Mazandaran University of Science and Technology

Available online at www.iiec2017.com

13th International Conference on Industrial Engineering (IIEC 2017)

Iran Institute of Industrial Engineering

Time Parameter Estimation Using Statistical Distribution of Weibull to Improve Reliability

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Abstract

Environmental stress screening program, which is used in development phase of manufacturing, causes decrease or even elimination of hidden faults of products before competency tests. This can lead to significant improvement in product reliability and decrease in maintenance costs. During environmental stress screening test, the product is exposed to a certain environmental conditions such as temperature, humidity and vibration in a certain level for some amount of times. The exposure time is a very important parameter that should be determined accurately using statistical methods. The aim of this paper is to calculate the optimal exposure time for an environmental stress screening process using Weibull distribution which has wide applications in reliability analysis. A case study of electronic sets is then applied to illustrate the process step by step. After determination of Weibull parameters, obtained results and charts are used to calculate the time parameter of screening test for the components.

Keywords:
Environmental stress screening, Weibull distribution, Reliability, Exposure time, Competency test

Introduction

The aim of screening electronic sets is actually to accelerate burnout of sets and make them reach to their useful life before customer delivery [1]. For many mechanical and electronic components, the failure rate function has a bathtub shape [2]. It is well-known that, because of design and manufacturing problems, the failure rate is high at the beginning of a product life cycle and decreases toward a constant level. After reaching a certain age, the product enters the wear-out phase and the failure rate starts to increase [3]. Despite the fact that this phenomenon has been presented in many reliability engineering texts, few practical models possessing this property have appeared in the literature [2]. In most of the previous studies, only a part of the bathtub curve is considered at any one time. Another common fact is that most engineers may be interested only in a part of the lifetime, because at component level, they only see one part of the failure rate function. However, it will be helpful to have a model that is reasonably simple and good for the whole product life cycle for making overall decisions. Furthermore, for tcomplex systems, both the decreasing and increasing parts of the failure rate fall into the ordinary product lifetime [2]. The idea is based on the conventional Weibull distribution which is widely used by reliability engineers today [2-4]. In practice, Weibull distributions have been shown to be very flexible in modeling various types of lifetime distributions and they have been used to model any of the three parts in a bathtub-curve [5-7]. The two-parameter version has the following form:

$$F(t) = 1 - exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right], t \geq 0 \tag{1}$$

Where η is called the scale parameter and β is called the shape parameter. The failure rate function that corresponds to (1) is given by.

$$R(t) = exp\left[-\left(\frac{t-t_0}{\eta - t_0}\right)^{\beta}\right] \tag{2}$$

It can be seen that when $\beta<1$, the failure rate function is decreasing and when $\beta > 1$, it is increasing. In the case of

β=1, we have the well-known exponential distribution which has a constant failure rate. Because of these interesting properties, Weibull distribution has been widely used for modeling different phases of lifetime [6, 8]. It is our intention in this paper to study some practical models for the bathtub-shaped failure rate function and to determine time parameters for screening test. The integration of screening process with robust statistical methods will facilitate the discovery of defects, ultimately improving the sensitivity and specificity of the screening process.
In this paper, drawing probability plot method is applied to determine Weibull parameters accurately. This method is the most proper one to show Weibull function's variety and functionality [7, 9]. To this order, at first essential process steps for determination of Weibull parameters will be explained. Then, actual failure behavior has employed to determine shape and life characteristic parameters of the Weibull function empirically. Then, using accurate Weibull parameters, the Weibull hazard rate function can be calculated. ultimately, obtained results and charts are used to determine the time parameter of screening test for an environmental stress screening process of electronic sets.

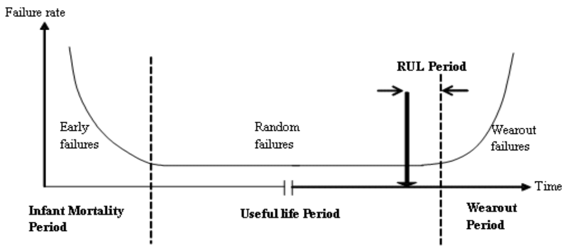

Fig. 1: Bathtub-curve

Parameter Estimation for Weibull Distribution

Weibull distribution has especial wide applications in reliability analysis. The reason for this matter is that determination of parameters for this distribution allows us to fit proper models of failure distribution in every span of life for a wide range of parts and products [4, 8, 10]. Weibull three- parameter probability density function generally is as follows:

$$f(t) = \frac{\beta}{\eta - t_0}\left[\left(\frac{t - t_0}{\eta - t_0}\right)^{(\beta-1)} \cdot exp\left[-\left(\frac{t - t_0}{\eta - t_0}\right)^{\beta}\right]\right] \qquad (3)$$

Three-parameter Weibull hazard rate function generally is as follow:

$$h(t) = \frac{\beta}{\eta - t_0}\left(\frac{t - t_0}{\eta - t_0}\right)^{(\beta-1)} \qquad (4)$$

