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Hybrid flow shop scheduling with robotic processing and AGV-based transportation system

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Abstract

This research, considers a flow shop manufacture line which has unrelated parallel machines at each stage. Machines numerically can vary in each stage and all machines are not able to process all parts. Each part must be assigned only to one machine in each stage and parts should not be waiting at machines (no buffer). Materials loading, transportation, unloading and packing are done by robots. In other words, transportation is done by Automated Guided Vehicles (AGV) and all tasks of parts (loading, unloading, packing and setup) on machines are done by Robotic Adaptive Grippers. It is not obligatory for one kind of parts to be processed on one machine. In other words, parts can be divided into parts families (Lot Split). The main contribution of this study is devising a mixed integer linear model which will be examined in an examples using GAMS software.

Keywords:
Hybrid Flow Shop, Parallel Unrelated Machines, Robotic Transportation, Lot Split, Makespan, Integer Linear Model, Automated Guided Vehicles, Entrance Tardiness, Exit Tardiness, Robotic Adaptive Grippers

1. Introduction

Technology-dependent advance caused emergence of the automatic devices in processing, inspecting and transporting fleet which play key roles in minimizing not only the overall cost, but also in total time. Hence, to meet the goals, a systematic plan must be adopted to employ the robot capability. In many assembly and manufacture factories, type and sequence of operations are equal for all products and parts. Therefore, all parts should pass the identical route to end up the process. This sequence of machines is referred as flow shop (FS). However, traditional flow shop could be applied in low capacity factories and this is why the hybrid flow shop (HFS) introduced. As a matter of fact, in the hybrid flow shop within each stage the number of machines can surpass one, parts backward movement is not allowed and machines are placed parallel which can be related / unrelated. Due to the fact that the hybrid flow shop system needs be completely accurate to avoid any mistakes in scheduling, robots are proposed as the best solution. Robotic equipment in this system can not only undertake the transportation, but also perform machine setup, loading and unloading. Robotic manufacturing environment can tremendously decrease manpower costs, increasing accuracy and lowering contamination in manufacturing. Moreover, robots have capability of working in harsh environments which are to some extent hard for human being.

In this study, we introduce an Integer linear mathematical model that is rather dissimilar to the hybrid flow shop classic model. In the following section we are going to mention our model general characteristics. firstly, we included transportation calculation accurately in this model which is a very striking factor in simulating real situation. Secondly, lot splitting (parts families) occurs. In other words, parts family takes place where identical parts can be divided into separate identities or different parts with similar process, so we put them in same parts family and it is noticeable that there is no setup time among processing sequence of parts from same family. Thirdly, travel rounds are allocated to AGVs which will be specified by the management of the factory (maximum number of travels permitted to each AGV). Fourthly, parts cannot be waiting at machines before and after being processed (no buffer). The objective is finding optimal makespan and optimum number of travels (minimizing travels time). To better understand the model's function, we provide an examples and solve that with GAMS software using CPLEX solver.

The HFS problems mostly are NP-hard. For example, a production line with two stages is considered even in the case of one stage involves two machines and the other one contains only a machine, it is NP-hard which is proved by Gupta and Tunc [1]. In another variant of HFS, Hunsucker and Shah [2] considered that system is permitted to cease the operations on parts before their completions occurred in order to resume them on different time frame which in turn identically strongly is NP-hard even with two machines. Nonetheless, Engin and Doyen [3] showed that with some specific properties and priority connections, the problem may be polynomially solvable.

