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A Maximum Profit Incomplete Hub Location Routing Problem

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Abstract

Location area is one of the most important research areas in decision-making due to the widely used. Hub location as a type of location problem has many applications such as transportation and telecommunication systems. The hub location problem (HLP) involves finding the location of hubs and allocates the other nodes to them. A significant issue that more should be noted that it is always minimizing cost is not a perfect solution for the survival. Therefore, other parameters such as price play significant role in the survival of the organization. Therefore, in this article, we consider profit at incomplete hub location and routing problem over the incomplete networks. We use two type datasets. First, Iranian Road Network (IRN) and second, Australia Post (AP). We run the model for different values of α *and* γ *on software* CPLEX 12.5 *and observe number of hubs, number of hub nodes, number of hub links and spoke links. The results show that the location of hubs is not usually altered by changing in the price. Moreover, the number of spoke links may be changed by increasing the price.*

Keywords:
Hub Location, Routing, Profit, Pricing

Introduction

Location studies play a critical role in the construction process of industrials or service units. Also, in the last few decades, hub networks have been very important because of widely used in the modern transportation [1]–[3] and telecommunication systems [4]. Hub location is one of the widely used problems of the location studies, and means that finding the location of hubs and non-hub nodes and allocates non-hub nodes to hubs in order to routing the commodities through the network, so that all nodes been served. Hub nodes join and distribute flows, when it is profitable. In the literature, HLPs have been studied in four aspects. The first one is objective function, that involves median, covering and center, which has been introduced well in [5]. Another objective is order median, which proposed by Puerto et al [6]. Second aspect, which are studied in [7] is the allocation strategy categorized single and multiple. Third aspect is number of hubs. This number is calculated by model or predefined by decision makers. When this number is predetermined, the model is called p-hub. And finally, fourth aspect is capacity of hubs, which is considered in [8]–[10].
In the classical hub location models, three main assumptions were often considered: (1) all hubs are connected that called complete network for example figure 1(d) but in some studies, this assumption is relaxed. (2) Non-hub nodes are only connected via hub node(s.). (3) On the hub links, are used a discount factor $0 \le \alpha \le 1$. In addition, classical HLPs ignore all arc setup costs.
One of the most widely used fields of incomplete hub location model is the transportation systems, especially public and urban transportation system. Also, in passenger transportation, linking all hubs are extremely costly. Consequently, using the incomplete hub network is inefficient [11]. In generally, the incomplete hub networks are classified depending on the topology of network into four categories.

- Tree shape. In this topology as shown in the figure 1(a), hub network connected without cycles. In fact, to go from one hub to the other hub, there is only one path. This network structure was originally introduced by Kim and Tcha [12] in HLPs.
- Ring network. In this topology as shown in figure 1(b), the number of hub links equals to the number of hubs, and there is just one cycle through the hub network. In addition, each hub node is connected exactly two hub nodes. This structure used in hub network problems by Chiu et al. [13]. Wang et al. [14] also used ring topology in the telecommunications systems. A single allocation hub-ring model formulation was proposed by Lee et al. [15].
- Special form. In this topology, the hub networks are restricted by decision-makers and the topology of them is different from the tree and ring structures. For example, Wasner and Zapfel

[16] introduced a model that all hubs just have been linked with one hub called as a central hub.

- General form. In this topology as shown in the figure 1(c), the shape of hub network is determined by model and each structure such as tree, ring and completed can be created. This structure introduced by Yoon et al. [17].

