$  that is assigned to facility  $j$  in scenario  $s$ .

**Formulation**

$$\min \sum_j f_j x_j + \sum_i \sum_j \sum_s q_{ijs} d_{ij} y_{ijs} h_i \quad (1)$$

*s.t.*

$$\sum_j y_{ijs} = 1 \quad \forall i, s \quad (2)$$

$$y_{ijs} \leq x_j \quad \forall i, j, s \quad (3)$$

$$y_{ijs} \geq 0 \quad \forall i, j, s \quad (4)$$

$$x_j \in \{0,1\} \quad \forall j \quad (5)$$

The objective function (1) minimizes the fixed cost plus the expected transportation cost across all scenarios. Constraints (2) show each customer is allocated to some facilities in each scenario. Constraints (3) indicate a customer being assigned to a facility that has been opened. Constraints (4) demonstrate assignment variables to be non-negative, and Constraints (5) ensure the location variables to be binary.

**Method of predicting the correlated failure probability of arcs**

We estimate the geographical failure probability based on database that consists of weather conditions of the region. Correlated geographical failure probability approach may be valid, when the facilities are distant from each other. When a disaster, flood for instance, occurs in a specific area, the infrastructure close to this area is more likely to be affected. In the other words, the probability that distant infrastructure is affected by this flood is much smaller [19].

In this paper, each region is divided into multiple sections, and the failure probability of each section is estimated. The failure probability of each route between two centers is different among the path due to different probability of failure sections. To address this challenge, each route is segmented into multiple exclusive sections, and the failure probability at each section of the route is considered.

This paper, estimate for arc failure probability by using the maximum failure probability of sections within each arc.

Assume that arc  $(i, j)$  can be divided into  $n_{ij}$  sections,

which has a failure probability of  $q_l$  for  $q_l = 1, \dots, n_{ij}$

.Thus, link failure probability  $q_{ij}$  can be estimated as

$$q_{ij} = \max q_l .$$

In what follows, we describe spatial statistic model to estimate the correlated failure probability of any part of arc  $q_l$ .

**Spatial model of arc failure probability**

We use a continuous spatial statistics model to characterize the geographical correlation. The model is described below:

$$q_s = \mu + e(s) \quad (6)$$

where

- $S$  represents the geographical coordinates of a place of interest such as latitude and longitude coordinates of a segment.
- $\mu$  is an unknown parameter to be estimated from the data, indicates the average probability of failure.
- $e(s)$  shows the variability in the failure probability and accounts for the geographical dependence in failure probability. In other words, the same discord is likely to fail with geographical correlation.

We assume that Dispersion in the process is a spatial  $e(s)$  probability of failure can be an indicator of the distribution called as “semi-variogram”. The semi-variogram function defined as follows:

$$\gamma(h) = \frac{1}{2|N(h)|} \sum_{N(h)} \{q(s_i) - q(s_j)\}^2 \quad (7)$$

Given the observed historical data of failure probability,  $q(s_1), \dots, q(s_n)$ , where  $N(h)$  is the set of location pairs  $(s_i, s_j)$  with a coordinate difference  $h$ , and  $|N(h)|$  is the number of distinct pairs in this set.

**Predicting the correlated failure probability**

Once the semi-variogram model of  $\mathcal{Y}$  is established, the resulting model, by means of Kriging is used. Since, the mean value  $\mu$  is unknown; the Ordinary Kriging can be used.

Let  $\mathbf{q} = q(s_1), \dots, q(s_n)$  be the set of available historical data set of failure probability. The Ordinary Kriging estimator for a location  $s_0$  is a linear predictor in the form of  $q_{ok}(s_0) = \lambda \mathbf{q}$ , where  $\lambda$  denotes the unknown weight coefficients to be estimated. To this end, we choose weight coefficients  $\lambda = (\lambda_1, \dots, \lambda_n)$  that minimize the loss function shown below:





$$L = E[(\lambda \mathbf{q} - q(s_0))^2] \quad (8)$$

subject to the weight constraint  $\sum_{i=1}^n \lambda_i = 1$ .

