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A new multi-objective mathematical model for supplier selection in uncertain environment

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Abstract

Nowadays, suppliers play an important role in development of sustainable Supply Chain Management (SCM). In this study, a novel multi-objective model has been proposed which takes into account the importance of objectives in an efficient way and considers the uncertainty of real world problems. In order to consider the mentioned uncertainty, the input data are defined as grey numbers. To solve the proposed mathematical multi-objective model based on grey numbers, it has been formulated in the form of Fuzzy Goal Programming (FGP); then, it has been converted to Grey Linear Programming (GLP). Eventually, a sensitivity analysis has been carried out in order to show the performance of the proposed model.

Keywords: Supply Chain Management, Supplier Selection, Quota Allocation, Fuzzy Goal Programming, Grey Linear Programming.

Introduction

In today's world, companies are confronted with complex difficulties in SCM. Considering the importance of purchasing function in the present competitive market, supplier's evaluation and selection, and finally, quota allocation to them can be called as one of the most important aspect of SCM (De Boer et al., 2001); beside its importance, it would be considered as a very difficult task due to existence of vast number of suppliers and multiple conflicting criteria for evaluating them. Figure (1) shows how noted developments made purchasing function much more complex. Since companies produce various products, they may be looking forward to optimize one or more specific objectives for purchasing some specific components, some other objectives for some other components, and all objectives for some other components. For instance, imagine a company which wants to evaluate suppliers of four components with respect to two objectives, such as cost of purchasing and evaluation score (or quality of products); this company may want to merely minimize costs of purchasing for two of components (i.e. quality for the specific components is not important or is less important comparing to their cost), maximize merely evaluation score for some specific components, and finally, optimize both mentioned objectives for the last components, simultaneously. To take into account the mentioned issue, a new multi-objective model has been proposed in this study. It is worth noting that the proposed method in literature for incorporating the importance of objective functions in multi-objective optimization (e.g. weighted sum method) assumes fixed importance for all objective functions and do not consider different settings of importance over various products. To best of our knowledge, most of previous papers have ignored this issue.

On the other hand, crisp data are not always accessible due to existing imprecision in real-world problems; unlike most of studies in the literature, we have studied supplier selection problem under uncertainty. Given that calculating an appropriate membership function in fuzzy sets or a precise probability distribution in stochastic programming is difficult, the proposed model has been constructed based on grey numbers. Regarding the fact that the input data are defined as grey numbers, the proposed model has been formulated in the form of FGP; then, it has been converted to GLP. Eventually, a sensitivity analysis has been carried out in order to show the performance of the proposed model.

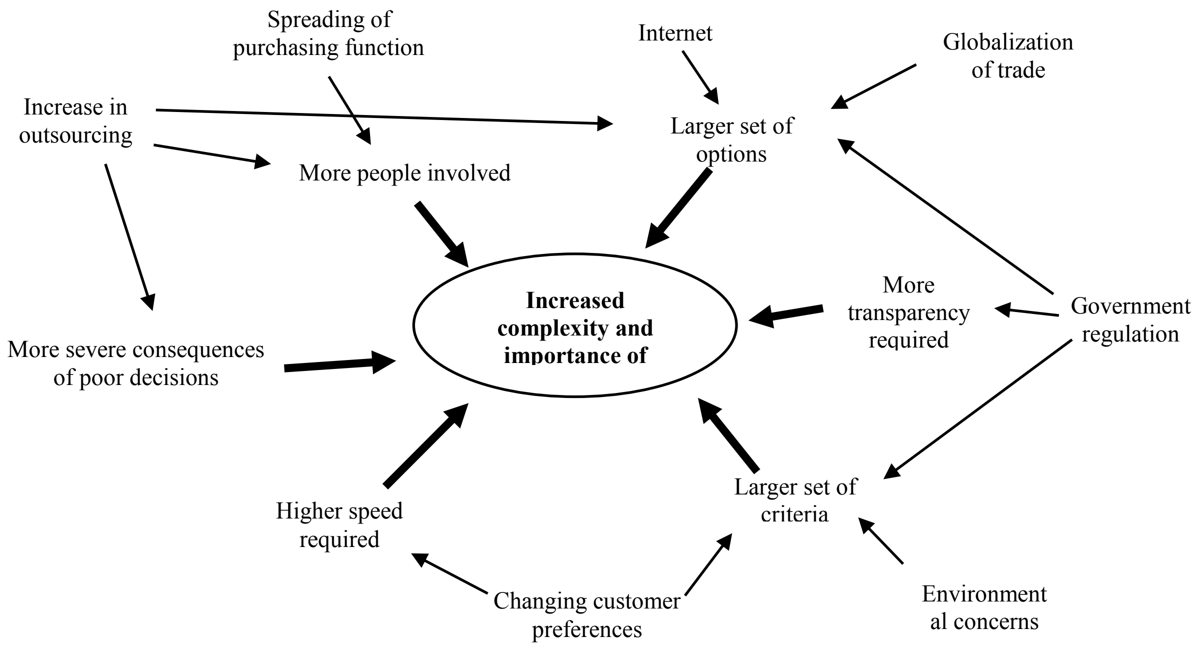

Figure 1. Impact of developments on the complexity of initial purchasing decisions (De Boer, 1998)

The remainder of this paper is as follows: In Section 2 a comprehensive literature review for the mentioned problem has been provided; In Sections 3 and 4, the FGP and GLP are presented completely, respectively. Model's structure is provided in Section 5. In Section 6, numerical study has been defined, and computational results and carried out sensitivity analysis have been provided; and finally, Conclusion has been presented in Section 7.

Literature Review

Chai et al. (2013) carried out a systematic literature review on the application of decision making methods for supplier's selection based on four following aspects: decision problems, decision makers, decision environment, and decision approaches. They studied the application of 26 decision making methods for supplier selection in the presence of uncertainty. They reviewed 26 aforementioned methods and analyzed the application of methods integration for suppliers' selection. Moreover, Ho et al. (2010) reviewed and provided a literature review for the problem of evaluation and selection of suppliers.