As other variants, there are several researches that have been done on the hybrid flow shop. Hurink and Knust [4] assumed an unrestricted buffer space and negligible empty moving times in flow shop scheduling with transportation problem. Engin *et al.* [5] minimized the makespan in the hybrid flow shop scheduling with multiprocessor task (HFSMIT) problem which is NP-hard and they introduced an efficient Genetic Algorithm to solve it. Pan and Haung [6] minimized the make span with consideration of no-wait job shop scheduling problems. They introduced NP-hard problem which is solved by a hybrid genetic algorithm. Marichelvam *et al*. [7] showed the multistage hybrid flow shop (HFS) problem and proved it is NP-hard. In the Study done by Soukhal *et al*. [8] it has been shown that even in condition of two machines with transportation and blocking, the problem is strongly NP-hard. Kashyrskikh *et al*. [9] studied on two-machine flow shop sequencing problem with minimum makespan criterion and arbitrary jobs release times. This study proved that problem in this state is NP-hard. Cheng *et al*. [10] investigated a problem of three-machine permutation flow shop scheduling with release where the objective is makespan. They synthesized an adaptive branching rule with a fuzzy search strategy to diminish the search area which lead to optimal solution as soon as possible. Sangsawang *et al*. [11] considered two-stage reentrant flexible flow shop (RFFS) with blocking constraint that the objective is minimizing the makespan. They applied hybrid genetic algorithm (HGA) with adaptive auto-tuning based on fuzzy logic controller and hybrid particle swarm optimization (HPSO) (Cauchy distribution) to solve the problem. Behnamian and Fatemi Ghomi [12] considered a PSO-SA hybrid meta-heuristic for a new comprehensive regression model to time-series predicting. Moslehi and Khorasanian [13] used blocking flow shop scheduling problem for minimizing the total completion time criterion. They introduced two mixed binary integer programming models, the first one is modeled based on the departure times of jobs from machines and the second is modeled based on the idle and blocking times of jobs. Wang *et al*. [14] to solve the blocking permutation flow shop scheduling problem with total flow time criterion proposed a hybrid modified global-best harmony search (hmgHS) algorithm. Zandieh and Karimi [15] concurrently minimized the total weighted tardiness and the maximum completion time with consideration of a multi-objective group scheduling in a hybrid flexible flow shop setting with sequence dependent setup times. They applied a multi-population genetic algorithm for the problem and juxtaposed it with the multi-objective genetic algorithm and the non-dominated sorting genetic algorithm. Wang *et al*. [16] suggested a novel hybrid discrete differential evolution (HDDE) algorithm to minimize makespan of blocking flow shop scheduling problems. Behnamian and Zandieh [17] minimized earliness and quadratic tardiness in the hybrid flow shop scheduling problem. They assumed each stage minimally has one machine. On the other hand, at least one of stages possesses more than one machine. Furthermore, there is no release time for jobs which are independent and each job must be allocated at most to one machine within one stage.

2. Problem description and formulation

In this study a hybrid flow shop system is considered which consists *p* parts that should be processed at *s* stages and all the *p* parts should pass all stages. In addition, *m* is attributed to stations within stages, except station of the first stage which is storehouse, the rest of stages incorporate $\mathrm{M_s}$ stations ($M_s \geq 1$), each station contains parallel processing machines which are supplied by *a* AGVs. All tasks on machines such as loading, unloading and setup are performed by Robotic Adaptive Grippers . Dissimilar to other works in this category, we considered limited *t* travels for AGVs (permitted travels). In other words, AGVs cannot do travel more than allowed ones this assumption is made firstly, because in the real situation transportation fleet are not allowed to do travel unlimitedly. Secondly, if we put the assumption of unlimited travels, it would increase computation of the model tremendously and unnecessarily. Therefore, donating limited travels to AGVs does make sense. Each *p* part which is transferred by one of the AGVs into each stage should be processed on one machine and consequently, each of *p* part should be transferred by one *a* AGV at one of its *t* travels. There is no buffer between stages. So, *p* parts cannot be stayed at machines and after they have been processed they should immediately be carried by AGV. Figure 1. illustrates a hybrid flow shop system with AGV-based transportation. Colorful rectangles show stations within stages in which rectangles with white interior illustrate related parallel machines and rectangles with grey interior demonstrate unrelated parallel machines and their borders color corresponds to the colors of stages in which they are installed. The black lines present routes on which AGVs can move forward (to convey loaded parts) and backward (to reload again). The green arrow heads on black routes show forward movements are allowed and the orange arrow heads on black routes show backward movements are allowed. Colorful lines (corresponding to the stages they stem out from) suggest that just backward movements are authorized and due to this they are specified by just orange arrow heads (because orange arrow symbolizes backward movement). The ideology behind this is that some routes (backward only) cannot be used for forward movements since forward movements can be implemented just to next stage (all parts should pass all stages) and in ***"just backward*** movements" routes AGVs can go to all machines regardless of that in which stage they are existed. Moreover, backward movement can be attributed to in-stage movements among interior-stage-machines in order to load parts and unload them in next stage.