Three-parameters Weibull reliability function is as follow:

$$R(t) = exp\left[-\left(\frac{t - t_0}{\eta - t_0}\right)^{\beta}\right] \qquad (5)$$

In which the parameters used in every three above equations are as follows:

- β: Shape parameter
- η: Scale parameter (Life characteristic)
- t_0: Minimum Life
- t: Time

Three-parameter Weibull distribution will turn into two-parameter one when it used during infant mortality period. As electronic sets generally has no minimum lifetime, thus for these sets $t_0 = 0$. Consequently, by $t_0 = 0$ in above equations, we will have two-parameter Weibull functions where its probability density function generally is as follow:

$$f(t) = \frac{\beta}{\eta}\left[\left(\frac{t}{\eta}\right)^{(\beta-1)} \cdot exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right]\right] \qquad (6)$$

Two-parameter Weibull hazard rate function generally is as follow:

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{(\beta-1)} \qquad (7)$$

Two-parameter Weibull reliability function is as follow:

$$R(t) = exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \qquad (8)$$

As $R(t) = 1 - F(t)$, Then by replacing it in Eq (8) we have:

$$1 - F(t) = exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \qquad (9)$$

The values of parameters β and η will be calculated via analyzing the results of screening process. There are three types of sets at the end of screening process: Passed (approved) sets, failed sets, and suspended sets. A suspended set is a non-corrupted set which its screening time will end before other members of the non-corrupted society. If screening of all of sets of society starts in a same time and finishes in a same time, there would be no suspended set [11, 12].
Extracted data of screening process includes part number, sample size, approval status, and elapsed time to fail. When a set is approved while its elapsed time is less than a failed set, it is considered a suspended one. When a set is under a test series, there is the possibility of choosing the number of loops leading to at least one successful pass. When a set is not able to have a successful pass, maybe it has failed before beginning the screen. These sets may follow different distributions. As products can have different behavior patterns and nature, thus different screening times and screening types are anticipated. In screening process, the extracted part number will apply to distinguish and classify data for Weibull analysis [12].
In Weibull analysis, outcome data of screening process will be transformed in a way that makes it capable to fit Weibull distribution on them. If successful, values of parameters β and η are calculated. By replacing these values in Weibull hazard rate function there is now a mathematical statement to explain the failure behavior. Now, screening process time is computable using a specific slope or a great value of hazard rate curve. After calculating the screening time, confidence limits of 90 percent can provide a range of certainty in prediction of screening time [7]. Surely analysis of the greater number of failed units concludes to prediction of screen time with higher certainty [13].

Steps to Determine Optimal Screening Time

In order to determine optimal screening time we have to follow thirteen steps which are described in detail in the following subsections respectively. Also data related to the case study of electronic sets is also calculated for each step [1, 9, 14].

Step1. In order to fit Weibull distribution, data should be collected from screened sets. In ideal situation, in order to achieve maximum accuracy in determination of Weibull distribution parameters, sample number must be greater than 31. Otherwise accuracy will reduce.

Step2. Samples include information as: Failed (F), passed (P), suspended (S), failure or suspension time, completed loops of the test and sample size [9]. Those points of the sets which have not passed even one successful loop of screening process loop series will be excluded and have no role in calculation of time parameter of screening process. The reason for this matter is that the failure distribution for these points is distinctively different from other sets of the society. These sets probably were defective before entering the screening process. These data from failed sets will be applied in ranking of all failed sets.

Step3. Failure and suspension data must be sorted ascending according to time then numerically reverse from up to down (by integers). The first number for reverse ranking is sample size. Samples in current study are shown at Table1. In this example, sample size is equal to 200.

Table 1: Ranked data

Set	reverse ranking	Loops	Time (hour)	Failed/ Suspended
A	200	0	0	F
B	199	0	0	F
C	198	1	0.2	F
D	197	2	0.8	F
E	196	3	1	S
F	195	6	1.3	F
G	194	11	2.1	F
H	193	24	5.8	F
I	192	25	7	S
J	191	40	8.9	F
K	190	69	12.7	F

Step4. Rank order numbers are considered only for failed sets. This ranking also includes sets which have failed in t=0 or their loop number is zero. Eq(10) shows the mathematical statement to rank order increment:

$$RI_i = \frac{(sample\ size + 1) - RN_{i-1}}{1 + RR_i} \quad (10)$$

Rank order number is calculated as follows:

$$RN_i = RI_i + RN_{i-1} \quad (11)$$

By combining Eq (10) and (11) we will have:

$$RN_i = \frac{(sample\ size + 1) - RN_{i-1}}{1 + RR_i} + RN_{i-1} \quad (12)$$

In which i, RR_i and RN_{i-1} are defined as follows:

i : a specific failed set which starts from the failed set with highest reverse rank number. In this example the highest reverse rank number (200) is related to set A.

RR_i: Reverse rank number for sets which their ranking will be calculated

RN_{i-1}: For the first considered set this variable is equal to zero. The reason is that there is no previous ranking order number.