Given that the manufacturer-supplier relationship would not be continuous if both sides cannot acquire benefit for themselves, Dath et al. (2009) recognized the critical dimensions of SCM for manufacturers and suppliers. Li et al. (2008) suggested a grey rough set approach for selecting suppliers in SCM. Their proposed approach is quite appropriate for decision making problem in the presence of uncertainty. Moreover, they provided a point of view for attribute values in the rough set decision table under the condition that all alternatives have been described through linguistic variables. In their paper, the best suppliers can be chosen by analyzing the grey relation. Razmi et al. (2009) provided an integrated two-stage framework which includes suppliers' evaluation and order allocation. In order to evaluate suppliers, they used a fuzzy TOPSIS model with combination of two validated coefficients. Then, they used an integer programming model with fuzzy objectives and constraints to allocate the optimal amount of order to suppliers. Ku et al. (2010) integrated fuzzy analytic hierarchy process (AHP) method and goal programming (GP) which was considered as a novel approach for supplier selection in SCM; by their proposed fuzzy AHP- FGP method, a manufacturer is able to integrate different principals' view to determine the weight of each goal, and also is able to determine order quantity for each supplier according to manufacturers' strategies. Gadakh (2014) evaluates efficiency of multi-criteria decision-making approach (i.e. a complex proportional assessment (COPRAS)) in supplier selection. Appltinh this method to solve two instances showed that this method provides efficient solutions as like as the previously proposed methods. Razmi and Maghool (2010) suggested a fuzzy two-objective model, which selects qualified, for some products and some periods suppliers subjected to capacity and budget constraints. Their model includes different kinds of discounts. They also considered different kinds of possible installment methods that can be suggested by different suppliers. Finally, they used $\varepsilon - constraint$ and *Tchebycheff* methods in order to solve the proposed two-objective model. Songhori et al. (2011) suggested an organized framework which has dependent phases. In order to determine the relative efficiency of suppliers and transport's alternatives, they used a DEA model in the selection phase. On the other hand, in the allocation phase, they provided a mixed integer programming model with two objectives (i.e. minimizing the total cost and maximizing the total efficiency). Fazlollahtabar et al. (2011) suggested an integrated AHP, TOPSIS, and multi-objective nonlinear programming approach that considers both of tangible and intangible factors for selection of the best suppliers; their proposed approach maximizes the amount of purchase, and minimizes the failure rate and related penalties to earliness or lateness. Their proposed determines priority of each supplier by AHP method. TOPSIS method has been applied to determine the rank of suppliers, and finally, the optimal quota allocation to each supplier is calculated by the obtained weights. Sadeghi et al. (2013) proposed a multi objective model for aggregated planning problem where parameters are defined by grey numbers. They solved the proposed model by GP approach with fuzzy aspiration levels.

Chen and Chao (2012) suggested a simple method for election of dealer that uses the structure of criteria of AHP model and applies compatible fuzzy preference relation for creating a decision matrix. Computational simplicity and high efficiency can be referred as manifest positive point of their proposed model. Ahmady et al. (2013) developed a novel fuzzy DEA approach with double frontiers for evaluating and selection of suppliers. By applying their developed DEA approach and traditional form of DEA to an instance, they reach to this conclusion that their developed DEA outperforms the traditional one. an Amin and Zhang (2012) designed a closed-loop supply chain network. They suggested a two-phase integrated model. In the first phase, they offered a framework which determines the relevant criterion for selection of suppliers in the reverse logistic. In addition, they designed a fuzzy method which can evaluate suppliers based on qualitative criteria. Output of this stage shows the weight of each supplier. In the second phase, they proposed a multi-objective mixed integer linear programming model in order to determine qualified suppliers and reconstruction locations of them (strategic decisions), and the optimal number of components and products in closed-loop supply chain (tactical decisions). Their objective functions maximize the earned benefit and suppliers' weight, and minimizing the failure rate. Tavana et al. (2016) proposed an integrated multi-criteria decision making approach for sustainable supplier selection problem. Their novel proposed approach distinguishes between all required sustainable factors and sub-factor. Tsai (2015) proposed a mixed integer non-linear programming model for supplier evaluation and selection. They assumed a multiple-item, multiple-supplier sourcing environment where purchasing costs, rejections, and late deliveries have to be minimized.

Assadipour and Razmi (2012) proposed a probabilistic mixed integer mathematical model with fuzzy objective function and soft constraints. Their proposed model determines what components and how much of them have to be allocated to each suppliers. They inspired the structure of their proposed model from one of the Iranian automobile manufacturers (i.e. Irankhodro Company). The objective function of their proposed model maximizes benefit while shortage is not allowed. Moreover, their model considers ambiguity of dynamic parameters, such as demand, available capacity of suppliers, prices, cost of maintenance, and shortage. In order to solve the mentioned problem, the probabilistic model is converted into a deterministic classic multi-objective model. Through an integrated fuzzy approach, the suggested multi-objective problem is converted into one-objective model. Eventually, they designed a Particle Swarm Optimization algorithm to provide a desired adaptive solution. Li et al. (2012) proposed a comprehensive model for suppliers' selection. In the first step, due to calculation of the weight of used criteria, the linguistic variables are defined as trapezoidal fuzzy numbers. In the second step, their proposed model determines the weight of qualitative and quantitative criteria by a fuzzy AHP model. In order to specify ranking of the suppliers, their model uses TOPSIS. Kubat and Yuce (2012) provided AHP, fuzzy AHP, and Genetic Algorithm (GA) that selects the best set of suppliers. While conventional methods cannot consider all of the important aspect in selecting suppliers, such as supplier's technical experience and knowledge, and inability of supplier management, Yang (2008) proposed an explicit knowledge evaluation model to resolve the aforesaid problem.

