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Multi-level Supply Chain Coordination with the Possibility of Credit Purchase

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Abstract

Today, with increased focus on sustainable supply chain management companies have found that the cost of the supply chain and inventory could be efficiently managed through greater cooperation and better coordination. This study provides a model for coordinating a three-level supply chain (suppliers, manufacturer and buyer) and the mechanisms of coordination in the supply chain is based on credit purchases. In this purchase the suppliers enables the manufacturer and also the manufacturer allows the buyer to pay his debt after the permissible delay and during this period the buyer does not pay any interest but after the end of the permissible time,he has to pay interest per late day. The permissible time for payment of debts is considered as a decision variable in the model. In the end, the overall costs of the supply chain are compared with and without delay and sensitivity analysis is conducted on the parameters.

Keywords:

Coordination, supply chain management, permissible delay in payment

1. Introduction

Supply chain management and decision-making within the chain is possible as centralized and decentralized modes. The most ideal state that leads to centralized management of the supply chain is that the whole supply chain is managed by a member and all decisions are made by the same member. In order to create such a state in the chain it is necessary to make the relevant members' information available to that member and information sharing between members is done in the best way possible. Decisions made in such chain reduce cost of the entire chain and ultimately increase the entire chain's gain. It should be noted that any decisions made in a centralized mode be optimized for the entire system but it will not necessarily achieve optimization for the individual members.

Of course in most cases this ideal mode does not occur for centralized management of the supply chain. It means that by the expansion of supply chain it is less possible for a member to take decision for the entire chain.

As mentioned earlier, another other methods of decision-making in the supply chain is decentralized decision-making. In such a case each individual member of the chain manages his operations and makes decision for his own such that he just increases profits and reduces costs of his own. Decentralized decisions will not optimize the whole system.

In such supply chains to take advantage of the benefits of centralized decisions, it is attempted to create coordination between members of the supply chain so that the total cost of the supply chain is minimized.

With the increased focus on sustainable supply chain management companies have found that inventory in the whole supply chain can be efficiently managed through greater cooperation and better coordination. Participatory model in the management of the supply chain can increase the effectiveness and efficiency of the supply chain and reduce inventory costs.

As mentioned above, the decisions that are taken centrally increase the total supply chain's profit but the individual members would not be optimal necessarily. In such case to encourage members to cooperate and create alignment

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between the goals of members which is the same as the entire system's increased profits, the obtained profit must be divided fairly among supply chain members.

To create coordination among supply chain members there are different mechanisms depending on the type and nature of activity of the members, the members' ownership level, type of product and supply chains' service that each of which can be useful.

There are many definitions of supply chain coordination but in general coordination is a joint effort among the supply chain members to perform the supply chain activities such as product development, shipping, information sharing and so on such that the risk is reduced and profit is increased and the profit is divided fairly between members of the chain [1].

In this study, the coordination mechanism is based on credit purchase which is one of the subsets of shared decision-making mechanism [2]. The mechanism is used to encourage the buyer (retailer) in order to buy more than the economic order quantity (EOQ) to reduce the cost of the seller (manufacturer) where the buyer can settle the accounts during a certain time after purchase without the additional interest.

In this case, the supplier provides the retailer with a certain amount of time in which the retailer must settle his accounts. No charge is added during this permissible period. But the interest for delay in payment is set that the retailer have to pay if he does not settle the accounts after the permissible period. Among the advantages of using this mechanism for the buyer is the possibility of a risk-free investment of debt during permissible period and the advantage for the seller is that the sales level increases, the cost of production is reduced and inventory is transferred to the buyer.

Literature review

Determining the Joint Economic Lot Sizing (JELS) among the members of the supply chain is one of the most effective mechanisms for reducing the total cost in the supply chain [3]. Goyal [4] was one of the first pioneers who conducted research on JELS. He presented a solution to a two-level problem under the premise of unlimited production rate for the seller and categorized transport policy for shipments from the seller to the buyer. Banerjee [5] in his article released the limiting premise of unlimited production in Goyal article but still studied the categorized transport policy in the two-level system. It was almost the first article that discussed the term JELS. In Giri et al [6] the premise of the sameness of ordered categories was removed and the production packages could be in different sizes. This paper aims to find the optimal order packages based on geometric transport policy in a two-level system.