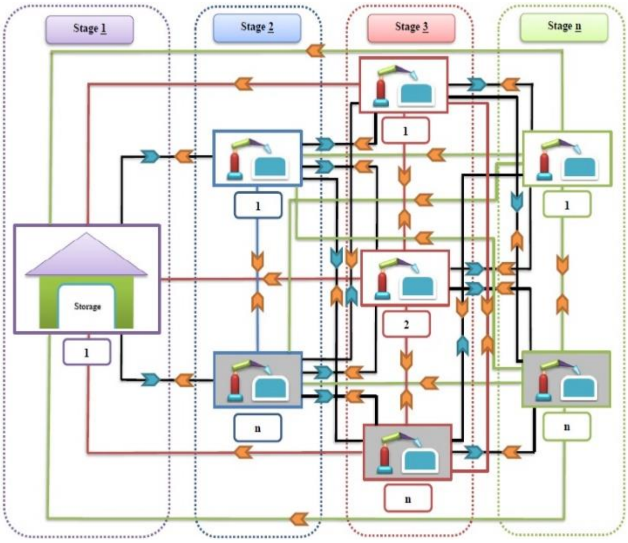

Figure 1- An example of Hybrid Flow Shop with robotic processing and AGV-based transportation system

2.1. The Mixed Integer linear programming (MILP) Model

In this section, a MILP model is formulated for Hybrid flow shop problem with Robotic Adaptive Gripper-based processing and AGV-based transportation system to minimize makespan and optimum number of travels (minimizing travels time). The developed model is discussed in the following.

2.2. Assumptions

- There is at least one machine at each stage.
- No part is exchanged between parallel machines (an AGV cannot go from a machine to another parallel machine unless for loading parts).
- All the required parts for being processed are provided entirely in storage.
- All parts should pass all stages.
- Sum of the loading and unloading time for each part is shown by one parameter (LT_p).

2.3. Indices

P: Set of parts where,
p: Index of part, $p \in \{1,2,\dots,P\}$
F: Set of parts families where,
f: Index of parts family, $f \in \{1,2,\dots,F\}$
S: Set of stages where,
s: Index of stages, $s \in \{1,2,\dots,S\}$
e: Index of last stage
A: Set of AGVs where,
a: Index of AGVs, $a \in \{1,2,\dots,A\}$
T: Number of travels for AGVs where,
t: Index of travels, $t \in \{1,2,\dots,T\}$
h: Index of last travel.
M: Set of machines with Robotic Adaptive Grippers where,
m:Index of machines with Robotic Adaptive Grippers, $m \in \{1,2,\dots,M\}$.
M_s *:* Set of machines with Robotic Adaptive Grippers existing in stage *s*.

2.4. Parameters

CAP_a Capacity of automated guided vehicles (AGV) *a*.
$ST_{p,m}$ Required setup time for part *p* on machine with Robotic Adaptive Gripper *m* .
$PT_{p,m}$ Required time for processing part *p* on machine with Robotic Adaptive Gripper *m* .
$FM_{m',m,s,a}$ Required time for AGV *a* to travel from existed machine m' to machine m in the stage s (forward movement).
$BM_{m,m',a}$ Required time for AGV *a* to travel from machine m to machine m' (backward movement).
LT_p Sum of the Required time for Robotic Adaptive Grippers to load and unload each part *p* on AGV.
D Total numbers of parts a last stage.
H A large number.

2.5. *Binary parameters*

$RRR_{m',m,s,a,t}$ Showing if AGV a is at machine m', to which next machine m at the stage s is permitted to go (it prevents assigning infeasible allocations of machine to stage).
$Q_{f,p}$ Showing which parts *p* are included to part family *f*
$PA_{p,m}$ Process ability, showing which machines *m* can process part *p*.

2.6. Positive variables

$RT_{m',m,s,a,t}$ The time (receipt time) at which AGV's load is received at machine m (located in stage *s*) by its t^{th} travel started from machine m' (located in stage *s-1*).
$PC_{p,m',m,s,a,t}$ The process completion time of part *p* on machine m which is transferred from machine m' (located in stage *s-1*) to machine m (located in stage *s*) by AGV *a* at its t^{th} travel.
$LPCT$ The time at which all parts reach the output stage, makespan.
$ET_{p,m',m,s,a,t}$ The waiting interval that part *p* may have at machine m to be processed on (Entrance Tardiness).
$EXT_{p,m',m,s,a,t}$ The waiting interval that part *p* may have at machine m (after being processed) to be transferred from (Exit Tardiness).