As mentioned before, Seifbarghy and Esfandiari (2013) modeled problem of supplier selection and quota allocation by considering five objectives: minimizing transaction costs, costs of purchase, number of late delivered products, number of returned products, maximizing evaluation score of suppliers. They designed two different Meta-heuristic algorithms including GA and Simulated Annealing. Pang and Bai (2013) developed an evaluation approach of suppliers based on fuzzy analytic network process (ANP) synthetic evaluation in fuzzy environment. They considered the importance weights of different criteria as linguistic variables, and also, assumed the linguistic rates as triangular fuzzy numbers by analysis of fuzzy space. In order to select an alternative from suppliers, fuzzy synthetic evaluation has been use; fuzzy ANP has been used to compute the importance weights of each criterion. Mendoza and Ventura (2013), studied the effect of transportation costs on supplier selection and inventory management decisions by a mixed integer nonlinear programming model.

Fuzzy Goal Programming

As mentioned, GP is a useful method for solving multi-objective models. Since GP is not able to deal with imprecise data and uncertainty, FGP is known as a useful tool. In FGP, goal values are calculated based on fuzzy numbers. After Narasimhan (1980), Hannan (1981a), and Tiwari (1987), Yang et al. (1991) suggested a FGP with fewer variables. If $G_k(x)$ denotes to the kth fuzzy goal with a triangular membership function, FGP can be formulated as like as Equation (1):

$$\mu_k = \begin{cases} 0 & if \;\; G_k(x) \geq b_k + d_{k2}; \\ 1 - \frac{G_k(x) - b_k}{d_{k2}} & if \;\; b_k \leq G_k(x) \leq b_k + d_{k2}; \\ 1 & if \;\; G_k(x) = b_k; \\ 1 - \frac{b_k - G_k(x)}{d_{k1}} & if \;\; b_k - d_{k1} \leq G_k(x) \prec b_k; \\ 0 & else \end{cases} \quad (1)$$

where b_k denotes to the aspiration level of the kth goal, and d_{k1}, and d_{k2} refer to the maximum permissible negative deviation, and positive deviation from b_k, respectively. The obtained linear programming formulation for FGP is as like as Equation (2):

$$\begin{aligned} & Max \;\; \beta \\ & s.t: \\ & \beta \leq 1 - \frac{G_k(x) - b_k}{d_{k2}} \\ & \beta \leq 1 - \frac{b_k - G_k(x)}{d_{k1}} \\ & \beta, X \geq 0; for \;\; all \;\; k \end{aligned} \quad (2)$$

Tiwari e al. (1986), formulated FGP by simple additive model in one another way. If the FGP problem contains n fuzzy goals ($G_k(x)$; $k = 1,2, \dots, n$) , their proposed FGP formulation will be written as follows:

$$\begin{aligned} & G_k(x) \succ\succ b_k (or \;\; G_k(x) \prec\prec b_k); \qquad k = 1,2, \dots, n \\ & s.t: \\ & AX \leq b; \qquad X \geq 0 \end{aligned} \quad (3)$$

where $G_k(x) \succ\succ (\prec\prec) b_k$ points to kth fuzzy goal that is approximately greater than or equal to (or smaller than or equal to) the b_k. As a result, when the goal is formulated as $G_k(x) \succ\succ b_k$, linear membership function for kth fuzzy goal can be written as Equation (4),

$$\mu_i = \begin{cases} 1 & if \;\; G_k(x) \geq b_k \\ \frac{G_k(x) - L_k}{b_k - L_k} & if \;\; L_k \leq G_k(x) \leq b_k \\ 0 & if \;\; G_k(x) \leq L_k \end{cases} \quad (4)$$

and while the goal is formulated as $G_k(x) \prec\prec b_k$, the mentioned linear membership function changes to the following equation:

$$\mu_i = \begin{cases} 1 & if \;\; G_k(x) \leq b_k \\ \frac{U_k - G_k(x)}{L_k - b_k} & if \;\; b_k \leq G_k(x) \leq U_k \\ 0 & if \;\; G_k(x) \geq U_k \end{cases} \quad (5)$$

wherein L_k(or U_k) refers to the lower limit of tolerance (or upper limit of tolerance) for the kth fuzzy goal of $G_k(x) \succ\succ (\prec\prec) b_k$. The simple additive model can be written as like as Equation (6):

$$\begin{aligned} & Max \;\; f(\mu) = \textstyle\sum_{k=1}^{n} \mu_k \\ & s.t: \\ & \mu_k = \frac{G_k(x) - L_k}{g_k - L_k} \qquad for \;\; some \;\; k, \\ & \mu_j = \frac{U_k - G_k(x)}{U_k - g_k} \qquad for \;\; some \;\; j, j \neq k \\ & AX \leq b \\ & \mu_k, \mu_j \leq 1 \\ & X, \mu_k, \mu_j \geq 0; \qquad k, j \in \{1,2, \dots, n\} \end{aligned} \quad (6)$$

Considering the above introduced FGP, the introduced FGP model has to be adopted for grey numbers. The following section will provide preliminary definitions about grey numbers and shows how the introduced FGP model will be solved in the event that grey numbers are applied.

Grey Linear Programming

Deng (1982) proposed grey systems and described how grey numbers are able to resolve the aforementioned ambiguity in real world problems. According to their study, data can be classified into three categories: white condition which shows the required data are completely known, grey condition which illustrates the required data are partially known and partially unknown, and black condition which shows the required data are thoroughly unknown. In the real-world, there are a lot of problems with partially known and unknown data. Applying the concept of grey numbers help decision makers to make decision more efficinetly in the presence of incomplete data.

The term of grey number designates to a number that does not have determined value; in fact, a grey number is defined in a range of numbers. Usually a grey number is shown by '⊗'. Where $\underline{a}$ mentions to the specified lower limit and $\overline{a}$ to specified upper limit, the grey number of ***a*** can be written as $\otimes(a) \in [\underline{a}, \overline{a}]$.