HLPs may be classified in terms of market into competitive and monopolized that in this article we consider a monopolized market. In this article, general hub network is considered and median objective, multiple allocation strategy, undetermined number of hubs and uncapacitated characteristics are used. On the other hand, because of the location is costly, appropriate quality and price are factors that cause more income. Therefore, calculating the profit is important for companies. A significant issue that should be noted that locations are chosen based on customers demand, and this demand depends on the price. Therefore, price is an important component in the location problem. Thus, we calculate profit at incomplete hub location and routing problem by considering multiple allocation strategy such that all nodes be serviced. It is also clear that accurate location and using the appropriate method for determining the price have interaction on each other. Hence, determining the pricing method is very important.
In this article, we use a γ coefficient to generating price that is equal to γ multiplied by the transportation cost. We use different values for γ, which are $\{1.7,1.9,2,2.2,2.5\}$. Also, we use different values for α, which are $\{0.5,0.6,0.7,0.8,0.9\}$.
In this paper two types of datasets are used that are *Iranian Road Network* (IRN) [18] and Australian Post (AP) [19]with 10, 20, 25, 40 ,50 nodes. IRN involves twenty nodes.
We run the model for different values of γ and α with CPLEX 12.5. We examine the result in terms of number of hub nodes, hub links, spoke links and number of hubs. The rest of the paper is organized as follows. In section 2, the related literature with this issue is briefly reviewed. Modeling is presented in section 3. In section 4, statistical observations are expressed. Finally, conclusion is presented in section 5.

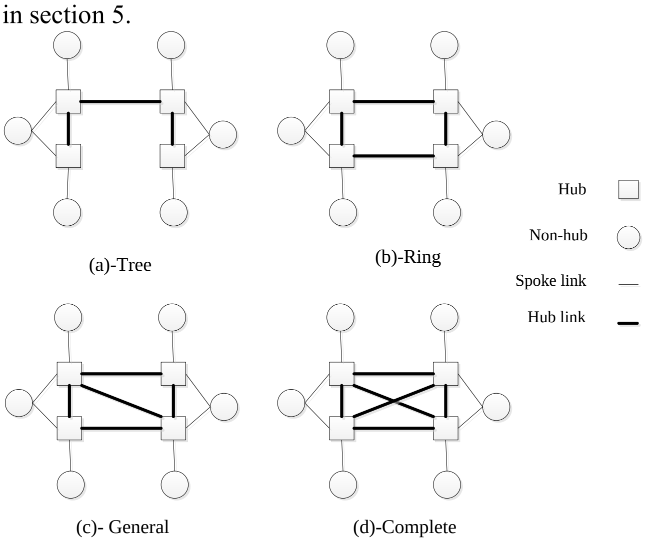

Figure 1-Example of Topology of Hub Networks [20].

Related literature review

The HLP was introduced by O'Kelly [21]. Over the past decades, most research in this area emphasized the use of hub location network with complete topology. A new approach in HLPs was introduced in [8, 16], and suggested hub arc location with incomplete hub network.
As mentioned, at first, the general form of hub networks was introduced by Yoon et al [17]. They proposed a model that did not consider fixed cost of locating the hub nodes. Then, they introduced a model that considered the cost of opening hub nodes, hubs links, spoke links and flow shipping cost, but their model had many variables and constraints. Next, Nickel et al. [22] introduced a MIP models for public transportation networks. Afterward, Gelareh and Nickel [23] improved model of Nickel et al. [22], and reduced number of variables and constraints, but their model could not make a network with one hub. Another model for urban public transportation with considering the proprietor and customer costs introduced by Setak and Karimi [20]. Also, they proposed an incomplete hub-routing model in urban transportation problem that they relaxed third assumption of classic HLPs (connection between non-hub nodes) [24]. This relaxation initially was introduced by Aykin [25], [26]. Liu et al. [27] proposed an improved approach based on multiple attribute group decision-making in public transportation (PT) hub location with stochastic demand. They considered quantitative and qualitative influenced factors in PTHL.
Also, there are many studies about pricing in the hub location. In the HLPs, pricing studies have been used at airports [28], [29] and parking [30]. Of course, [30] analyzes the influence factors of comprehensive passenger transport hub parking pricing. Obviously, O'Kelly et al [31] presented hub location problems with price sensitive demand. To the best of our knowledge, until 2013, there is no literature on hub location problems that involve simultaneously location and pricing problem. For example, Lüer-Villagra and Marianov presented a competitive hub location and pricing problem (CHLPP) [32]. Both pricing decision and location-network design were considered. They used Genetic algorithm (GA) for solving their model, because of the NP-hardness. Afterward, Abbasi and Niknam proposed a hybrid algorithm of GA and simulated annealing (SA) for pricing in HLP [33]. Their aim was to compare their method with GA method in terms of solution quality and time.
Some papers such as [33] and [34] present a class of hub network design problems with profit oriented objectives that employed in the design of air and ground transportation networks. Some studies (for example [34]) calculate the profit with predetermined price in hub networks.