2.7. Binary variables

$w_{m',m,s,a,t}$ 1 if AGV a at its t^{th} travel goes from machine m' to machine m, 0 otherwise.

$Y_{p,m',m,s,a,t}$ 1 if AGV *a* at its t^{th} travel transfers part *p* from machine m' to machine m, 0 otherwise.

$U_{m,m',a,t}$ 1 if AGV a goes from machine m to machine m' at its t^{th} travel's return to reload again, 0 otherwise.

$SS_{p,p',s}$ 1 if part p be processed before part p' in stage s, 0 otherwise.

2.8. Objective function and constraints

Minimize:

$$LPCT + \sum_{m'\in M_{s-1}} \sum_{m\in M_s} \sum_{a\in A} \sum_{s\in S} \sum_{t\in T} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \times FM_{m',m,s,a}$$

$$\sum_{s\in S-\{1\}} \sum_{m'\in M_{s-1}} \sum_{m\in M_s} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \leq 1 \qquad \forall \quad t\in T \ , a\in A \qquad (1)$$

$$\sum_{m'\in M_{s-1}} \sum_{m\in M_s} \sum_{t\in T} \sum_{a\in A} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \leq 1 \qquad \forall \quad p\in P \ , s\in S-\{1\} \qquad (2)$$

$$\sum_{m'\in M_{s-1}} \sum_{m\in M_s} \sum_{s\in S-\{1\}} \sum_{p\in P} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} \leq CAP_a \qquad \forall \quad a\in A \ , t\in T \qquad (3)$$

$$\sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} \leq \sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad \forall \quad p\in P \ , a\in A \ , m\in M_s \ , t\in T \qquad (4)$$

$$\sum_{m\in M_s} \sum_{s\in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} \leq \sum_{m\in M_s} \sum_{s\in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad \forall \quad p\in P \ , a\in A \ , m'\in M_{s-1} \ , t\in T \qquad (5)$$

$$\sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} \times H \geq \sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad (6)$$

$$\forall \quad p\in P \ , a\in A \ , t\in T \ , m\in M_s$$

$$\sum_{m\in M_s} \sum_{s\in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} \times H \geq \sum_{m\in M_s} \sum_{s\in S-\{1\}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad (7)$$

$$\forall \quad p\in P \ , a\in A \ , t\in T \ , m'\in M_{s-1}$$

$$\sum_{m\in M_s} \sum_{s\in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \times H \geq \sum_{m\in M_s} \sum_{s\in S-\{1\}} RT_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad (8)$$

$$\forall \quad a\in A \ , t\in T \ , m'\in M_{s-1}$$

$$\sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \times H \geq \sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} RT_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \qquad (9)$$

$$\forall \quad a\in A \ , t\in T \ , m\in M_s$$

$$\sum_{m\in M_s} \sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \leq \sum_{m\in M_s} \sum_{m'\in M_{s-1}} \sum_{s\in S-\{1\}} W_{s,m,m',a,t-1} \times RRR_{s,m,m',a,t-1} \qquad (10)$$

$$\forall \quad t\in T \ , t>1 \ , a\in A$$

$$M \times \Big(2 - \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} - \sum_{m' \in M-\{m\}} \sum_{m \in M_s} U_{m,m',a,t-1}\Big) + \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} RT_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} RT_{m',m,s,a,t-1} \times RRR_{m',m,s,a,t-1} + \sum_{m' \in M-\{m\}} \sum_{m \in M_s} U_{m,m',a,t-1} \times BM_{m,m',a} + \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \times FM_{m',m,s,a} + \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} \sum_{p \in P} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times LT_p \quad \forall \; t > 1, a \in A \tag{11}$$

$$M \times \Big(1 - \sum_{m \in M_s} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m}\Big) + \sum_{m \in M_s} RT_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m'' \in M_{s-2}} PC_{p,m'',m',s-1,a',t'} \times RRR_{m'',m',s-1,a',t'} + \sum_{m \in M_s} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \times FM_{m',m,s,a} + \sum_{m \in M_s} \sum_{p' \in P} Y_{p',m',m,s,a,t} \times RRR_{m',m,s,a,t} \times LT_{p'}$$

$$\forall \; t,t' \in T, a,a' \in A, s \in S-\{1,2\}, p \in P, m' \in M_{s-1} \tag{12}$$