If $\otimes_1 \in [a, b]$ and $\otimes_2 \in [c, d]$, some preliminary definitions of grey numbers, which have been provided by Liu and Forrset (2010), are as follows:

Definition 1. For $\otimes_1 \in [a, b]$ we have:

$$\begin{aligned} & \otimes_1 \geq 0 \qquad if \;\; a \geq 0 \;\;\; and \;\; b \geq 0 \\ & \otimes_1 < 0 \qquad if \;\; a < 0 \;\;\; and \;\; b < 0 \end{aligned} \quad (7)$$

Definition 2. For the addition and subtraction operations we have:

$$\begin{aligned} & \otimes_1 + \otimes_2 \in [a + c, b + d] \\ & \otimes_1 - \otimes_2 \in [a - d, b - c] \end{aligned} \quad (8)$$

Definition 3. For the multiplication and division operations we have:

$$\begin{aligned} & \otimes_1 * \otimes_2 \in [\min(a.c, a.d, b.c, b.d), \max(a.c, a.d, b.c, b.d)] \\ & \otimes_1 / \otimes_2 \in [\min(a/c, a/d, b/c, b/d), \max(a/c, a/d, b/c, b/d)] \end{aligned} \quad (9)$$

Definition 4. If k is a positive real number, the scalar multiplication is defined as Equation (10):

$$k.\otimes_1 \in [k.a, k.b] \quad (10)$$

Definition 5. For the grey number of $\otimes_1$, $Sign(\otimes_1)$ is defined as follows:

$$Sign(\otimes_1) = \begin{cases} 1, & if\ \otimes_1 \geq 0 \\ -1, & if\ \otimes_1 < 0 \end{cases} \quad (11)$$

Definition 6. For the grey number $\otimes_1$, the absolute value is defined as follows:

$$|\otimes_1| \in \begin{cases} \otimes_1, & if\ \otimes_1 \geq 0 \\ -\otimes_1, & if\ \otimes_1 < 0 \end{cases} \quad (12)$$

Definition 7. The length of each grey number is calculated as bellow:

$$l(\otimes_1) = b - a \quad (13)$$

Definition 8. If $\otimes_1$ is a continuous grey number and $\otimes_1 \in [a, b]$, its kernel is calculated as Equation (14):

$$\hat{\otimes}_1 = \frac{1}{2}(a + b) \quad (14)$$

Definition 9. If $\otimes_1$ is a discrete grey number and $e_i \in [a, b]$ $(i = 1,2, \dots, n)$, its kernel is calculated as Equation (15):

$$\hat{\otimes}_1 = \frac{1}{n}\sum_{i=1}^{n} e_i \quad (15)$$

Definition 10. If the background which makes $\otimes_1$ coming into being is Ω, and $\mu(\otimes_1)$ refers to the measure of $\otimes_1$ (i.e. length of $\otimes_1$), degree of greyness is equal to:

$$g^{\circ}(\otimes_1) = \mu(\otimes_1)/\mu(\Omega) \quad (16)$$

A linear programming problem, which its objective function's and constraints' coefficients are defined by grey numbers, can be called as GLP. Mostly, GLP is formulated as Equation (17):

$$\begin{aligned} &Max(min)\otimes\ f = \otimes C.\otimes X \\ &s.t: \\ &\otimes A.\otimes X \leq (\geq)\otimes B \\ &\otimes X \geq 0 \end{aligned} \quad (17)$$

wherein $\otimes B \in (R)^{m\times 1}$, $\otimes C \in (R)^{1\times n}$, and $\otimes X \in (R)^{n\times 11}$. Huang (1994) has proposed an approach for this kind of problems. In order to find the optimal value of $\otimes f \in [\underline{f}, \overline{f}]$, they proposed two following sub-models. The first sub-model calculates the lower and upper limits of $\otimes f$ according to Equations (18-19), respectively:

$$\bar{f} = \sum_{j=1}^{k_1} \overline{c_j}.\overline{x_j} + \sum_{j=k_1+1}^{n} \overline{c_j}.\underline{x_j} \quad (18)$$

$$\underline{f} = \sum_{j=1}^{k_1} \underline{c_j}.\underline{x_j} + \sum_{j=k_1+1}^{n} \underline{c_j}.\overline{x_j} \quad (19)$$

where $\otimes(c_j) \in [\underline{c_j}, \overline{c_j}]$ $\forall j = 1,2, \dots, k_1$ refers to the positive coefficients of objective function, and $\otimes(c_j) \in [\underline{c_j}, \overline{c_j}]$ $\forall j = k_1 + 1, \dots, n$ denotes to the negative coefficients of the objective function. In addition, the constraints of Equation (17) will be calculated as follows:

$$\sum_{j=1}^{k_1} \underline{\otimes}(|a_{ij}|).Sign(\underline{a}_{ij})\,\overline{x}_j/\overline{b}_i + \sum_{j=k_1+1}^{n} \overline{\otimes}(|a_{ij}|).Sign(\overline{a}_{ij})\,\underline{x}_j/\underline{b}_i \leq 1, \quad \forall i \quad (20)$$

$$\sum_{j=1}^{k_1} \overline{\otimes}(|a_{ij}|).Sign(\overline{a}_{ij})\,\underline{x}_j/\underline{b}_i + \sum_{j=k_1+1}^{n} \underline{\otimes}(|a_{ij}|).Sign(\underline{a}_{ij})\,\overline{x}_j/\overline{b}_i \leq 1, \quad \forall i \quad (21)$$

Therefore, $\underline{f}$ value will be calculated by Equations (18 and 20), and $\overline{f}$ will be calculated by Equations (19 and 21). Given that the values of $\underline{x}_j$ and $\overline{x}_j$ have been incorporated in the first sub-model, the values of $\underline{x}_j$ and $\overline{x}_j$ are calculated by adding the second sub-model:

$$\underline{x}_j \leq \overline{x}_j^*, \ \forall j = 1,2, \dots, k_1 \quad (22)$$

$$\overline{x}_j \geq \underline{x}_j^*, \ \forall j = k_1 + 1, k_1 + 2, \dots, n \quad (23)$$

Model's structure

Since companies produce various products, they may be looking forward to optimize one or more specific objectives for purchasing some specific components, some other objectives for some other components, and all objectives for some other components. For instance, imagine a company which wants to evaluate suppliers of four components with respect to two objectives, such as cost of purchasing and evaluation score (or quality of products); this company may want to merely minimize costs of purchasing for two of components (i.e. quality for the specific components is not important or is less important comparing to their cost), maximize merely evaluation score for some specific components, and finally, optimize both mentioned objectives for the last components, simultaneously. To take into account the mentioned issue, a new multi-objective model has been proposed in this study. It is worth noting that the proposed method in literature for incorporating the importance of objective functions in multi-objective optimization (e.g. weighted sum method) assumes fixed importance for all objective functions and do not consider different settings over various products.