Model Formulation

As mentioned, in this article, the aim is to calculate the profit at the incomplete hub network that involves hub(s), hub link(s) and spoke link(s) for different values of α and γ.

Parameters are hub nodes, hub links and spoke links fixed costs. Also, customers' location and distance and flows are given. Another assumption will be explained in the following.

- All nodes can be a hub.
- There is no connection between non-hub nodes.
- A discount factor on hub links is considered
- Multiple allocation strategy is used.
- All flows travel through the shortest path.

Indices and sets:
N : Set of all nodes.
i, j : Origin and destination.
k, m : Hub nodes.

Parameters:
w_{ij} : Transitional flow from origin i to destination j.
c_{ij} : Customer transshipment cost from origin i to destination j.
I_{km} : Hub link fixed cost between hub nodes k and m.
J_{ik} : fixed cost of spoke link between non-hub node i and hub node k.
F_k : Fixed cost of locating a hub at node k.
α : Discount factor ($0 \le \alpha \le 1$).
γ : Percent that is multiplied at transportation cost and forms price.

Decision variables:
z_{ik} : Flow from node i to hub k.
x_{ij}^{k} : flow which originated from node i to destination j via hub node k.
y_{km}^{i} : Originating flows from node i and routed between hub nodes k and m.
a_{ik} : It is one, when there is exactly one hub at nodes k and i $(k \ne i)$, otherwise is zero. Also, $a_{kk} = 1$, if a hub is opened at node k.
b_{km} : It is one, when a hub link is used between hub nodes k and m, otherwise $b_{km} = 0$.

The mathematical model formulation.

$$\max Z = \sum_{i\in N}\sum_{k\in N}((p_{ik}-c_{ik})z_{ik} + \sum_{j\in N}(p_{kj}-c_{kj})x_{ij}^{k}) + \sum_{i\in N}\sum_{k\in N}\sum_{m\in N}(p_{km}-\alpha c_{km}y_{km}^{i}) + \sum_{k\in N}F_k a_{kk} + \sum_{k\in N}\sum_{\substack{m\in N\\ m>k}} I_{km}.b_{km} + \sum_{i\in N}\sum_{k\in N} J_{ik}.a_{ik} \quad (1)$$

$$\sum_{k\in N} z_{ik} = \sum_{j\in N} w_{ij} \qquad \forall i \in N \quad (2)$$

$$\sum_{k\in N} x_{ij}^{k} = w_{ij} \qquad \forall i, j \in N \quad (3)$$

$$\sum_{\substack{m\in N\\ m\ne k}} y_{km}^{i} \sum_{j\in N} x_{ij}^{k} = \sum_{\substack{m\in N\\ m\ne k}} y_{mk}^{i} + z_{ik} \qquad \forall i, K \in N, i \ne k \quad (4)$$

$$z_{ik} \le a_{ik} \sum_{j\in N} w_{ij} \qquad \forall i, K \in N \quad (5)$$

$$\sum_{i\in N} x_{ij}^{k} \le a_{jk} \sum_{i\in N} w_{ij} \qquad \forall k, j \in N \quad (6)$$

$$y_{km}^{i} + y_{mk}^{i} \le b_{km} \sum_{j\in N} w_{ij} \qquad \forall i, k, m \in N \quad (7)$$