$$\sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} + \Big(1 - \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m}\Big) \times H \geq \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} RT_{m',m,s,a,t} \times RRR_{m',m,s,a,t} + \sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{s \in S-\{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PT_{p,m}$$

$$\forall \; p \in P, \; a \in A, \; t \in T \tag{13}$$

$$\sum_{m' \in M} \sum_{m \in M-\{1\}, m \neq m'} U_{m,m',a,t} \leq 1 \quad \forall \; a \in A, t \in T, t < h \tag{14}$$

$$\sum_{m \in M_s} \sum_{s \in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \leq \sum_{m'' \in M, m'' \neq m'} U_{m'',m',a,t-1} \quad \forall \; a \in A, \; t \in T, \; t > 1, m' \in M_{s-1} \tag{15}$$

$$\sum_{m' \in M_{s-1}} \sum_{s \in S-\{1\}} W_{m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m'' \in M, m \neq m''} U_{m,m'',a,t} \quad \forall \; a \in A, \; t \in T, \; t < h, m \in M_s \tag{16}$$

$$\sum_{m' \in M_{s-1}} \sum_{m \in M_s} \sum_{a \in A} \sum_{t \in T} \sum_{p \in P} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PA_{p,m} = D \quad \forall \; s = e \tag{17}$$

$$M \times \Big(2 - \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times Q_{f,p} - \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times Q_{f,p'}\Big) + (1 - SS_{p,p',s}) \times H + \sum_{m' \in M_{s-1}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m'' \in M_{s-1}} PC_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} + \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PT_{p,m} \tag{18}$$

$$\forall \; a,a' \in A, \; t,t' \in T, \; m \in M_s, \; p,p' \in P, p \neq p', \; f \in F \;, s \in S-\{1\}$$

$$M \times (2 - \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times Q_{f,p} - \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times Q_{f,p'}) + (SS_{p,p',s}) \times H + \sum_{m'' \in M_{s-1}} PC_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \geq \sum_{m' \in M_{s-1}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} + \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times PT_{p',\mathrm{m}} \quad (19)$$

$\forall \; a, a' \in A \;, \; t, t' \in T, \; m \; \in M_s \;, \; p, p' \in P, p \neq p', \; f \in F \;, s \in S - \{1\}$

$$M \times (2 - \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times Q_{f,p} - \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times Q_{f',p'}) + (1 - SS_{p,p',s}) \times H + \sum_{m' \in M_{s-1}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m'' \in M_{s-1}} PC_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} + \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times PT_{p,m} + ST_{p,m} \quad (20)$$

$\forall \; a.a' \in A \;, t, t' \in T \;, m \in M_s \;, \; p, p' \in P, p \neq p' \;, f, f' \in F \;, f \neq f' \;, s \in S - \{1\}$

$$M \times (2 - \sum_{m' \in M_{s-1}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times Q_{f,p} - \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times Q_{f',p'}) + (SS_{p,p',s}) \times H + \sum_{m'' \in M_{s-1}} PC_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \geq \sum_{m' \in M_{s-1}} PC_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} + \sum_{m'' \in M_{s-1}} Y_{p',m'',m,s,a',t'} \times RRR_{m'',m,s,a',t'} \times PT_{p',\mathrm{m}} + ST_{p',m} \quad (21)$$

$\forall \; a.a' \in A \;, t, t' \in T \;, m \in M_s \;, \; p, p' \in P, p \neq p' \;, f, f' \in F, f \neq f' \;, s \in S - \{1\}$

$$\sum_{m' \in M_{s-1}} \sum_{s \in S - \{1\}} EXT_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} + ET_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m' \in M_{s-1}} \sum_{s \in S - \{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times H \quad (22)$$

$\forall \; p \in P \;, a \in A \;, t \in T \;, m \in \; M_s$

$$\sum_{m \in M_s} \sum_{s \in S - \{1\}} EXT_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} + ET_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \geq \sum_{m \in M_s} \sum_{s \in S - \{1\}} Y_{p,m',m,s,a,t} \times RRR_{m',m,s,a,t} \times H \quad (23)$$