Also, sample sizes include failed or suspended samples.
For sets "A" to "D" as there are no suspended data between them, increments are equal to 1. Ranking order numbers for sets A, B, C and D are 1, 2, 3, 4 respectively. As sets "E" and "J" have been suspended, thus they won't have rank order numbers. The set "F" would have a decimal rank order number since there are suspended sets between it and the next early failure. The rank order number for set "F" is calculated as below:

$$RN_i = \frac{(200 + 1) - 4}{1 + 195} + 4 = 5.0051$$

Data of Table1 with all rank order values are shown in Table 2.

Table 2: Sample data with rank order values

Set	reverse ranking	Rank order number	Loops	Time (hour)	Failed/ Suspended
A	200	1	0	0	F
B	199	2	0	0	F
C	198	3	1	0.2	F
D	197	4	2	0.8	F
E	196	-	3	1	S
F	195	5.0051	6	1.3	F
G	194	6.0102	11	2.1	F
H	193	7.0153	24	5.8	F
I	192	-	25	7	S

J	191	8.0256	40	8.9	F
K	190	9.0359	69	12.7	F

Step5. In this step, for all of failed sets, median rank values will be added to Table2. An approximation for median rank values can be calculated as follow:

$$MR_i = \frac{RN_i - 0.3}{sample\ size + 0.4} \tag{13}$$

In which RN_i is the rank order number for the set which its median rank is calculated. Also sample size includes failed or suspended samples. So median rank value for "E" is calculated as follow:

$$MR_i = \frac{3.29 - 0.3}{10 + 0.4} = 28.8\ \%$$

All median rank values of data in table2 is shown in Table3:

Table 3. sample data with median rank values

Set	reverse ranking	Rank order number	Median rank	Loops	Time (hour)	Failed/ Suspended
A	200	1	0.0035	0	0	F
B	199	2	0.0085	0	0	F
C	198	3	0.0135	1	0.2	F
D	197	4	0.0185	2	0.8	F
E	196	-	-	3	1	S
F	195	5.0051	0.0235	6	1.3	F
G	194	6.0102	0.0285	11	2.1	F
H	193	7.0153	0.0335	24	5.8	F
I	192	-	-	25	7	S
J	191	8.0256	0.0386	40	8.9	F
K	190	9.0359	0.0436	69	12.7	F

Step6. In this step, 90% confidence limit band for median rank values is considered. This confidence band is consisted with a 5% lower confidence limit and a 95% upper confidence limit. Eqs (14) and (15) calculate the 5% and 95% confidence limits for non-decimal rank order numbers respectively. If there is a decimal rank order number, then the confidence limits will be calculated by interpolation of nearest upper and lower integer numbers. In this case study, the rank order number of "5.0051" is a result of interpolation of 5.0000 and 6.0000.

For 5% confidence limit we have:

$$CL_{5\%} = \frac{\frac{j}{n-j+1}}{[F_\alpha, 2(n-j+1), 2j] + \frac{j}{n-j+1}} \tag{14}$$

And for 95% confidence limit we have:

$$CL_{95\%} = \frac{\frac{j}{n-j+1} * [F_\alpha, 2j, 2(n-j+1)]}{1 + \frac{j}{n-j+1} * [F_\alpha, 2j, 2(n-j+1)]} \tag{15}$$

j : Rank order number (integer)

n : Number of set of samples

$[F_\alpha, v_1, v_2]$:Value of F_α when the area of distribution function is α at the right side of F_α

α :The right side area of F value

v_1 : Numerator degree of freedom

v_2: Denominator degree of freedom

We have distribution function of "F" as follows:

$$h(f) = \frac{\Gamma[(V_1+V_2)/2] * (V_1/V_2)^{(V_1/2)}}{\Gamma(V_1/2) * \Gamma(V_2/2)} * \frac{f^{\left(\frac{V_1}{2}-1\right)}}{(1+V_1 f/V_2)^{(V_1+V_2)/2}} \tag{16}$$

f : Independent variable in function

Γ :Gamma function

$$\Gamma(n) = \int_0^\infty X^{n-1} * e^{-X}\, dX \tag{17}$$

Gamma function for integer operands:

$$\Gamma(n) = (n-1)! \tag{18}$$

When combining Eqs.(17) and (18) we have:

$$h(f) = \frac{\left[\frac{(V_1+V_2)}{2} - 1\right]! * (V_1/V_2)^{(V_1/2)}}{\left(\frac{V_1}{2}-1\right)! * \left(\frac{V_2}{2}-1\right)!} * \frac{f^{\left(\frac{V_1}{2}-1\right)}}{(1+V_1 f/V_2)^{(V_1+V_2)/2}} \tag{19}$$