$$a_{ik} \le a_{ii} + a_{kk} \qquad \forall i, K \in N, i \ne k \quad (8)$$

$$a_{ik} \le 2 - a_{ii} - a_{kk} \qquad \forall i, K \in N, i \ne k \quad (9)$$

$$a_{mk} \le b_{mk} + a_{kk} \qquad \forall k, m \in N, k \le m \quad (10)$$

$$a_{km} \le b_{mk} + a_{mm} \qquad \forall k, m \in N, k \le m \quad (11)$$

$$p_{ij} = \gamma . c_{ij} \qquad \forall i, j \in N \quad (12)$$

$$z_{ik} \ge 0 \qquad \forall i, k \in N \quad (13)$$

$$x_{ij}^{k} \ge 0 \qquad \forall i, j, k \in N \quad (14)$$

$$y_{km}^{i} \ge 0 \qquad \forall i, k, m \in N \quad (15)$$

$$a_{ik} \in \{0,1\} \qquad \forall i, k \in N \quad (16)$$

$$b_{ik} \in \{0,1\} \qquad \forall k, m \in N \quad (17)$$

In this model, equation (1) maximize the profit. Constraints (2) ensure that all originated flows from node *i* leaving it. Constraints (3) ensure that the hub node *j* receive the originated flow from node *i*. Constraints (4) calculate flows that routed from node *i* via hub nodes *m* and *k*.
If a non-hub node is allocated to a hub node, it can send (receive) to (from) it, which is ensure by constraints (5) and (6). If there is one link between hub nodes *m* and *k*, variables y_{km}^{i} are positive, which is display by constraints (7). Constraints (8) and (9) ensure that only when a link used between nodes *i* and *k*, just one of them is one hub. The endpoints of hub link are hub nodes. And also, one of the endpoints of spoke link is a hub that ensured by constraints (10) and (11). Constraints (12) shows the price structure. Eventually, constraints (13)-(17) represent the domains of decision variables.

Statistical analysis

This model has been run on software CPLEX 12.5 for two type datasets that are AP with 10, 20, 25, 40, 50 nodes and IRN. IRN involves twenty nodes. Also, results for different values of α and γ that their values expressed in introduction and results are compare in Tables 1-3. In this tables, NoH, HN and NoSL are acronyms of number of hub, hub nodes and number of spoke link, respectively. This comparisons are in terms of objective function value (OFV), NoH, NH, and NoSL.
As observed, for AP in all instances, the OFV logically is increased by increasing the value of γ. In AP 10, that results showed in table 1, for $\gamma = 1.7$, the number of hubs and spoke links decrease in $\alpha = \{0.8, 0.9\}$. For $\gamma = \{1.9, 2.2, 2.5\}$, there is no change in the number of hubs and spoke links for different values of α. For $\gamma = 2$, the number of hubs

increase in $\alpha = 0.9$ and number of spoke links are fixed.
In AP 20, for all values of γ and α, the number of spoke links are the same. For $\gamma = \{1.7, 1.9, 2, 2.2\}$, the number of hub nodes are not changed. However, in $\gamma = 2.5$ for all values of α, the location of hubs are changed.
In AP 25, for $\gamma = \{1.9, 2.2, 2.5\}$, the number of hub nodes and the number of spoke links are the same. However, for $\gamma = \{1.7, 2\}$, the number of hubs and hub nodes are different, but the number of spoke links are the same.
In AP 40 and 50, respectively, the number of hubs and hub nodes are the same for all values of γ and α, but the number of spoke links in $\gamma = \{1.7, 1.9, 2\}$ are different for different values of α.
In IRN dataset, the number of hubs are equal and the number of hub nodes and spoke links are the same.

Table 1 – Results of solved instances (part 1).