Supplier selection model

Since a product can be supplied by a large numberr of suppliers, evaluating and selecting the best supplier regarding to conflicting criteria is a complicate task. Furthermore, quota allocation to different suppliers is complex too. The most important objective of this study is to evaluate and select the qualified suppliers, and determine the allocated quota. For this purpose, a multi-objective model has been developed. The objectives of developed model are as follows:

1) Minimizing transaction costs
2) Minimizing costs of purchase
3) Maximizing evaluation scores

The following indices, parameters and variables are used in the proposed model:

Indices:

i	Index of supplier
j	Index of product

t	Index of time

Parameters:

$\otimes p_{ijt}$	Price of j^{th} product in the t^{th} period by i^{th} supplier
$\otimes E_i$	Evaluation score of i^{th} supplier
E_j^{min}	Minimum required evaluation score for j^{th} product which eliminate the constraint on purchase
$1-u_j$	Coefficient which shows that how much of all demand of j^{th} product can be purchased from suppliers with less evaluation score than predetermined threshold (i.e. E_j^{min})
$\otimes Q_j$	Maximum allowable percentage of returned j^{th} product
$\otimes D_{jt}$	Demand for j^{th} product in t^{th} period
$\otimes Cap_{ijt}$	Maximum production capacity of i^{th} suppliers for j^{th} product in t^{th} period
$\otimes a_{ij}$	Transaction cost of buying j^{th} product from i^{th} supplier

$$\beta_{ijt} = \begin{cases} 1 & \text{if } i^{th} \text{ supplier produces } j^{th} \text{ product in } t^{th} \\ 0 & \text{otherwise} \end{cases}$$

$$V_j = \begin{cases} 1 & \text{if quality is an imporatnt factor in purch } j^{th} \text{ product} \\ 0 & \text{otherwise} \end{cases}$$

$$F_j = \begin{cases} 1 & \text{if price is an imporatnt factor in purchasi } j^{th} \text{ product,} \\ 0 & \text{otherwise} \end{cases}$$

Decision Variables

$\otimes X_{ijt}$	Order Quantity of j^{th} product from i^{th} supplier in t^{th} period

$$Y_{ijt} = \begin{cases} 1 & \text{if } j^{th} \text{ product hsa been purchased from } i^{t} \text{ supplier in } t^{th} \text{ period,} \\ 0 & \text{otherwise} \end{cases}$$

As mentioned, the objective function includes five objectives which are equal to Equations (24-26):

$$Min\ \ Z_1 = \sum_j \sum_i \sum_t V_j.\left(1-F_j\right).\otimes a_{ij}.Y_{ijt} + \sum_j \sum_i \sum_t \left(1-V_j\right).F_j.\otimes a_{ij}.Y_{ijt} + \sum_j \sum_i \sum_t V_j.F_j.\otimes a_{ij}.Y_{ijt} \quad (24)$$

$$Min\ \ Z_2 = \sum_j \sum_i \sum_t V_j.\left(1-F_j\right).\otimes p_{ijt}.\otimes X_{ijt} + \sum_j \sum_i \sum_t \left(1-V_j\right).F_j.\otimes p_{ijt}.\otimes X_{ijt} + \sum_j \sum_i \sum_t V_j.F_j.\otimes p_{ijt}.\otimes X_{ijt} \quad (25)$$

$$Max\ \ Z_3 = \sum_j \sum_i \sum_t V_j.\left(1-F_j\right).\otimes E_i.\otimes X_{ijt} + \sum_j \sum_i \sum_t \left(1-V_j\right).F_j.\otimes E_i.\otimes X_{ijt} + \sum_j \sum_i \sum_t V_j.F_j.\otimes E_i.\otimes X_{ijt} \quad (26)$$

Equations (24-25) minimizes the transaction and variable costs of purchasing from suppliers, respectively. Moreover, Equation (27) indicates the total acquired evaluation scores.

The above-proposed objective functions are subjected to the following constraints:

$$\sum_i \otimes X_{ijt} \geq \otimes D_{jt} \qquad \forall j,t \quad (27)$$

$$\sum_i \sum_t \otimes q_{ij}.\otimes X_{ijt} \leq \otimes Q_j.\sum_t \otimes D_{jt} \qquad \forall j \quad (28)$$

$$\otimes X_{ijt} \leq (1-\left[u_j \times (1-\frac{\max\left(0, E_i - E_j^{min}\right)}{\max\left(1, E_i - E_j^{min}\right)}\right]).D_{jt} \qquad \forall i,j,t \quad (29)$$

$$\otimes X_{ijt} \leq \otimes Cap_{ijt}.Y_{ijt} \qquad \forall i,j,t \quad (30)$$

$$Y_{ijt} \leq \beta_{ijt} \qquad \forall i,j,t \quad (31)$$

$$\otimes X_{ijt} \geq 0 \quad (32)$$

$$Y_{ijt} = \{0,1\} \quad (33)$$

Constraint (27) ensures that the given orders to the i^{th} supplier in t^{th} period can be satisfied. Constraint (28) guarantees that the total number of returned j^{th} products are less than the maximum allowable number of returned products, and constraint (29) ensures that more than $(1-u_j).D_{jt}$ number of j^{th} product cannot be purchased from suppliers with less evaluation score than predetermined evaluation score (i.e. E_j^{min}). Constraint (30) indicates that the given order to i^{th} supplier has to be less than its production capacity. Constraint (31) guarantees that the j^{th} product can be ordered in t^{th} period to supplier i^{th} when they produce j^{th} product in t^{th} period. The constraint (32) shows that $\otimes X_{ijt}$ is nonnegative, and finally, constraint (33) shows the integrality of variables.