$\forall \quad p \in P \;, a \in A \;, t \in T \;, m' \in M_{s-1}$

$$\sum_{m\in M_s}\sum_{m'\in M_{s-1}} PC_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t} - \sum_{m\in M_s}\sum_{m'\in M_{s-1}} RT_{m',m,s,a,t}\times RRR_{m',m,s,a,t} - \sum_{m\in M_s}\sum_{m'\in M_{s-1}} Y_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t}\times PT_{p,m} \le \sum_{m\in M_s}\sum_{m'\in M_{s-1}} ET_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t}\times PA_{p,m} + \left(1 - \sum_{m\in M_s}\sum_{m'\in M_{s-1}} Y_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t}\times PA_{p,m}\right)\times H \quad \forall\ a\in A,\ t\in T,\ p\in P,\ s\in S-\{1\} \tag{24}$$

$$\sum_{m\in M_s}\sum_{m''\in M_{s+1}} RT_{m,m'',s+1,a,t}\times RRR_{m,m'',s+1,a,t} - \sum_{m\in M_s}\sum_{m'\in M_{s-1}}\sum_{a'\in A}\sum_{t'\in T} PC_{p,s,m',m,s,a',t'}\times RRR_{s,m',m,s,a',t'}\times PA_{p,m} - \sum_{m\in M_s}\sum_{m''\in M_{s+1}}\sum_{p'\in P} Y_{p',m,m'',s+1,a,t}\times RRR_{m,m'',s+1,a,t}\times PA_{p',m''}\times LT_{p'} - \sum_{m\in M_s}\sum_{m''\in M_{s+1}} Y_{p,m,m'',s+1,a,t}\times RRR_{m,m'',s+1,a,t}\times PA_{p,m''}\times FM_{m,m'',s+1,a} \le \sum_{m\in M_s}\sum_{m'\in M_{s-1}}\sum_{a'\in A}\sum_{t'\in T} EXT_{p,s,m',m,s,a',t'}\times RRR_{s,m',m,s,a',t'}\times PA_{p,m} + \left(2 - \sum_{m\in M_s}\sum_{m''\in M_{s+1}} Y_{p,m,m'',s+1,a,t}\times RRR_{m,m'',s+1,a,t}\times PA_{p,m''} - \sum_{m\in M_s}\sum_{m'\in M_{s-1}}\sum_{a'\in A}\sum_{t'\in T} Y_{p,s,m',m,s,a',t'}\times RRR_{s,m',m,s,a',t'}\times PA_{p,m}\right) \quad \forall\ a\in A,\ t\in T,\ p\in P,\ s\in S-\{1,e\} \tag{25}$$

$$\sum_{t\in T}\sum_{a\in A}\sum_{m'\in M_{s-1}} Y_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t} = \sum_{t\in T}\sum_{a\in A}\sum_{m''\in M_{s+1}} Y_{p,m,m'',s+1,a,t}\times RRR_{m,m'',s+1,a,t} \quad \forall\ p\in P,\ m\in M_s,\ s\in S-\{1,e\} \tag{26}$$

$$\left(\sum_{m'\in M_{s-1}}\sum_{m\in M_s}\sum_{a\in A}\sum_{t\in T}\sum_{p\in P}\sum_{s\in S} PT_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t}\times PA_{p,m} + \sum_{m'\in M_{s-1}}\sum_{m\in M_s}\sum_{a\in A}\sum_{t\in T}\sum_{p\in P}\sum_{s\in S} TT_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t}\times PA_{p,m}\right)\times \mathrm{H} \le \sum_{m'\in M_{s-1}}\sum_{m\in M_s} PC_{p',m',m,s',a',t'}\times RRR_{m',m,s',a',t'}\times PA_{p',m} \quad \forall\ p'\in P,\ s'=e,\ a'\in A,\ t'\in T \tag{27}$$

$$LPCT \ge \sum_{m'\in M_{s-1}} PC_{p,m',m,s,a,t}\times RRR_{m',m,s,a,t} \quad \forall\ p\in P,\ m'\in M_{s-1},\ s=e,\ a\in A,\ t\in T \tag{28}$$

$$PC_{p,m',m,s,a,t},\ RT_{m',m,s,a,t},\ LPCT,\ SS_{p,p',s},\ ET_{p,m',m,s,a,t},\ EXT_{p,m',m,s,a,t} \ge 0 \quad \forall\ p\in P,\ m'\in M_{s-1},\ a\in A,\ t\in T,\ m\in M_s,\ s\in S \tag{29}$$

$$Y_{p,m',m,s,a,t},\ W_{m',m,s,a,t},\ U_{m,m',a,t},\ SS_{p,p',s} = Binary \quad \forall\ p\in P,\ m'\in M_{s-1},\ a\in A,\ t\in T,\ m\in M_s,\ s\in S \tag{30}$$