AP10							AP20						
γ	α	**Time (s)**	**OFV**	**NoH**	**HN**	**NoSL**	γ	α	**Time (s)**	**OFV**	**NoH**	**HN**	**NoSL**
	0.5	0.863	3504454000	2	1,9	16		0.5	2.278	6498644000	2	4,17	36
	0.6	1.635	3499913000	2	1,9	16		0.6	4.264	6492429000	2	4,17	36
1.7	0.7	2.512	3495373000	2	1,9	16	1.7	0.7	6.017	6486215000	2	4,17	36
	0.8	3.448	3491993000	1	1	9		0.8	8.078	648000000	2	4,17	36
	0.9	4.393	3491993000	1	1	9		0.9	10.171	6473785000	2	4,17	36
	0.5	0.770	6445647000	2	1,9	16		0.5	2.253	7297484000	2	4,17	36
	0.6	1.591	6441106000	2	1,9	16		0.6	4.579	7290830000	2	4,17	36
1.9	0.7	2.425	6436565000	2	1,9	16	1.9	0.7	6.803	7284616000	2	4,17	36
	0.8	3.231	6432025000	2	1,9	16		0.8	8.751	7278401000	2	4,17	36
	0.9	4.042	6427484000	2	1,9	16		0.9	10.947	7272186000	2	4,17	36
	0.5	0.667	4162930000	2	1,9	16		0.5	2.502	7697040000	2	4,17	36
	0.6	1.650	4158389000	2	1,9	16		0.6	5.372	7690031000	2	4,17	36
2	0.7	2.440	4153849000	2	1,9	16	2	0.7	7.475	7683816000	2	4,17	36
	0.8	3.249	4149308000	2	1,9	16		0.8	9.420	7677601000	2	4,17	36
	0.9	4.257	4144768000	4	1,6,9,10	16		0.9	11.513	7671387000	2	4,17	36
	0.5	0.834	4601914000	2	1,9	16		0.5	2.676	8496420000	2	4,17	36
	0.6	1.751	4597373000	2	1,9	16		0.6	4.817	8488765000	2	4,17	36
2.2	0.7	2.633	4592833000	2	1,9	16	2.2	0.7	6.793	8482229000	2	4,17	36
	0.8	3.513	4588292000	2	1,9	16		0.8	8.916	8476014000	2	4,17	36
	0.9	4.236	4583752000	2	1,9	16		0.9	11.019	8469800000	2	4,17	36
	0.5	0.921	5237687000	2	1,9	16		0.5	3.593	727736800	2	1,4	36
	0.6	1.713	5255849000	2	1,9	16		0.6	6.975	727381200	2	1,4	36
2.5	0.7	2.572	5251309000	2	1,9	16	2.5	0.7	10.483	727272200	2	1,4	36
	0.8	3.504	5246768000	2	1,9	16		0.8	14.097	727246100	2	1,4	36
	0.9	4.336	5242228000	2	1,9	16		0.9	16.816	727244600	2	1,4	36

Table 2 – Results of solved instances (part 2).

AP25							AP40						
γ	α	**Time (s)**	**OFV**	**NoH**	**HN**	**NoSL**	γ	α	**Time (s)**	**OFV**	**NoH**	**HN**	**NoSL**
	0.5	4.519	9831635.785	2	5,21	46		0.5	46.983	27003310	3	1,8,33	74
	0.6	8.868	9822603.416	2	5,21	46		0.6	85.472	27003310	3	1,8,33	74
1.7	0.7	12.532	9813651.265	2	5,21	46	1.7	0.7	115.440	26988480	3	1,8,33	74
	0.8	17.396	9805556.033	5	2,3,5,14,21	46		0.8	148.162	26982490	3	1,8,33	75
	0.9	22.012	9798015.448	2	5,21	46		0.9	189.257	26976720	3	1,8,33	75
1.9	0.5	3.677	11034970	2	5,21	46	1.9	0.5	44.870	30234170	3	1,8,33	74
	0.6	7.084	11025930	2	5,21	46		0.6	83.723	30225990	3	1,8,33	74