This problem can be defined with respect to the Lagrange multipliers  $\theta$  as follows, and function (9) instead function (8) is minimized.

$$\arg \min_{\lambda, \theta} L = \arg \min_{\lambda, \theta} \left\{ E[(\lambda \mathbf{q} - q(s_0))^2] - 2\theta \left( \sum_{i=1}^n \lambda_i - 1 \right) \right\} \quad (9)$$

The parameter  $\theta$  is a Lagrange multiplier that ensures  $\sum_{i=1}^n \lambda_i = 1$ . Let  $\Gamma = [\gamma(s_i - s_j)]$  be the  $n \times n$  semi-variogram matrix of the existing data points and  $\gamma(s_0) = [\gamma(s_0 - s_1), \dots, \gamma(s_0 - s_n)]$  be the semi-variogram vector between  $s_0$  and  $\{s_1, \dots, s_n\}$ . The resulting Ordinary Kriging is expressed in terms of the semi-variogram  $\gamma$ , and it is as follows:

$$q_{ok}(s_0) = \lambda \mathbf{q}(s) \quad (10)$$

where

$$\lambda = \left( \gamma(s_0) + \mathbf{I} \frac{1 - \mathbf{I} \Gamma^{-1} \gamma(s_0)}{\mathbf{I} \Gamma^{-1} \mathbf{I}} \right) \Gamma^{-1} \quad (11)$$

## Numerical results

In this section, a numerical example is presented in order to demonstrate the applicability of the presented model. Consider a bi-echelon supply chain network similar to the one depicted in Figure 1. In this example, spatial statistics method has been used to obtain the failure probability of arcs. Then, we solved the model using CPLEX solver version 12.5. The application of the RLAP is satisfied the demand of a distribution company in North-Khorasan province in Iran. North-Khorasan area is 28.434 km<sup>2</sup> that consists of 26 towns (nodes) depicted in Figure 2.

### Data description

There is a set of six distributions (shown by rectangles) that satisfy their demands and customers (shown by circles) described in Figure 3. The transportation costs are defined corresponding to the distance between nodes in supply chain network. Also, the failure probability of each section is uncertain parameters that for the four scenarios (seasons) values is obtained.

Weather conditions of each section have a significant impact on transport. In Iran, the number of accidents increases in rainfall by 75 percent. More than 60 percent of all accidents in the months of rainfall has been recorded that 40 percent of accidents happen in the winter, because

the road surface is covered by ice or snow or is wet<sup>1</sup>. For this reason, in this example, every season considers a scenario and we estimated the failure probability for each section. Our area of North-Khorasan province was divided into 36 sections that shown in Figure 2. This division is based on the increasing distance from left to right and down to up. This division is based on geographic coordinates. In Figure 2 show the coordinates of longitude and latitude of each section.

To model the spatial distribution of failure probability and predict it in locations, where historical data is unavailable, we first estimate the semi-variogram of failure probability  $\mathbf{q}(s)$  based on expression (7). The resulting empirical semi-variogram is demonstrated in Figure 4. In this plot, the vertical axis represents the calculated  $\gamma$  and the horizontal axis represents the distance between any pair of locations  $(s_i - s_j)$ . Once the semi-variogram model is established, we use the Ordinary Kriging method, as shown in Equation (10), to interpolate failure probability of the arcs.