As noted above, the use of permissible delay in payments by JELS through coordination leads to profitability.

The first article that has analyzed permissible delay in payment by JELSP was Goyal [7] where the length of permissible period was considered fixed.

Among other studies in this area includes Osman [8] who developed two-level supply chain model with permissible delay in payments and revenue sharing. They showed that coordination will lead to more retailers' order and brings savings for the entire supply chain.

Jaber [9] has criticized the EOQ model because this model ignores the hidden costs of the inventory. Accordingly, to estimate the hidden costs this paper applies the first and second law of thermodynamics to reduces entropy (disorder) of the system in terms of costs. Chung [10] has studied the inventory system using the discounted cash flow (DCF) including credit purchases and as the buyer orders more, he enjoys more discounts. This article has integrated the DCF and credit transactions approaches for the optimal order quantity. In Luo [11] the optimum period of vendor replenishment and financing is considered as mechanism to coordinate the supply chain that the buyer by increasing the order can reduce the costs of the parties (buyer and seller). Balkhi [12] has proposed an EOQ model with narrow horizon of a trade credit for an inventory policy of sensitive items under inflation and time value of money and solved it.

Aljazzar et al [13] which is considered as a basic research and the project development has been based on this paper has proposed the two-level supply chain coordination (manufacturer - retailer) with permissible delay in the payment as a decision variable. They have studied the implementation of three production policies of "Hill, Goyal and Jaber" and concluded that the minimum total cost of production is achieved when the Hill's policy is applied. In Hill's policy packets are sent during production.

Another part of the articles have released the assumption of constant set-up costs. In fact, set up cost can be reduced and controlled by investment. Porteus [14] studied the impact of investment to reduce the set up cost in EPQ model for the first time. Billington [15] studied the impact of this investment on EPQ model. Several relationships are studied between the investment and the set up cost in Keem et al [16] and Nasiri et al [17]. The common assumption among all articles is the continuous relationship between the investment and set up cost.

Sarker et al [18] proposed a method to find the optimal amount of investment to reduce set up costs when there is a non-continuous relationship between the investment and the set-up cost.

Huang et al [19] studied the impact of investment to reduce the set-up cost with the premise of permissible shortage as overdue order and possible demand during the delivery in a two-level system with transportation policy of equal magnitude.

In Jha [20] a model is investigated in which the assumption of a single buyer is removed and the model is discussed by a seller and multiple buyers with transport policy in the same size. The buyer demand is based on normal distribution and the buyers control their inventory based on the continuous review policy and the shortage is based on the overdue demand. This model minimizes the expected cost of one seller -multi-buyer. Since it is difficult to estimate the cost of inventory shortage, instead of entering a cost shortfall in the model, the service level of each every buyer is included in the model. The main scientific contribution of this paper is to study three-level supply chain coordination (supplier-manufacturer-retailer) by permissible delay in the payment as a decision variable. Although in reality the permissible delay in the payment of a decision variable is not considered, this assumption minimizes the whole supply chain costs. Three-level supply chain coordination with permissible delay in the payment between both levels has not been studied in the literature. This situation is very common in real life and comprehensive analysis of real projects allows providing useful advice that can portray three-level structure and supply chain practices. In this model, the manufacturer purchases the raw materials from suppliers and converts it to the final product that follows Hill's policy based on which the manufacturer produces the product and transfers to retailer during production. In particular, supplier, manufacturer and retailer are assumed that their decisions will be coordinated by the delay in payment. During any period the supplier offers the manufacturer a delay in payment after which the manufacturer must pay interest for each day late. The manufacturer also proposes the retailer delay in payment period in which the retailer does not pay interest for the balance of the flow and after that the manufacturer is allowed to impose interest on any current account. Here the centralized decision-making with delay in payment is considered where three members of the supply chain coordinate their decisions on the order and determining the length of permissible delay in payment. In this article delay in payment is a decision variable to be determined by the upstream member (upstream member for manufacturer is the supplier and the upstream member for the retailer is the manufacturer). For a comprehensive review and consider all scenarios, 9 scenarios should be considered. Since in the basic paper three different scenarios are proposed based on the interval between the main variables and parameters and for the development each scenario should be classified into 3 scenarios. Because of the vastness and complexity of the model by applying two constraints to develop the model 9 scenarios are reduced to 1 scenario.