	0.7	10.838	11016900	2	5,21	46		0.7	114.027	30217870	3	1,8,33	74
	0.8	14.174	11008080	2	5,21	46		0.8	147.445	30211620	3	1,8,33	74
	0.9	17.978	10999980	2	5,21	46		0.9	184.615	30205640	3	1,8,33	75
2	0.5	4.454	11636600	2	5,21	46	2	0.5	58.885	31769000	3	1,8,33	76
	0.6	8.767	11627570	2	5,21	46		0.6	144.975	31841420	3	1,8,33	74
	0.7	12.358	11618540	2	5,21	46		0.7	189.026	31833240	3	1,8,33	74
	0.8	16.954	11609510	2	5,21	46		0.8	219.879	31826280	3	1,8,33	74
	0.9	20.898	11601210	3	5,10,21	46		0.9	251.278	31820120	3	1,8,33	74
2.2	0.5	4.328	12839900	2	5,21	46	2.2	0.5	59.504	35080470	3	1,8,33	74
	0.6	8.780	12830870	2	5,21	46		0.6	146.932	35072290	3	1,8,33	74
	0.7	13.869	12821830	2	5,21	46		0.7	190.438	35064110	3	1,8,33	74
	0.8	19.227	12812800	2	5,21	46		0.8	223.412	35055930	3	1,8,33	74
	0.9	23.158	12803770	2	5,21	46		0.9	274.564	35049390	3	1,8,33	74
2.5	0.5	4.827	14495760	2	5,21	46	2.5	0.5	72.175	39926820	3	1,8,33	74
	0.6	8.366	14635820	2	5,21	46		0.6	157.777	39918580	3	1,8,33	74
	0.7	12.116	14626790	2	5,21	46		0.7	239.455	39910400	3	1,8,33	74
	0.8	15.822	14617760	2	5,21	46		0.8	301.302	39902220	3	1,8,33	74
	0.9	19.582	14608720	2	5,21	46		0.9	376.603	39894040	3	1,8,33	74

Table 3 – Results of solved instances (part 3).

AP50							IRN						
γ	α	Time (s)	OFV	NoH	HN	NoSL	γ	α	Time (s)	OFV	NoH	HN	NoSL
1.7	0.5	130.213	47852540	3	1,10,41	94	1.7	0.5	2.373	9217507.209	2	5,20	36
	0.6	265.404	47844230	3	1,10,41	94		0.6	4.662	9208915.606	2	5,20	36
	0.7	390.788	47836090	3	1,10,41	95		0.7	7.768	9200777.955	2	5,20	36
	0.8	549.988	47828260	3	1,10,41	96		0.8	10.821	9193064.225	2	5,20	36
	0.9	651.143	47820580	3	1,10,41	96		0.9	13.470	9185350.494	2	5,20	36
1.9	0.5	155.805	53537590	3	1,10,41	94	1.9	0.5	3.253	10368280	2	5,20	36
	0.6	312.667	53529270	3	1,10,41	94		0.6	6.705	10359690	2	5,20	36
	0.7	468.930	53520960	3	1,10,41	94		0.7	9.932	10351100	2	5,20	36
	0.8	623.742	53512900	3	1,10,41	95		0.8	12.145	10343130	2	5,20	36
	0.9	779.027	53505080	3	1,10,41	96		0.9	15.219	10335410	2	5,20	36
2	0.5	180.064	56380110	3	1,10,41	94	2	0.5	2.788	10943550	2	5,20	36
	0.6	360.312	56371790	3	1,10,41	94		0.6	5.430	10934960	2	5,20	36
	0.7	539.663	56363470	3	1,10,41	94		0.7	8.705	10926370	2	5,20	36
	0.8	717.465	56355240	3	1,10,41	94		0.8	11.363	10918040	2	5,20	36
	0.9	898.495	56347380	3	1,10,41	95		0.9	14.856	10910330	2	5,20	36
2.2	0.5	178.403	62065360	3	1,10,41	94	2.2	0.5	2.903	12094320	2	5,20	36
	0.6	356.43	62056870	3	1,10,41	94		0.6	5.665	12085730	2	5,20	36
	0.7	534.644	62048550	3	1,10,41	94		0.7	8.808	12077140	2	5,20	36
	0.8	713.940	62040240	3	1,10,41	94		0.8	11.607	12068550	2	5,20	36
	0.9	892.049	62032060	3	1,10,41	94		0.9	14.967	12060390	2	5,20	36
2.5	0.5	154.130	70593470	3	1,10,41	94	2.5	0.5	3.364	13820610	2	5,20	36
	0.6	311.266	70584450	3	1,10,41	94		0.6	6.803	13812020	2	5,20	36
	0.7	466.740	70576130	3	1,10,41	94		0.7	9.995	13803430	2	5,20	36
	0.8	620.947	70567810	3	1,10,41	94		0.8	12.404	13794840	2	5,20	36
	0.9	775.564	70559500	3	1,10,41	94		0.9	15.276	13786240	2	5,20	36