Now this model attempts to figure out optimum solution through following constraints:

Constraint (1) states that each travel of one AGV can be allocated to one route. Constraint (2) argues that each part through each stage can be processed just on one machine. Constraint (3) shows loading capacity of each AGV. Constraint (4) and (5) state that if certain travel of AGV occurs (variabl*e W)*, the model can decide whether part *p* be loaded on AGV at that travel or not (variabl*e Y)*. Constraint (6) and (7) show binary variable *(Y)* should take 1 before (*PC)* (part completion time) can take value. In other words, a certain part *(Y)* should be loaded on machine (*m)* then we can expect to have a process completion time *(PC)*. Constraint (8) and (9) mention that, load receipt time (variable *RT*) depends on binary variable *W* (firstly, travel should occur then receipt time can take value). Constraint (10) implies that t^{th} travel of certain AGV can take place, when AGV's $t^{th}-1$ travel has occurred. Constraint (11) proves that delivery time *(RT)* for AGV at its travel (*t >1*) should be equal or greater than sum of receipt time *(RT)* at pervious travel *(t-1)*, return time *(BM)* from pervious travel to one of the machines for loading, loading time of loaded parts on AGV *(LT)* and going time *(FM)* from one of the machines to one of origin machines in next stage. Constraint (12) is to some extent identical to constraint (11) but unlike constraint (11) it does not calculate interval between two sequential travels of certain AGV, constraint (12) considers that receipt time *(RT)* should be at least equal to sum of completion process time of loaded parts on AGV which have been processed on pervious machine $\left(\sum_{m''\in M_{s-2}} PC_{p,m'',m',s-1,a',t'} \times RRR_{m'',m',s-1,a',t'}\right)$, going time *(FM)* from pervious machine to origin machine in current and loading time of loaded parts on AGV *(LT)* (we used p' instead of p because maybe loads of certain AGV include parts other than p and we should take into account their loading times). Constraint (13) points out that process completion time *(PC)* should be at least equal to sum of receipt time *(RT)* and required time for processing certain parts ($PT_{p,m}$). Constraint (14) shows each AGV per each its travel can return from just one machine to just another one. Constraint (15) states that each AGV can start new travel (*t >1*), when return from pervious travel took place. Constraint (16) implies that AGV can return from its travel, when that travel happened. Constraint (17) represents total parts at last stage. Constraint (18) and (19) show process priority for parts at a certain machine where parts are from identical parts family ($f \in F$) (due to similarity between parts family, setup time is eliminated). Constraint (20) and (21) illustrate process priority for parts at a certain machine where parts are not from identical parts family ($f, f' \in F \ \ , f \neq f'$) (parts families are different and due to dissimilarity between parts family, setup time exists)). Constraints (22) and (23) show dependency of both *(ET)* and *(EXT)* variables on corresponding binary variable *(Y)* (Part p should be permitted to travel to certain machine, then corresponding *(ET)* and *(EXT)* can take value). Constraint (24) shows entrance tardiness of each part at corresponding machine. To calculate entrance tardiness *(ET)*, it Subtracts sum of receipt time *(RT)* and processing time ($PT_{p,m}$) from process completion time *(PC)*. Constraint (25) calculates exit tardiness of each part after being processed at certain machine (time it waits to be allocated to a AGV). To calculate exit tardiness *(EXT)*, receipt time of that part in next stage *(RT)* is subtracted from sum of completion process time *(PC)* of that part in present stage, loading time of all parts loaded on that carrier AGV to move into next stage ($LT_{p'}$) (we should consider loading time for not only that part, but also all loaded parts) and travel time from present machine to origin machine (next stage) ($FM_{m,m'',s+1,a}$). Constraint (26) balances input and output for each part *(p)* at the machine *(m)*. Constraint (27) makes a condition in which both tardiness *(PT)* and *(TT)* cannot take value but zero because waiting for parts is no allowed. Constraint (28) points out that *(LPCT)* should be at least equal to maximum of *(PCs)* at the last machine/machines. Constraints (29) shows mentioned variables are positive and constraint (30) limits noted variables to binary value.