By integrating Eq. (19) in range of 0 to F_α, the area of "F" distribution curve is calculated. F_α is the value that specifies the result of 95% integration. This integration calculates the left side area of F_α. Due to integration complexity, we use numerical techniques (Trapezoidal method) to solve the following statement:

$$0.95 = \int_0^{F_\alpha} \frac{\left[\frac{(V_1+V_2)}{2} - 1\right]! * (V_1/V_2)^{(V_1/2)}}{\left(\frac{V_1}{2}-1\right)! * \left(\frac{V_2}{2}-1\right)!} * \frac{f^{\left(\frac{V_1}{2}-1\right)}}{(1+V_1 f/V_2)^{(V_1+V_2)/2}} * df \tag{20}$$

As an example, 5% and 95% confidence limits for set "B" we have:

$n = 200$: sample size

$j = 2$:Rank order number for set "B"

$$CL_{5\%} = \frac{\frac{2}{200-2+1}}{[0.05, 2(200-2+1), 2*2] + \frac{2}{200-2+1}} = 0.18\ \%$$

$$CL_{95\%} = \frac{\frac{2}{200-2+1} * [0.95, 2*2, 2(200-2+1)]}{1+\frac{2}{200-2+1} * [0.95, 2*2, 2(200-2+1)]} = 2.36\ \%$$

For set "F" (Rank order number=5.0051), we interpolate between "5" and "6". Then, 5% and 95% confidence limits for rank order numbers 5 and 6 is shown in Table4.

Table 4. Interpolation limits

	Rank order number of 5	*Rank order number of 6*
5%	*0.0099*	*0.0131*
95%	*0.0453*	*0.0519*

For a simple linear interpolation between two points, Eq. (21) is applied as follow:

$$y = y_1 + \frac{(y_2 - y_1)}{(x_2 - x_1)} * (x - x_1) \qquad (21)$$

Using above information, for 5% limit we have:

$(x_1, y_1) = (5, 0.0099)$

$(x_2, y_2) = (6, 0.0131)$

Using Eq (21) we have:

$$CL_{5\%} = 0.0099 + \frac{(0.0131 - 0.0099)}{(6-5)} * (x-5)$$

$$CL_{5\%} = 0.0032x - 0.0061$$

Interpolated 5% confidence limit for rank order number "5.0051" we have:

$$CL_{5\%} = 0.0032(5.0051) - 0.0061 = 0.99\ \%$$

Using above information, for 95% limit we have:

$(x_1, y_1) = (5, 0.0453)$

$(x_2, y_2) = (6, 0.0519)$

Using Eq (21):

$$CL_{95\%} = 0.0453 + \frac{(0.0519 - 0.0453)}{(6-5)} * (x-5)$$

$$CL_{5\%} = 0.0066x - 0.0123$$

Interpolated 95% confidence limit for rank order number "5.0051" is calculated as follow:

$$CL_{95\%} = 0.0066(5.0051) - 0.0123 = 4.53\ \%$$

Table5 shows data of Table3 with all 5% and 95% confidence limit values. Unnecessary information has been removed.

Table 5. Sample data with 5% and 95% confidence limits

Set	*Median rank*	$CL_{5\%}$	$CL_{95\%}$	*Loops*	*Time (hour)*	*Failed/ Suspended*
A	*0.0035*	*0.0003*	*0.0154*	*0*	*0*	*F*
B	*0.0085*	*0.0018*	*0.0263*	*0*	*0*	*F*
C	*0.0135*	*0.0041*	*0.0313*	*1*	*0.2*	*F*
D	*0.0185*	*0.0069*	*0.0385*	*2*	*0.8*	*F*
E	*-*	*-*	*-*	*3*	*1*	*S*
F	*0.0235*	*0.0099*	*0.0453*	*6*	*1.3*	*F*
G	*0.0285*	*0.0131*	*0.052*	*11*	*2.1*	*F*
H	*0.0335*	*0.0166*	*0.0585*	*24*	*5.8*	*F*
I	*-*	*-*	*-*	*25*	*7*	*S*
J	*0.0386*	*0.0201*	*0.0649*	*40*	*8.9*	*F*
K	*0.0436*	*0.0237*	*0.0716*	*69*	*12.7*	*F*

Step7. In order to smooth data and determine whether sample data fit Weibull distribution or not, it is better to convert data obtained from samples and Weibull reliability function in a way that makes converted data be drawn in a straight line form. Since Weibull reliability function is a quadratic exponential function, it is necessary to get inverse logarithm from both sides of the equation twice in order to linearize it. Using the main function we have:

$$R(t) = 1 - F(t) = exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \xrightarrow{1-F(t)=1-MR_i} 1 - MR_i = exp\left[-\left(\frac{t}{\eta}\right)^{\beta}\right] \qquad (22)$$

After simplifying the above statement we will have:

$$\ln\ln[1/(1-MR_i)] = \beta \ln(t) - \beta \ln(\eta) \qquad (23)$$

MR_i :Median rank values

t: Time

β: Shape parameter (It should calculated from failure data)

η: Characteristic's life (It should calculated from failure data)

Also, we use below equations to convert median rank, time and confidence limits before depiction as follows:

$$Y = \ln\ln\left[\frac{1}{1-MR_i}\right] \qquad (24)$$

$$X = \ln(t) \qquad (25)$$

$$Y_{5\%} = \ln\ln\left[\frac{1}{1-CL_{5\%}}\right] \qquad (26)$$

$$Y_{95\%} = \ln\ln\left[\frac{1}{1-CL_{95\%}}\right] \qquad (27)$$

In which:

MR_i :Median rank

t : time to fail

$CL_{5\%}$: 5% confidence limit

$CL_{95\%}$:95% confidence limit

Sample data after conversion are shown at Table 6. Unnecessary columns have been removed. In addition, failure data for t=0 and zero loop or suspended will not be used in Weibull depiction. Hence, conversions for these cases have not been done.