Concluding remarks

In this paper, we suggested a mathematical model that calculates the profit for a kind of HLP. We run the model for two datasets: (I) AP and (II) IRN. Results show that logically the profit is increased by increasing prices. Moreover, by changing in the prices, the location of hubs commonly do not change and the number of spoke links may be changed. Furthermore, by changing the value of α in the same γ, number of hubs is increased. It should be noted that some of them are isolated and number of hub nodes are the same with number of hub nodes in other values of α in the same γ. Also, naturally, time solve increase by increasing number of nodes. The suggested model will be extended by making an allowance for direct link and considering capacity in hub nodes and hub links.

References

[1] Sarkis, J. and Sundarraj, R. P., "Hub location at Digital Equipment Corporation: A comprehensive analysis of qualitative and quantitative factors," *Eur. J. Oper. Res.*, vol. 137, no. 2, pp. 336–347, 2002.

[2] Çetiner, S., Sepil, C. and Süral, H., "Hubbing and routing in postal delivery systems," *Ann. Oper. Res.*, vol. 181, No. 1, pp. 109–124, Dec. 2010.

[3] Yaman, H., Kara, B. Y. and Tansel, B. Ç., "The latest arrival hub location problem for cargo delivery systems with stopovers," *Transp. Res. Part B Methodol.*, vol. 41, no. 8, pp. 906–919, 2007.

[4] Lin, C.-C., Lin, J.-Y. and Chen, Y.-C., "The capacitated p-hub median problem with integral constraints: An application to a Chinese air cargo network," *Appl. Math. Model.*, vol. 36, No. 6, pp. 2777–2787, 2012.

[5] Campbell, J. F., "Integer programming formulations of discrete hub location problems," *Eur. J. Oper. Res.*, vol. 72, No. 2, pp. 387–405, 1994.

[6] Puerto, J., Ramos, A. B. and Rodríguez-Chía, A. M., "Single-allocation ordered median hub location problems," *Comput. Oper. Res.*, vol. 38, No. 2, pp. 559–570, 2011.

[7] Yaman, H., "Allocation strategies in hub networks," *Eur. J. Oper. Res.*, vol. 211, No. 3, pp. 442–451, 2011.

[8] Ernst, A. T. and Krishnamoorthy, M., "Solution algorithms for the capacitated single allocation hub location problem," *Ann. Oper. Res.*, vol. 86, No. 0, pp. 141–159, 1999.

[9] Marín, A., "Formulating and solving splittable capacitated multiple allocation hub location problems," *Comput. Oper. Res.*, vol. 32, no. 12, pp. 3093–3109, 2005.

[10] Correia, I., Nickel, S. and Saldanha-da-Gama, F., "Hub and spoke network design with single-assignment, capacity decisions and balancing requirements," *Appl. Math. Model.*, vol. 35, No. 10, pp. 4841–4851, 2011.

[11] Contreras, I., Fernández, E. and Marín, A., "Tight bounds from a path based formulation for the tree of hub location problem," *Comput. Oper. Res.*, vol. 36, No. 12, pp. 3117–3127, 2009.

[12] Kim, J. and Tcha, D., "Optimal design of a two-level hierarchical network with tree-star configuration," *Comput. Ind. Eng.*, vol. 22, No. 3, pp. 273–281, 1992.

[13] "A Steiner Ring Star Problem in Designing Survivable Telecommunication Networks." [Online]. Available: http://meetings2.informs.org/NO95/TALKS/TA24.3.html. [Accessed: 14-Dec-2016].

[14] Wang, Z., Lin, C. and Chan, C., "Demonstration of a single-fiber self-healing CWDM metro access ring network with unidirectional OADM," *IEEE Photonics Technol. Lett.*, Vol. 18, No. 1, pp. 163–165, Jan. 2006.

[15] Lee, C., Ro, H. and Tcha, D., "Topological design of a two-level network with ring-star configuration," *Comput. Oper. Res.*, Vol. 20, No. 6, pp. 625–637, 1993.