After creating the model and its solution, the cost of the entire supply chain in credit and cash purchases is compared based on numerical examples and it is observed that when they do not offer delay in payment, the supply chain costs are at the highest level. Then the effects of each independent and dependent variables on total supply chain costs are analyzed under each plan and the parameters with negligible effect on the total cost are detected. Finally, the sensitivity analysis is carried out and the interest rate is variable and the cost of each actor and the demand to production ratio are kept constant. For each case of sensitivity the total costs are obtained for credit and cash purchases and the lowest cost is identified. This provides different members with identifying the best plan that would optimize the total cost of the supply chain which is based on the specific value of interest rate, fixed costs and the ratio of demand to production.

Other sections of this paper have been developed as follows. Section 2 introduces the concepts and assumptions. Section 3 describes the mathematical model of the plan. Section 4 provides numerical examples and sensitivity analysis. Section 5 deals with the conclusion and discussion.

2. Notations and assumptions

Table 1- Notations table

i	Indicates the chain member (m for manufacturer and r for retailers)
A_i	Cost of setting up / order for a member "i"
C_i	Cost of production / purchase per item for member "i"

h_i	Financial maintenance costs per item for member "i"
S_i	Physical maintenance costs per item for member "i"
Q	Order quantity
τ	When the buyer pays his debts to manufacturer
t_i	Permissible delay in payment period by player "i"
K_i	Return on investment for member "i"
P	Annual production rate of producer
D	Retailer's annual demand rate
j	Indicates the level of inventory in the supply chain members (" w "For the raw materials and " f "For the final product). When the member of the chain has one kind of inventory, j = 0. For example A_s represents the start-up cost for suppliers and $A_{m,w}$ indicates the cost of ordering the manufacturer's raw materials.
n_1	The number of packets sent from suppliers to manufacturer per raw materials' cycle
n_2	The number of packets sent from manufacturer to retailer per retailer's cycle
x	The number of raw materials needed to produce a unit of final product
T	Shared cycle length (n_2Q/D)
T_s	Supplier's cycle length (n_2Q/P)
T_w	Manufacturer raw materials' cycle length (n_2Q/n_1P)
T_m	Manufacturer's final product cycle length (n_2Q/P)
T_r	Retailer's cycle length (Q/D)

Assumptions:

- Model is single-product
- Packets have the same size
- Demand is assumed deterministic
- Shortage is unpermissible
- The rate of production is higher than demand
- The cost of inventory maintenance consists of 2 parts:
    1. Financial cost of maintaining inventory
    2. Physical cost of maintaining inventory
- The manufacturer allows the buyer to pay off the debt without payment of interest and financial expenses after a certain period of time
- If the debt pay off exceeds the permissible time, the buyer should pay the interest and debt financial fees for the late payment
- Retailer has a risk-free investment opportunity with the value of C_rQ within the period of clearance with the manufacturer
- The retailer must pay off his debts over one payment
- The problem has one supplier, one manufacturer and one retailer
- Supplier's production rate is higher than the manufacturer's need and the manufacturer's production rate is higher than the retailer's demand
- The supplier permits the manufacturer to delay in payment and the manufacturer provides such permit to the retailer
- Delay in payment is intended as a decision variable
- The manufacturer has 2 separate storages: one for raw materials and the other for the final product
- The length of manufacturer in the production of final product is longer than the rest of length of supply chain member cycle. Therefore the manufacturer cycle length is considered as the shared cycle length to calculate the total annual cost.

In this model at the beginning of each buyer cycle (Q / D), the buyer gives an order as big as Q to the manufacturer and sends them at the end of each buyer cycle (Q / D) in the packages with the same size and the manufacturer inventory becomes zero after the first shipment (based on Hill transport policy). The buyer must settle the accounts until the end of period t_m and over this period he does not pay interest during this period. The manufacturer presents an order as big as $\frac{\alpha n_2 Q}{n_1}$ to the supplier at the beginning of each raw material's cycle and the supplier sends the orders in the packages of the same size. And the manufacturer must seetle the accounts until the end of period t_s otherwise he has to pay interest for each day of delay.