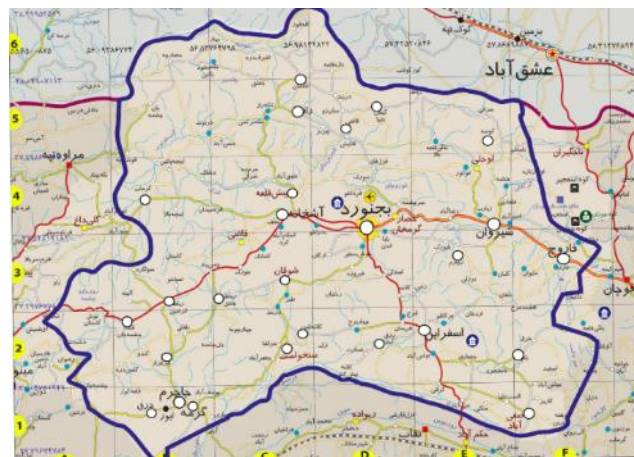


Figure 2 - Geographical map of North-Khorasan province



Figure 3 -Candidate location in distribution centers

<sup>1</sup> www.ebtekarnews.com



**Results and discussion**

The results obtained from the data in the CPLEX solver version 12.5 are shown in Figure 5. In solution of model two cities were selected for distribution centers and customers allocation. Total network costs are 3895069.056 that contain fixed costs of distribution centers (120000) and transportation cost (2695069.056).

Now, the failure probability of each section is changed in order to analyze its impact on the result. With an increase of 10%, 20% or 30% in the failure probability, the structure of the supply chain network does not change in this example.

Figure 6 presents the results obtained from the instance with an increase of 40% in the failure probability. The result of increase of 50% to 140% in the failure probability are presented in Figure 7, and the result obtained from the increase of 150% and more in the failure probability arcs are shown in Figure 8.

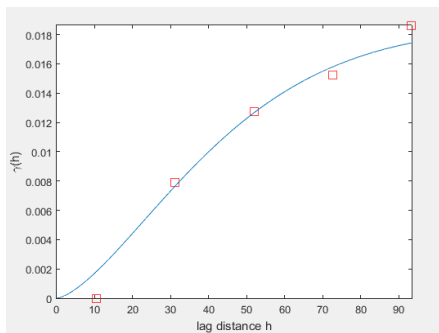


Figure 4 -Empirical plot of the semi-variogram

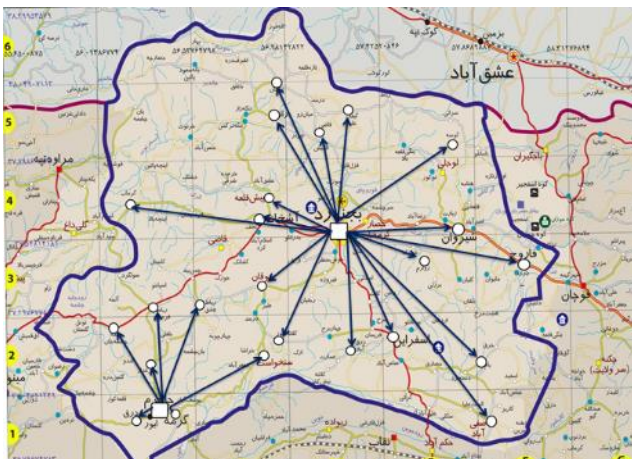


Figure 5 -Allocate customers to distribution centers

The results obtained from 10% to 40% reduction in the arc failure probability presented in Figure 6 as well as an increase of 40% in the probability of failure. The results of decrease of 50% to 100% in the failure probability are presented in Figure 9.

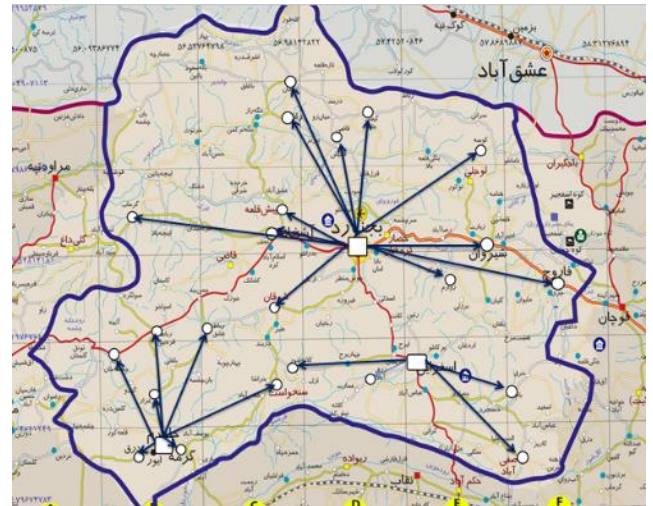


Figure 6 -increase of 40% and decrease of 10% to 40% in the failure probability of arcs