[16] Wasner, M. and Zäpfel, G., "An integrated multi-depot hub-location vehicle routing model for network planning of parcel service," *Int. J. Prod. Econ.*, vol. 90, No. 3, pp. 403–419, Aug. 2004.

[17] Yoon, M.G., Beak, Y.H. and Tcha, D.W., "Optimal design model for a distributed hierarchical network with fixed-charged facilities," *J. Manag. Sci*. Vol. 18, No. 1, JANUARY 1, 2006

[18] Karimi, H. and Setak, M., "Flow shipment scheduling in an incomplete hub location-routing network design problem," *J. of Comput. Appl. Math.*, No. 7, 2016.

[19] Ernst, A. T. and Krishnamoorthy, M., "Efficient algorithms for the uncapacitated single allocation p-hub median problem," *Locat. Sci.*, Vol. 4, No. 3, pp. 139–154, 1996.

[20] Karimi, H. and Setak, M., "Proprietor and customer costs in the incomplete hub location-routing network topology," *Appl. Math. Model.*, Vol. 38, No. 3, pp. 1011–1023, 2014.

[21] O'Kelly, M. E., "The Location of Interacting Hub Facilities," *Transp. Sci.*, Vol. 20, No. 2, pp. 92–106, May 1986.

[22] Nickel, S., Schöbel, A. and Sonneborn, T., "Hub Location Problems in Urban Traffic Networks," Springer US, 2001, pp. 95–107.

[23] Gelareh, V. and Nickel, S., "Hub location problems in transportation networks," *J of Transp. Res. Part E Logist. Transp. Rev.*, Vol. 47, No. 6, pp. 1092–1111, 2011.

[24] Setak, M., "Designing Incomplete Hub Location-routing Network in Urban Transportation Problem," *Int. J. Eng.*, Vol. 26, No. 9, pp. 997–1006, 2013.

[25] Aykin, T., "Lagrangian relaxation based approaches to capacitated hub-and-spoke network design problem," *Eur. J. Oper. Res.*, Vol. 79, No. 3, pp. 501–523, Dec. 1994.

[26] Aykin, T., "The hub location and routing problem," *Eur. J. Oper. Res.*, Vol. 83, No. 1, pp. 200–219, 1995.

[27] Liu, S., Song, Z. and Zhong, S., "Public Transportation Hub Location with Stochastic Demand : An Improved Approach Based on Multiple Attribute Group Decision-Making," *J. of Discrete Dynamics in Nature and Society*, Vol. 2015.

[28] Lin, M. H. and Zhang, Y., "Airline pricing and airport charges in hub-spoke networks with congestion," October 6, 2011," pp. 1–25, 2011.

[29] Daniel, J. I. and Harback, K. T., "Pricing the major US hub airports," *J. Urban Econ.*, Vol. 66, No. 1, pp. 33–56, 2009.

[30] Wang, D. D. Ji, C. X. and Xu, L., "Analysis of Influence Factors of Comprehensive Passenger Transport Hub Parking Pricing," *Appl. Mech. Mater.*, Vol. 644–650, pp. 5857–5859, 2014.

[31] O'Kelly, M. E., Luna, H. P. L., de Camargo, R. S., and de Miranda, G., "Hub Location Problems with Price Sensitive Demands," *Networks Spat. Econ.*,

Vol. 15, No. 4, pp. 917–945, 2015.

[32] Lüer -Villagra, V. and Marianov, V., “A competitive hub location and pricing problem,” *Eur. J. Oper. Res.*, Vol. 231, No. 3, pp. 734–744, 2013.

[33] Abbasi, M. and, Niknam, R., “A Combined Genetic Algorithm and Simulated Annealing Approach for Solving Competitive Hub Location and Pricing Problem,” *Int. j. of applied management science*.,pp. 1–23.

[34] Alibeyg, A., Contreras, I. and Fernández, E., “Hub network design problems with profits,” *Transp. Res. Part E Logist. Transp. Rev.*, Vol. 96, pp. 40–59, 2016.