The chart Q versus t is shown below. The area under the curve shows the inventory levels and to calculate the cost of maintaining the inventory in the cost function, the area under the curve for each member of the chain is calculated and multiplied by the rate of maintenance cost.

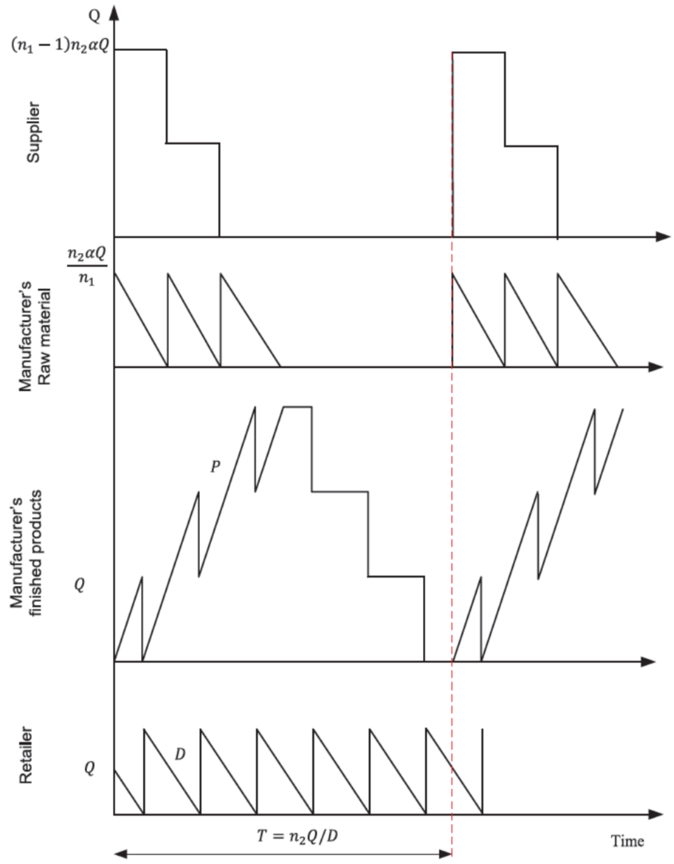

Figure 1- Q-T chart chart

3. Mathematical model

As noted above this research is developed based on Aljazzar et al [13]. In the basic paper three different models of Goyal, Hill and Jaber are studied and each of these policies is considered in three different scenarios and after solving and conclusions Hill's transport model has the lowest cost. Therefore Hill's model is used to develop the model. Here a brief description of Hill's transport model is presented.

Hill's transport model: in this model the packages are sent to the buyer while manufacturing and the buyer will not wait for the process to be completed and manufacturer's inventory becomes zero after the first shipment.

In the basic paper [13], based on the period t and τ three scenarios are discussed as follows:

$$\begin{aligned} &\text{caseI}: \ 0 \le t = \tau \le T_r \\ &\text{caseII}: \ 0 \le t < \tau \le T_r \\ &\text{caseIII}: \ 0 \le t \le T_r \le \tau \end{aligned} \quad (1)$$

In the first case settling time should be less than the length of the retailer cycle. In the second case settling time is between the length of the retailer cycle and the time period and in the third case the settling time is longer than the retailer of the retailer cycle. To be developed each scenario should be classified into three other scenarios because in the basic scenario there is only one τ (the period that the manufacturer allows the buyer to settle his account) but on the developed model there are two τ variables (one for the supplier and manufacturer and the other for the manufacturer and buyer). Three scenarios are considered to develop the base model based on the first case (case1) of the base model.

$$\left.\begin{aligned} &\text{case I-I} && 0 \le t_s = \tau_m \le \frac{n_2 Q}{Pn_1} \ \text{and} \ 0 \le t_m = \tau_r \le \frac{Q}{D} \\ &\text{case I-II} && 0 \le t_s = \tau_m \le \frac{n_2 Q}{Pn_1} \ \text{and} \ 0 \le t_m < \tau_r \le \frac{Q}{D} \\ &\text{case I-III} && 0 \le t_s = \tau_m \le \frac{n_2 Q}{Pn_1} \ \text{and} \ 0 \le t_m < \frac{Q}{D} \le \tau_r \end{aligned}\right\} \ \text{caseI}: 0 \le t = \tau \le T_r \quad (2)$$