Table 6. Sample and converted data

Set	Median rank	Y	$CL_{5\%}$	$Y_{5\%}$	$CL_{95\%}$	$Y_{95\%}$	Loops	$X=\ln(t)$	Time (hour)
A	0.0035	-	0.0003	-	0.0154	-	0	-	0
B	0.0085	-	0.0018	-	0.0263	-	0	-	0
C	0.0135	-4.289	0.0041	-5.495	0.0313	-3.448	1	-1.61	0.2
D	0.0185	-3.981	0.0069	-4.973	0.0385	-3.238	2	-0.22	0.8
E	-	-	-	-	-	-	3	-	1
F	0.0235	-3.739	0.0099	-4.61	0.0453	-3.071	6	0.26	1.3
G	0.0285	-3.543	0.0131	-4.329	0.052	-2.93	11	0.74	2.1
H	0.0335	-3.379	0.0166	-4.09	0.0585	-2.809	24	1.76	5.8
I	-	-	-	-	-	-	25	-	7
J	0.0386	-3.235	0.0201	-3.897	0.0649	-2.702	40	2.19	8.9
K	0.0436	-3.11	0.0237	-3.73	0.0716	-2.6	69	2.54	12.7

Step8. The best line fit for median rank points (when time and loop are greater than zero) will be determined using linear regression (Least squares method) [15]. Resulted line has a familiar shape as below:

$$Y = AX + B \qquad (28)$$

In which the values of A and B are calculated as follows:

$$A = \frac{n\sum X_i Y_i - \sum X_i \sum Y_i}{n\sum X_i^2 - (\sum X_i)^2} \qquad (29)$$

$$B = \frac{\sum Y_i \sum X_i^2 - \sum X_i \sum X_i Y_i}{n\sum X_i^2 - (\sum X_i)^2} \qquad (30)$$

In which:

n : allocated number to the data point

X_i : X value for data points (i= 1 to n)

Y_i : Y value for data points (i= 1 to n)

Data points for linear regression and the total data for linear regression are presented in Tables 7 and 8 respectively.

Table 7. Data points for linear regression

i	X_i	Y_i
1	-1.61	-4.289
2	-0.22	-3.981
3	0.26	-3.739
4	0.74	-3.543
5	1.76	-3.379
6	2.19	-3.235
7	2.54	-3.11

Table 8. Total data for linear regression

i	X_i	Y_i	X_iY_i	X_i^2
1	-1.61	-4.289	6.9198	2.5921
2	-0.22	-3.981	0.8758	0.0484
3	0.26	-3.739	-0.9721	0.0676
4	0.74	-3.543	-2.6218	0.5476
5	1.76	-3.379	-5.947	3.0976
6	2.19	-3.235	-7.0847	4.7661
7	2.54	-3.11	-7.8994	6.4516
$\sum$	5.66	-25.285	-16.7294	17.601

$$A = \frac{7 * (-16.7294) - (5.66 * (-25.285))}{(7 * 17.6010) - (5.66)^2} = 0.285$$

$$B = \frac{(-25.285 * 17.6010) - (5.66 * (-16.7294))}{(7 * 17.6010) - (5.66)^2} = -3.843$$

Accordingly:

$Y_r = 0.285X - 3.843$

After applying linear regression to depict confidence limits, new X values can be calculated using Y values. To this order, the linear regression for *x* variable must be solved as follow:

$$X_r = \frac{Y - B}{A} = \frac{Y + 3.843}{0.285}$$

Step9. Using linear regression statement, in order to depict data on a chart, a new set of points will be added to the table of data values. The values of X & Y ($\ln\ln\left[\frac{1}{1-MR_i}\right]$) are used to calculate another set of X values (X_r) which helps us to draw the regression line on the chart. These data and the other data which are necessary to draw the cart are represented in Tables 9 and 10 respectively.