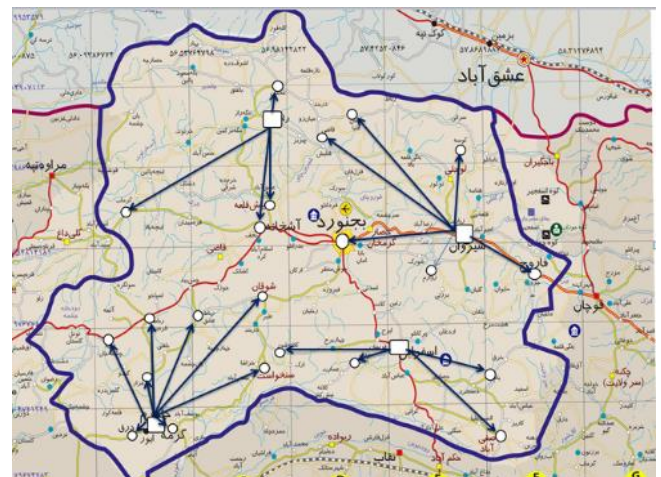


Figure 7 -increase of 50% to 140% in the failure probability of arcs

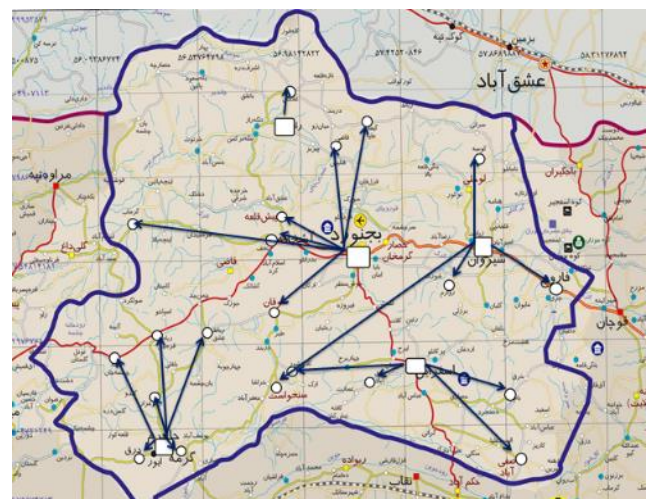


Figure 8 -increase of 150% and more in the failure probability of arcs





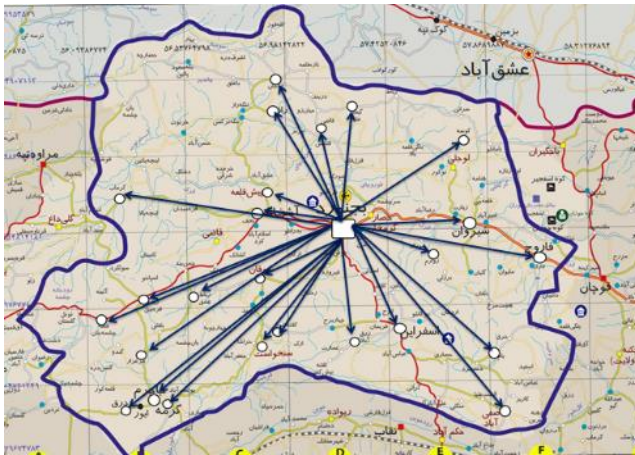


Figure 9 -decrease of 50% to100% in the failure probability of arcs

## Conclusions

This paper studies a location-allocation problem that seeks to strengthen the arcs between the distribution centers in the network. A mixed-integer programming model is developed to determine the optimal locations for distribution centers and customer allocation. The failure probability of the arcs between distribution centers is estimated with a geographical statistics model.

The North-Khorasan province is used as a sample for this model. We present the results obtained from different failure probability. By increasing step by step in the failure probability, the transport costs are minimized. Moreover, to minimize the cost of transportation, the number of distribution centers is increased. Decreasing the failure probability causes lower total transportation cost.

This work can be extended in several directions. Our work can further be extended by increasing the echelons of the supply chain network. On the other hand, failure probability of centers can be considered in the model.

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