As mentioned in the previous section model development is only based on case I-I and due to the development of the model the rest of cases are ignored. In fact by applying two restrictions $t_s = \tau_m \le \frac{n_2 Q}{Pn_1}$ and $0 \le t_m = \tau_r \le \frac{Q}{D}$ the developed model is reduced to one scenario. To obtain the overall cost function the cost of each member of the chain should be calculated and by summing the obtained functions for each member the overall cost function is obtained.

The cost function of any member includes five sections as follows:

1- Cost of ordering/setup, 2- cost of purchasing / manufacturing, 3- cost of maintaining the inventory, 4- cost of investment opportunity before account settlement (based on Riggs, Bedworth, & Randhawa) and 5- cost of debt interest for late payment (based on Riggs, Bedworth, & Randhawa)

3.1 The annual cost of supplier function

$$\begin{aligned} \mathbf{Z}_s = {} & \frac{A_s D}{n_2 Q} + C_s xD + \frac{n_1(n_1-1)}{2}(h_s + S_s)\frac{x n_2 QD}{Pn_1^2} \\ & + h_s \tau_m xD + (C_{m,w} - C_s) xDe^{k_s t_s} - C_{m,w} xDe^{k_s(\tau_m - t_s)} \end{aligned} \quad (3)$$

$\frac{A_s D}{n_2 Q}$: suppliers' cost of ordering

$C_s xD$: suppliers' cost of purchase (manufacture)

$\frac{n_1(n_1-1)}{2}(h_s+S_s)\frac{xn_2QD}{Pn_1^2}$: Supplier's cost of maintaining inventory which is obtained by the area under the curve in Figure 1.

$h_s \tau_m xD$: The financial cost of the part of inventory that the supplier has sent to the manufacturer but has received no money for it (the financial expenses of this inventory is on the supplier but the physical cost of the inventory is the responsibility of the manufacturer).

$(C_{m,w}-C_s)xDe^{k_s t_s}$: The opportunity cost of that part of the inventory that has been sent to the manufacturer but no money is received for it (this amount of the fee should be deducted from the cost of production because the suppliers will incur these costs).

$C_{m,w}xDe^{k_s(\tau_m-t_s)}$: The amount of interest received by the supplier after the expiration of the permissible time from the manufacturer.

3.2 Annual cost of the manufacturer for purchasing the materials from supplier

$$\mathbf{Z}_{m,w} = n_1 A_{m,w} + \frac{n_1 C_{m,w} xn_2 Q}{n_1} + \frac{h_{m,w} n_1 x^2 n_2^2 Q^2}{2xPn_1^2} - \frac{h_{m,w} n_1 xn_2 Qt_s}{n_1} + \frac{n_1 S_{m,w} xn_2^2 Q^2}{2Pn_1^2} + \frac{n_1 C_{m,w} xn_2 Q\left(e^{k_s(\tau_m-t_s)} - e^{k_m\tau_m}\right)}{n_1} \quad (4)$$

$n_1 A_{m,w}$: The cost of ordering from supplier

$\frac{n_1 C_{m,w} xn_2 Q}{n_1}$: The cost of purchasing raw materials from supplier

$\frac{h_{m,w} n_1 x^2 n_2^2 Q^2}{2xPn_1^2}$: Financial cost of the inventory (This cost does not include the amount of inventory received from suppliers as credit)

$\frac{h_{m,w} n_1 xn_2 Qt_s}{n_1}$: Financial cost of the inventory received from suppliers as credit

$\frac{n_1 S_{m,w} xn_2^2 Q^2}{2Pn_1^2}$: Physical cost of inventory on hand

$\frac{n_1 C_{m,w} xn_2 Q\left(e^{k_s(\tau_m-t_s)} - e^{k_m\tau_m}\right)}{n_1}$: Cost of opportunity