Table 9. new data points to draw regression line

Set	Y	X_r
A	-	-
B	-	-
C	-4.289	-1.596
D	-3.981	-0.484
E	-	-
F	-3.739	0.365
G	-3.543	1.053
H	-3.379	1.628

I	-	-
I	*-3.235*	*2.133*
J	*-3.11*	*2.572*

Table 10. Necessary data to draw

X	X_r	Y	$Y_{5\%}$	$Y_{95\%}$
-۱/٦۱	-۱/٥۹٦	-٤/۲۹۸	-٥/٤۹٥	-۳/٤٤۸
-۰/۲۲	-۰/٤۸٤	-۳/۹۸۱	-٤/۹۷۳	-۳/۲۳۸
۰/۲٦	۰/۳٦٥	-۳/۷۳۹	-٤/٦۱	-۳/۰۷۱
۰/۷٤	۱/۰٥۳	-۳/٥٤۳	-٤/۳۲۹	-۲/۹۳
۱/۷٦	۱/٦۲۸	-۳/۳۷۹	-٤/۰۹	-۲/۸۰۹
۲/۱۹	۲/۱۳۳	-۳/۲۳٥	-۳/۸۹۷	-۲/۷۰۲
۲/٥٤	۲/٥۷۲	-۳/۱۱	-۳/۷۳	-۲/٦

The columns of Table 10 which are used to draw Weibull graph are given in Tabe11. In general, Weibull graph has the following parts:

- A set of points
- A regression line
- Upper confidence limit curve
- Lower confidence limit curve

Weibull graph with all of the above mentioned parts is presented in Fig. 2.

Table 11. Needed X & Y columns

	X axis	*Y axis*
Set of points	X	Y
Regression line	X_r	Y
Upper confidence limit	X_r	$Y_{95\%}$
Lowe confidence limit	X_r	$Y_{5\%}$

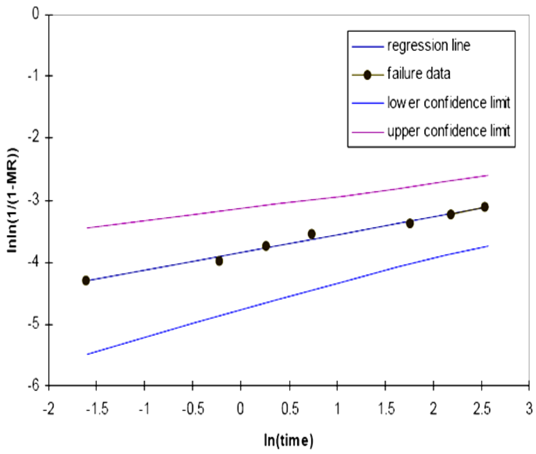

Fig. 1: Weibull graph

When data points are close to regression line, it is possible to describe data behavior using Weibull distribution. Confidence limits demonstrate the accuracy of estimation according to drawn points. The more drawn data points give the closer confidence limits for regression line. There are two ways in application of confidence limits. The first application is to calculate the range for expected failure percentage in a specific time. To this order, a vertical line which cuts off "ln (time)" axis and confidence limit curves must be drawn. Then horizontal lines from each of intersection points with confidence limit curves to the "y" axis must be drawn. Now there are two resulted "y" values. Using Eq. (25) these values must be converted to median rank values.

$$rank\ \% = 100 * (1 - 1/\exp(\exp(y))) \quad (31)$$

y : the value on "y" axis where horizontal line has intersected it.

$rank\ \%$: cumulative failures

Rank percentages ($rank\ \%_{low}$ & $rank\ \%_{high}$) indicate that for a specific screening time with 90% confidence limit, the society failure percentage will happen between $rank\ \%_{low}$ and $rank\ \%_{high}$.

The second application of confidence limits is to calculate ranges for expected time of a specific failure percentage. To do so, horizontal line must be drawn in a way that cuts off the cumulative failure percentage (converted by $\ln\ln\left[\frac{1}{1-MR}\right]$) and both confidence limit curves. Then vertical line must be drawn from each of the intersection points of confidence limit curves to "x" axis. Using Eq. (26) these values are converted to time values.

$$time = \exp(x) \quad (32)$$

x : the value on "x" axis where vertical line has intersected it

$time$: screening time

Time values ($time_{low}$&$time_{high}$) indicate that the failure percentage with 90% confidence will happen in range of $time_{low}$ and $time_{high}$.

Step10. In this step using linear regression the values of β **&** η can be calculated according to following steps: According to the previous steps, the values of A and B in the linear regression is calculated as follow:

$Y_r = AX + B$

$A = 0.285 B = -3.843$ ،

Also the converted Weibull function is as follow:

$$\ln\ln[1/(1 - MR)] = \beta \ln(t) - \beta \ln(\eta) \qquad (33)$$

By comparing above statement with linear regression, it is concluded that:

$$Y_r = \ln\ln\left[\frac{1}{1 - MR}\right], A = \beta, X = \ln(t), B = -\beta\ln(\eta) \qquad (34)$$

So, the value of β is calculated using the above statement. At the other hand, the parameter of η or life criterion is equivalent to the condition in which *F(t)* = 0.632 that means 63.2% of society fail to operate. Since $R(t) = 1 - F(t)$ and $1 - MR = R(t)$, Then *R(t)* can be calculated as follow:

$R(t) = 1 - 0.632 \Rightarrow 1 - MR = 0.368$

By replacing above statement in Eq (33):

$$\ln\ln\left[\frac{1}{0.368}\right] = \beta\ln(t) - \beta\ln(\eta) \Rightarrow$$

$$0 = \beta\ln(t) - \beta\ln(\eta)$$

After solving the above equation for η, we will have:

$$\beta\ln(t) = \beta\ln(\eta) \Rightarrow \ln(t) = \ln(\eta) \Rightarrow \eta = t \qquad (35)$$

Using linear regression statement, the values of *X*, *t* and finally η can be calculated as follows:

$$0 = AX + B \Rightarrow X = -\frac{B}{A} \Rightarrow X = 13.484$$

As we have the following relationship between *X* and *t*:

$$\mathrm{Ln}(t) = X \Rightarrow t = e^X \Rightarrow t = e^{13.484} \Rightarrow$$

$t = 717989$

Therefore the value of η is calculated as follow:

$\eta = 717989$

Step11. After calculation of β and η, Weibull hazard rate function can be obtained with accuracy. Then, Weibull hazard rate function is used to create a set of points to draw Weibull hazard function as below.

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{(\beta-1)} = \frac{0.285}{717989}\left(\frac{t}{717989}\right)^{(0.285-1)} \qquad (36)$$

In hazard rate function, horizontal axis (*x*) is over time and vertical axis (*y*) is over momentary hazard rate (*h(t)*). We will not assess the hazard rate values when t=0. Data which are used to draw Weibull hazard function and finally hazard rate chart are given in Table 12 and Fig 3 respectively.

Table 12. Data points to draw Weibull hazard rate

Set	*Time (hour)*	$h(t)$
A	-	-
B	-	-
C	۰/۲	۰/۰۱۹۳۰۳
D	۰/۸	۰/۰۰۷۱٦٤
E	۱	۰/۰۰٦۱۰۷
F	۱/۳	۰/۰۰۵۰٦۳
G	½	۰/۰۰۳۵۹۳
H	۵/۸	۰/۰۰۱۷۳۸
I	۷	۰/۰۰۱۵۱۹
J	۸/۹	۰/۰۰۱۲۷۹
K	۱۲/۷	۰/۰۰۰۹۹۲

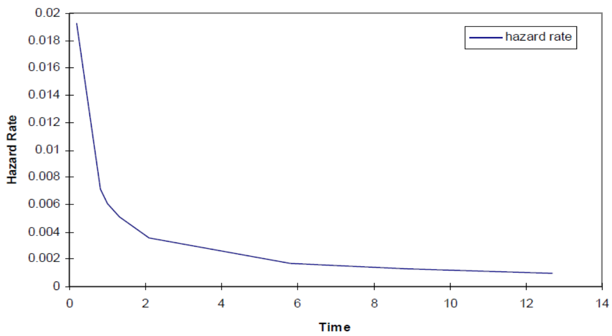

Fig. 2: hazard rate chart

Step12. In this step Weibull hazard rate is used in order to calculate the time of screening in a screening process. There are two methods to calculate the screen time. The first method is based on using the mean time between failure values of products (MTBF). MTBF is actually the converse of the hazard rate which is modeled by Weibull function when the shape parameter is equal to 1. The end of screening is when the descending exponential statement of hazard rate function ($\beta < 1$) intersects with a horizontal line describing 1/MTBF. This method can lead to a long and unreasonable screening time.

The other method considers the end of screening time that is a moment in which the slope of hazard rate curve inclines to a small negative number. This small number indicates that the hazard rate remains constant approximately. The smaller negative number concludes the longer screening time. Generally, values which are equal or less than "-0.00005" are used as small negative number for slope of curve.

Finally, in order to find the proper screening time for a specific hazard rate, differential of the hazard rate statement must be taken and solved over the time. To this order, hazard rate function is given as follows:

$$h(t) = \frac{\beta}{\eta}\left(\frac{t}{\eta}\right)^{(\beta-1)} \qquad (37)$$

The differential of the hazard rate function is calculated as follow:

$$\frac{dh}{dt} = h'(t) = \frac{\lfloor \beta * (\beta - 1) * t^{(\beta-2)} \rfloor}{\eta^\beta} \qquad (38)$$

After solving the differential of hazard rate function over time, the following statement is obtained:

$$t = \left[\frac{(\eta^{\beta} * \acute{h}(t))}{(\beta * (\beta - 1))}\right]^{\frac{1}{\beta-2}} \quad (39)$$

In which:

β: Constant (shape parameter calculated from failure data)

η: Constant (life characteristic calculated from failure data)

$\acute{h}(t)$: of hazard rate (a small negative number)

t : Screening time

Finally, using the above statement for *t*, the optimal screening time for sample data in the presented case study can be calculated as follow:

$$t = \left[\frac{(717989^{0.285} * (-0.00005))}{(0.285 * (0.285 - 1))}\right]^{\frac{1}{0.285-2}} \Rightarrow \quad (40)$$

$$t = 13.55\ Hour$$

Step13. A very complicated and accurate method to determine the screening time is Bayesian analysis [16-18]. Exerting Bayesian analysis, failure probability assessment for weak subpopulation and main population will be used to find the cumulative failure percentage which has been demolished at that population. This percentage value after conversion will be replaced in regression formula in order to determine the proper screening time.