3. Computational experiment

In this section, the outputs of computational experiments are presented to evaluate performance of the model. Due to exactness of solutions produced by GAMES software, the time it takes is more in comparison with metaheuristics, but dissimilar to metaheuristics it produces deterministic solutions. We are going to express mentioned example in detail which is solved by GAMS software using CPLEX solver.

3.1. Depicted instance

Consider a hybrid flow shop line with three stages, four parts, three parts families, two AGVs, six stations (one storage and five machines), five travels for each AGV and capacity for two parts in each travel of AGV. The objective is to minimize the makespan and number of travels simultaneously. In this example we have one station (storage) in first stage, three stations (three machines) in second stage and two stations (machines) in third station. In the second stage two machines are related and the third one is not related and in third station both machines are not related to each other. Parts 1 and 2 can be processed on machines 2 (stage 2), 3(stage 2) and 5 (stage 3), parts 3 and 4 can be processed on machine 4 (stage 2) and machine 5 (stage 3).

Table 1 – Binary variables

Table 2 – Positive variables

Table 3 – Objective

$w_{m',m,s,a,t}$	$Y_{p,m',m,s,a,t}$	$U_{m,m',a,t}$	$SS_{p,p',s}$
$W_{1,2,2,1,3} = 1$	$Y_{1,1,2,2,1,3} = 1$	$U_{5,1,2,2} = 1$	$SS_{1,2,2} = 1$
$W_{1,2,2,2,1} = 1$	$Y_{2,1,2,2,2,1} = 1$	$U_{6,1,1,2} = 1$	$SS_{1,2,3} = 1$
$W_{1,4,2,1,1} = 1$	$Y_{3,1,4,2,1,1} = 1$	$U_{6,2,2,4} = 1$	$SS_{1,3,2} = 1$
$W_{1,4,2,2,3} = 1$	$Y_{4,1,4,2,2,3} = 1$	0	$SS_{1,3,3} = 1$
$W_{2,5,3,1,4} = 1$	$Y_{1,2,5,3,1,4} = 1$	0	$SS_{1,4,3} = 1$
$W_{2,5,3,2,2} = 1$	$Y_{2,2,5,3,2,2} = 1$	0	$SS_{2,3,3} = 1$
$W_{4,6,3,1,2} = 1$	$Y_{3,4,6,3,1,2} = 1$	0	$SS_{2,4,3} = 1$
$W_{4,6,3,2,4} = 1$	$Y_{4,4,6,3,3,2} = 1$	0	$SS_{4,3,2} = 1$
			$SS_{4,3,3} = 1$

4.Conclusion

$RT_{m',m,s,a,t}$	$PC_{p,m',m,s,a,t}$	$ET_{p,m',m,s,a,t}$	$EXT_{p,m',m,s,a,t}$
$RT_{1,2,2,1,3} = 8$	$PC_{1,1,2,2,1,3} = 25$	0	0
$RT_{1,2,2,2,1} = 4$	$PC_{2,1,2,2,2,1} = 8$	0	0
$RT_{1,4,2,1,1} = 5$	$PC_{3,1,4,2,1,1} = 8$	0	0
$RT_{1,4,2,2,3} = 23$	$PC_{4,1,4,2,2,3} = 25$	0	0
$RT_{2,5,3,1,4} = 28$	$PC_{1,2,5,3,1,4} = 29$	0	0
$RT_{2,5,3,2,2} = 12$	$PC_{2,2,5,3,2,2} = 14$	0	0
$RT_{4,6,3,1,2} = 13$	$PC_{3,4,6,3,1,2} = 15$	0	0
$RT_{4,6,3,2,4} = 30$	$PC_{4,4,6,3,3,2} = 15$	0	0

LPCT(second part of objective)	**Total travel time** (second part of objective)	**Obj**
33	**24**	**57**

In this research, the hybrid flow shop scheduling with robotic processing and AGV-based transportation system is considered in which there are related and unrelated parallel machines in each stage. A mixed integer linear programming (MILP) model is proposed to minimize the makespan and total AGV travels. To compute the model, GAMS software using CPLEX solver is employed and to better understand the solution, we depict all steps of solving problem. As future research, limited buffer can be considered for each machine within each stage and routing constraints can be applied for AGVs to find unoccupied routs.

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