It should be noted that the proposed method for incorporating the importance of criteria for each product, can be incorporated for more objective easily. Moreover, V_j and F_j can acquire other values apart from zero and one by slight modifications in order to incorporate various settings.

The problem's solving methodology

Considering the explained FGP, GLP, and model's structure for supplier selection in the previous sections, four following steps have to be taken to distinguish qualified suppliers and determine the quota of suppliers:

1. Converting the proposed model of supplier selection to FGGP: Considering the study of Tiwari et al. (1986), the proposed mathematical model for supplier selection will be converted to a FGGP model.
2. Determining the membership function for fuzzy goals: To do this, solve the proposed model by deterministic parameters firstly; then, used the obtained values for each goal as benchmark to create membership function.
3. Converting the FGGP to GLP: By incorporating the provided basic concepts in the study of Chen and Tsai (2001), the FGGP will be reformulated to GLP model.
4. Solving the GLP problem: Finally, the GLP model will be solved by the proposed solving approach of GLP in the study of Huang (1994).

In order to evaluate the efficiency of the proposed approach, a numerical study has been provided in the following section.

Numerical study

In order to evaluate the proposed model, a comprehensive numerical example has been presented in this section. In this numerical study, three suppliers have to supply the demand of 4 various products in three periods. The values of V_j, F_j, and $\otimes Q_j$, and the demand of aforesaid products has been provided in Tables (1) and (2), respectively. All of the other input parameters have been provided in Tables (1A-3A) in Appendix. The values of evaluation score for each supplier is considered to be equal to $E_1 = [85, 95]$, $E_2 = [75, 85]$, and $E_3 = [65, 75]$, and to sake for simplicity, E_j^{min} assumed to be equal to **0**.

Table 1. Values of V_j, F_j, and $\otimes Q_j$

Products / Parameters	$j = 1$	$j = 2$	$j = 3$	$j = 4$
V_j	0	1	1	1
F_j	1	1	0	0
$\otimes Q_j$	[0.065, 0.075]	[0.085, 0.095]	[0.115, 0.125]	[0.090, 0.10]

Considering the presented numerical study, the proposed model has to be solved one time by Equations (18 and 20), and one another time by Equations (19 and 21) in order to obtain the values of $\bar{f}$ and $\underline{f}$, respectively. Therefore, the ordinal linear programming will be formulated as Equation (34):

$$\begin{cases} Min\ ro\ Max\ Z_k & \forall\ k = 1,2,3 \\ Subject\ to: \\ Constraints\ (28-34) \end{cases} \qquad (34)$$

Table 2. Demand of products within various periods

$\otimes D_{jt}$	$j = 1$	$j = 2$	$j = 3$	$j = 4$
$t = 1$	[295, 305]	[230, 240]	[245, 255]	[145, 155]
$t = 2$	[95, 105]	[195, 205]	[95, 105]	[95, 105]
$t = 3$	[95, 105]	[375, 385]	[135, 145]	[215, 225]
$t = 4$	[145, 155]	[95, 105]	[345, 355]	[345, 355]

By solving Equation (34), values of Z_1, Z_2, and Z_3 become equal to [940, 1180], [1050665, 1135900], and [224855, 270275], respectively. Afterward, membership function of three objective functions will be calculated according to Equations (4-5):

$$\mu_1 = \begin{cases} 1 & if\ Z_1 \le 940 \\ \frac{1180 - Z_1}{1180 - 940} & if\ 940 \le Z_1 \le 1180 \\ 0 & if\ Z_1 \ge 1180 \end{cases}$$

$$\mu_2 = \begin{cases} 1 & if\ Z_2 \le 1050665 \\ \frac{1135900 - Z_2}{1135900 - 1050665} & if\ 1050665 \le Z_2 \le 1135900 \\ 0 & if\ Z_2 \ge 1135900 \end{cases}$$

$$\mu_3 = \begin{cases} 1 & if\ Z_3 \ge 270275 \\ \frac{Z_3 - 224855}{270275 - 224855} & if\ 224855 \le Z_3 \le 270275 \\ 0 & if\ Z_3 \le 224855 \end{cases}$$

Based on abovementioned equations, Figures (2-4) illustrate the membership function of three discussed objective functions.

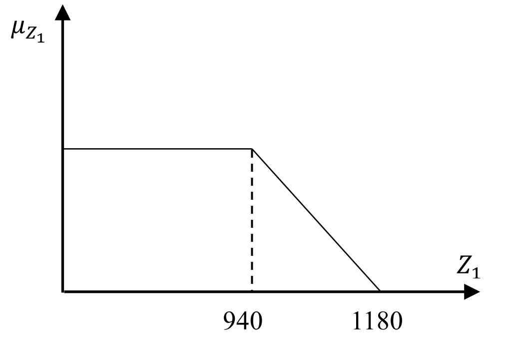

Figure 2. Membership function of Z_1 (minimizing the transaction costs)

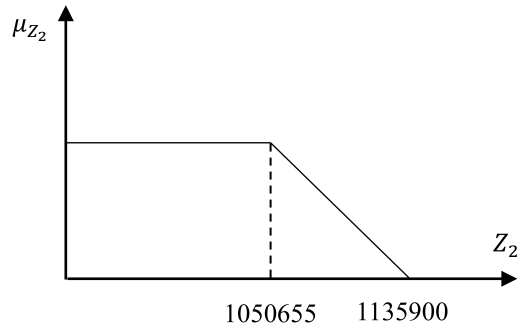

Figure 3. Membership function of Z_2 (minimizing the variable costs of purchasing)