Weak subpopulation parameters are β_1 as shape parameter and η_1 as characteristic life (derived from Weibull analysis). Main population parameters are β_2 as shape parameter for constant hazard rate of "*-1*" and η_2 as mean time to failure (MTTF) which is considered 100000 hours.

Weak population failure probability "*i*" is as follow:

$$P_i = \frac{\frac{f_1}{t_i}}{\frac{f_1}{t_i} + \frac{f_2}{t_i}} \quad (41)$$

In which there are the following statements:

$$\frac{f_1}{t_i} = \frac{\beta_1}{\eta_1} exp\left[-\left(\frac{t_i}{\eta_1}\right)^{\beta_1}\right]\left(\frac{t_i}{\eta_1}\right)^{(\beta_1-1)} \quad (42)$$

$$\frac{f_2}{t_i} = \frac{\beta_2}{\eta_2} exp\left[-\left(\frac{t_i}{\eta_2}\right)^{\beta_2}\right]\left(\frac{t_i}{\eta_2}\right)^{(\beta_2-1)} \quad (43)$$

In which:

t_i : time left to failure of a member in the failed society.

By combination of three above equations, we have:

$$P_i = \frac{\frac{\beta_1}{\eta_1} exp\left[-\left(\frac{t_i}{\eta_1}\right)^{\beta_1}\right]\left(\frac{t_i}{\eta_1}\right)^{(\beta_1-1)}}{\left[\frac{\beta_1}{\eta_1} exp\left[-\left(\frac{t_i}{\eta_1}\right)^{\beta_1}\right]\left(\frac{t_i}{\eta_1}\right)^{(\beta_1-1)}\right] + \left[\frac{\beta_2}{\eta_2} exp\left[-\left(\frac{t_i}{\eta_2}\right)^{\beta_2}\right]\left(\frac{t_i}{\eta_2}\right)^{(\beta_2-1)}\right]} \quad (44)$$

On the other hand, Subpopulation ratio is given as follow:

$$\bar{P} = \frac{\sum_i P_i}{N} \quad (45)$$

By combination of Eqs (44) and (45), following statement is obtained:

$$\bar{P} = \frac{\sum_i \left[\frac{\frac{\beta_1}{\eta_1} exp\left[-\left(\frac{t_i}{\eta_1}\right)^{\beta_1}\right]\left(\frac{t_i}{\eta_1}\right)^{(\beta_1-1)}}{\left[\frac{\beta_1}{\eta_1} exp\left[-\left(\frac{t_i}{\eta_1}\right)^{\beta_1}\right]\left(\frac{t_i}{\eta_1}\right)^{(\beta_1-1)}\right] + \left[\frac{\beta_2}{\eta_2} exp\left[-\left(\frac{t_i}{\eta_2}\right)^{\beta_2}\right]\left(\frac{t_i}{\eta_2}\right)^{(\beta_2-1)}\right]}\right]}{N} \quad (46)$$

Where:

N: Sample size (approved, rejected, and suspended units)

$\bar{P}$: sample proportion of weak subpopulation

β_1: Shape parameter calculated from Weibull analysis

η_1: Characteristic life calculated from Weibull analysis

β_2 : shape parameter for constant hazard rate of 1

η_2: Product MTTF = 100000 hours

t_i : time left to the failure of a member in failed society

The numerical values of $\frac{f_1}{t_i}$ ، $\frac{f_2}{t_i}$ and P_i for different failure times are given in Table13.

Table 13. Median rank for Bayesian analysis

Failure time	$\frac{f_1}{t_1}$	$\frac{f_2}{t_1}$	P_1
0	*0*	*0*	*1*
0	*0*	*0*	*1*
0.2	*0.019043*	*0.00001*	*0.999475*
0.8	*0.007021*	*0.00001*	*0.998578*
1	*0.004947*	*0.00001*	*0.997983*
1.3	*0.003499*	*0.00001*	*0.99715*
2.1	*0.001677*	*0.00001*	*0.994072*
5.8	*0.001229*	*0.00001*	*0.991929*
7	*0.00944*	*0.00001*	*0.989572*

Conclusion

Wibull distribution function is able to simulate the failure behavior during a routine mission or under accelerated conditions effectively. In this paper, drawing probability plot method is applied to determine Weibull parameters accurately. This method is the most proper one to show Weibull function's variety and functionality. To this order, actual failure behavior is applied to determine shape and life characteristic parameters of Weibull function empirically. After determination of Weibull parameters, the hazard rate function is obtained. Ultimately, using this hazard function, the optimal exposure time for an environmental stress screening process is calculated. Moreover, a case study of electronic sets is applied to prove each step of the process. For further investigation, using other statistical distributions like lognormal distribution and also considering more practical conditions in calculating the failure rate of components can be interesting subjects for researchers.

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