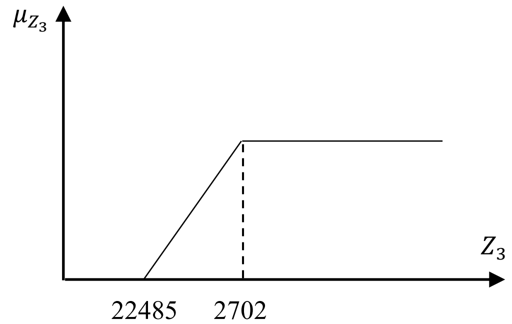

Figure 4. Membership function of Z_3 (maximizing the evaluation score of suppliers)

Then, the FGGP problem has to be converted to its equivalent in the form of GLP with the objective that maximizes the summation of achievement degree. The GLP form of FGGP of presented instance is as follows:

$$Max\ f(\mu) = \sum_{k=1}^{n} \otimes\mu_k$$

$Subject\ to:$

$$\mu_1 \le \frac{1180 - Z_1}{1180 - 940}$$

$$\mu_2 \le \frac{1135900 - Z_2}{1135900 - 1050665}$$

$$\mu_3 \geq \frac{Z_3 - 224855}{270275 - 224855}$$
$$\textstyle\sum_i \otimes X_{ijt} \geq \otimes D_{jt} \qquad \forall j,t$$
$$\textstyle\sum_i \sum_t \otimes q_{ij} . \otimes X_{ijt} \leq \otimes Q_j . \sum_t \otimes D_{jt} \qquad \forall j \qquad (35)$$
$$\otimes X_{ijt} \leq (1 - \left[u_j \times (1 - \frac{\max(0, E_i - E_j^{min})}{\max(1, E_i - E_j^{min})} \right]). D_{jt} \qquad \forall i,j,t$$
$$\otimes X_{ijt} \leq \otimes Cap_{ijt} . Y_{ijt} \qquad \forall i,j,t$$
$$Y_{ijt} \leq \alpha_{ijt} \qquad \forall i,j,t$$
$$\otimes \mu_k \leq 1 \qquad \forall k$$
$$\otimes \mu_k, \otimes X_{ijt} \geq 0$$
$$Y_{ijt} = \{0,1\}$$

Eventually, Equation (35) has been solved, and the obtained results have been provided in the following. Tables (3) shows order allocation to suppliers where both V_j and F_j are set equal to **1**. It is worth noting that grey values of decision variable (i.e. $\otimes X_{ijt}$) fortify decision makers to make decisions in uncertain environment.

Table 3. Values of the decision variable

$\otimes X_{ijt}$		$i = 1$	$i = 2$	$i = 3$
$j = 1$	$t = 1$	[75, 85]	[130, 130]	[90, 90]
	$t = 2$	[35, 45]	[60, 60]	-
	$t = 3$	-	-	[95, 105]
	$t = 4$	[145, 155]	-	-
$j = 2$	$t = 1$	-	[95, 95]	[135, 145]
	$t = 2$	[57, 67]	[138, 138]	-
	$t = 3$	[175, 185]	[200, 200]	-
	$t = 4$	-	[95, 105]	-
$j = 3$	$t = 1$	[150, 150]	-	[95, 105]
	$t = 2$	-	[95, 105]	-
	$t = 3$	-	[135, 145]	-
	$t = 4$	-	[200, 200]	[145, 155]
$j = 4$	$t = 1$	-	[145, 155]	-
	$t = 2$	-	-	[95, 105]
	$t = 3$	[110, 110]	[105, 115]	-
	$t = 4$	[200, 200]	-	[145, 155]

It should be noted that the proposed approach has been coded in GAMS 24.1.2, and has been solved in a laptop with Core i7 CPU and 8 GB of RAM.

Sensitivity Analysis

In order to clarify the performance of the proposed model, a sensitivity analysis over settings of V_j and F_j have been done. The values V_j and F_j have been changed in comparison to what had defined in Table (1), and have set equal to $V_j = 1$ and $F_j = 1$ (the new setting); therefore, Equations (24-26) (i.e. objective functions of th proposed model) will be converted to Equation (36). It should be noted that the new setting has solved by applying goal programing approach.

$$\begin{cases} Min \ \ Z_1 = \sum_j \sum_i \sum_t \otimes a_{ij} . Y_{ijt} \\ Min \ \ Z_2 = \sum_j \sum_i \sum_t \otimes p_{ijt} . X_{ijt} \\ Max \ \ Z_3 = \sum_j \sum_i \sum_t \otimes E_i . X_{ijt} \\ Subject \ \ to: \\ Constraints \ \ (28-34) \end{cases} \qquad (36)$$

The new setting wants to show that how values of $\otimes X_{ijt}$ will change comparing to the old settings (i.e. what had defined in Table (1)). As shown in Table (4), values of $\otimes X_{ijt}$ have provided for the new settings.

As Table (4) shows, values of $\otimes X_{ijt}$ have not been changed for the second product; since the setting for second products has not changed in the new settings, unchanged values of $\otimes X_{ijt}$ in Table (4) seems to be rational. To seek for clarity, occurred changes in values of $\otimes X_{ijt}$ have been shown in color for the rest of products. Given that comparison the values of Tables (3) and (4) is so difficult, differences of these values have been studied more in Table (5).

As considered in the old settings, cost of purchasing (i.e. transaction costs + variable costs) is the important criterion. Therefore, changing the setting has increased the purchasing costs for the first product. As show in the Table (5), in the case of third and fourth products, evaluation scores have decreased if the setting modifies from the old one to the new one. All discussed changes indicate that the proposed model performs efficiently in such environments. Since the difference between old and new settings will be more significant in large scale instances (i.e. the proposed model in the old settings outperforms the proposed model in the new settings much more significantly), it will be obvious that taking different importance weights for different products is important.

Table 4. Obtained results where $V_j = 1$ and $F_j = 1$

$\otimes X_{ijt}$		$i = 1$	$i = 2$	$i = 3$
$j = 1$	$t = 1$	**[90, 95]**	**[115, 120]**	[90, 90]
	$t = 2$	**[45, 55]**	**[50, 55]**	-
	$t = 3$	-	-	[95, 105]
	$t = 4$	[145, 155]	-	-
$j = 2$	$t = 1$	-	[95, 95]	[135, 145]
	$t = 2$	[57, 67]	[138, 138]	-
	$t = 3$	[175, 185]	[200, 200]	-
	$t = 4$	-	[95, 105]	-
$j = 3$	$t = 1$	**[125, 130]**	-	**[120, 125]**
	$t = 2$	-	[95, 105]	-
	$t = 3$	-	[135, 145]	-
	$t = 4$	-	**[180, 180]**	**[165, 175]**
$j = 4$	$t = 1$	-	[145, 155]	-
	$t = 2$	-	-	[95, 105]
	$t = 3$	[110, 110]	[105, 115]	-
	$t = 4$	**[175, 180]**	-	**[170, 175]**

Table 5. Comparison between variable costs and evaluation scores over various settings

	Products	Settings	Values
Variable costs	$j = 1$	$V_j = 0$ and $F_j = 1$	[60750, 71550]
		$V_j = 1$ and $F_j = 1$	[61000, 72275]
Evaluation scores	$j = 3$	$V_j = 1$ and $F_j = 0$	[60600, 72000]
		$V_j = 1$ and $F_j = 1$	[59900, 71400]
	$j = 4$	$V_j = 1$ and $F_j = 0$	[60700, 71900]
		$V_j = 1$ and $F_j = 1$	[60200, 71500]

Conclusion

In this paper, the problem of supplier selection and quota allocation to them was investigated. For this purpose, a new mathematical model which consolidates the importance of different objective function's weight for different products has been proposed. Because data are mostly imprecise in most of real-world problems, FGP model based on grey numbers was applied to resolve the mentioned problem. Therefore, GLP and FGP were incorporated to solve the proposed model. In order to figure out the efficiency of the proposed model, a numerical study was evaluated. The obtained value for decision variables indicated that applying grey numbers fortifies decision makers to make decisions in uncertain environment. Finally, a sensitivity analysis was done in order to show the performance of the proposed model over various setting of V_j and F_j. As showed in Tables (4-5), setting desired values of each product to V_j and F_j provides better solution in terms of costs of purchasing and evaluation scores comparing to the case that both V_j and F_j were set equal to **1** (i.e. converting the Equations (24-26) to Equation (36)).

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Appendix

In the following Tables, all of the input parameters of the presented numerical study have been provided.

Table 1A. Percentage of returned products for each supplier

$\otimes q_{ij}$	$i = 1$	$i = 2$	$i = 3$
$j = 1$	[0.045, 0.055]	[0.025, 0.035]	[0.075, 0.085]
$j = 2$	[0.095, 0.105]	[0.045, 0.055]	[0.145, 0.155]
$j = 3$	[0.055, 0.065]	[0.005, 0.015]	[0.015, 0.25]
$j = 4$	[0.055, 0.065]	[0.075, 0.085]	[0.095, 0.105]

Table 2A. Transaction costs of purchasing form suppliers

$\otimes a_{ij}$	$i = 1$	$i = 2$	$i = 3$
$j = 1$	[35, 45]	[15, 25]	[10, 20]
$j = 2$	[55, 65]	[25, 35]	[45, 55]
$j = 3$	[65, 75]	[35, 45]	[55, 65]
$j = 4$	[75, 85]	[30, 40]	[20, 30]

Table 3A. Price of each product, capacity of each supplier, and ability of suppliers to supply each product

$\otimes p_{ijt}$		$i=1$	$i=2$	$i=3$
$j=1$	$t=1$	[100,110]	[90,100]	[80,90]
	$t=2$	[105,115]	[95,105]	-
	$t=3$	-	-	[95,105]
	$t=4$	[110,120]	-	-
$j=2$	$t=1$	-	[290,300]	[280,290]
	$t=2$	[305,315]	[295,305]	-
	$t=3$	[315,325]	[305,315]	-
	$t=4$	-	[300,310]	-
$j=3$	$t=1$	[400,410]	-	[380,390]
	$t=2$	-	[395,405]	-
	$t=3$	-	[405,415]	-
	$t=4$	-	[400,410]	[390,400]
$j=4$	$t=1$	-	[490,500]	-
	$t=2$	-	-	[485,495]
	$t=3$	[515,525]	[505,515]	-
	$t=4$	[510,520]	-	[490,500]

Cap_{ijt}		$i=1$	$i=2$	$i=3$
$j=1$	$t=1$	100	130	90
	$t=2$	50	60	-
	$t=3$	-	-	120
	$t=4$	190	-	-
$j=2$	$t=1$	-	95	160
	$t=2$	80	150	-
	$t=3$	200	200	-
	$t=4$	-	105	-
$j=3$	$t=1$	150	-	125
	$t=2$	-	110	-
	$t=3$	-	155	-
	$t=4$	-	200	200
$j=4$	$t=1$	-	175	-
	$t=2$	-	-	170
	$t=3$	110	130	-
	$t=4$	200	-	200

β_{ijt}		$i=1$	$i=2$	$i=3$
$j=1$	$t=1$	1	1	1
	$t=2$	1	1	0
	$t=3$	0	0	1
	$t=4$	1	0	0
$j=2$	$t=1$	0	1	1
	$t=2$	1	1	0
	$t=3$	1	1	0
	$t=4$	0	1	0
$j=3$	$t=1$	1	0	1
	$t=2$	0	1	0
	$t=3$	0	1	0
	$t=4$	0	1	1
$j=4$	$t=1$	0	1	0
	$t=2$	0	0	1
	$t=3$	1	1	0
	$t=4$	1	0	1