3.3 Annual cost of manufacturer for the production of the final product

$$\mathbf{Z}_m = \frac{A_{m,f} D}{n_2 Q} + C_{m,f} D + (h_{m,f} + S_{m,f})\left[\frac{Q(2D_r + (P-D)n_2 - P)}{2P}\right] + h_{m,f}\tau_r D + (C_r - C_{m,f})De^{k_m t_m} - C_r De^{k_m(\tau_r - t_m)} \quad (5)$$

$\frac{A_{m,f} D}{n_2 Q}$: The cost of setup (ordering) of manufacturer to produce the final product

$C_{m,f} D$: The cost of the final product

$(h_{m,f} + S_{m,f})\left[\frac{Q(2D_r + (P-D)n_2 - P)}{2P}\right]$: Physical and financial cost of inventory which is calculated based on area under the curve of Figure 1.

$h_{m,f}\tau_r D$: Financial cost of that part of the inventory sent as credit to the buyer and no money is received for

$(C_r - C_{m,f})De^{k_m t_m}$: Opportunity cost for that part of the inventory that has been sent as credit to the buyer

$C_r De^{k_m(\tau_r - t_m)}$: The amount of interest received by the manufacturer for late payment after the expiry of the permissible time.

3.4 Annual cost of the retailers

$$\mathbf{Z}_r = \frac{A_r D}{Q} + C_r D + h_r \frac{(Q - Dt_m)^2}{2Q} + \frac{s_r}{2}Q + C_r D\left(1 - e^{k_r t_m}\right) \quad (6)$$

$\frac{A_r D}{Q}$: cost of ordering

$C_r D$: cost of buying from the manufacturer

$h_r \frac{(Q - Dt_m)^2}{2Q}$: Financial cost of inventory

$\frac{s_r}{2} Q$: Physical cost of inventory on hand

$C_r D\left(1 - e^{k_r t_m}\right)$: Opportunity cost

3.5 The overall cost function

The overall cost function is calculated by summing 3 functions as follows:

$$\mathbf{Z}_t = \mathbf{Z}_s + \mathbf{Z}_{m,w} + \mathbf{Z}_{m,f} + \mathbf{Z}_r$$

$$\begin{aligned}\mathbf{Z}_t &= \left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q} \\ &+ \left(C_s x + C_{m,w} x + C_{m,f} + C_r\right) Da + \left(h_s \tau_m - h_{m,w} t_s\right) xD \\ &+ \frac{(n_1 - 1)(h_s + S_s)\, xn_2 QD + h_{m,w} n_2 QD + h_{m,w} xn_2 QD}{2Pn_1} \\ &+ \frac{Q\left(2D + (P - D)n_2 - P\right)\left(h_{m.f} + S_{m.f}\right) + S_r PQ}{2P} \\ &+ h_{m.f}\tau_r D + \left(C_r - C_{m.f}\right) De^{k_m t_m} - C_r De^{k_r t_m} - C_{m.w} xDe^{k_m \tau_m}\end{aligned} \quad (7)$$

3.6 Solution

After gaining the overall cost function in the next stage to obtain optimal values of Q, n_1, n_2, T_S and T_M the partial derivative of the objective function should be obtained with respect to each of these variables and after analyzing the convex function at these points the optimal values are calculated by equalizing the partial derivative with zero.

3.6.1 Calculating the optimal amount of Q

The partial derivative of the general function is obtained versus Q and equalized with 0.

$$\begin{aligned}\frac{\partial}{\partial Q}\mathbf{Z}_t &= -\left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q^2} \\ &+ \frac{(n_1 - 1)(h_s + S_s)\, xn_2 D + h_{m,w} xn_2 D + s_{m,w} xn_2 D}{2Pn_1} \\ &- \frac{h_{m,w} xPn_1 Dt_s^2}{2n_2 Q^2} + \frac{h_r n_2 Q}{2n_2} + \frac{\left(2D + (P - D)n_2 - P\right)\left(h_{m,f} + S_{m,f}\right) + S_r P}{2P}\end{aligned} \quad (8)$$

$$\frac{\partial}{\partial Q}\mathbf{Z}_t = 0$$

$$Q = \sqrt{\frac{\left[A_s + A_{m,f} + n_1 A_{m,w} + n_2 A_r + \frac{xn_1 h_{m,w} Pt_s^2}{2} + \frac{n_2 h_r Dt_m^2}{2}\right]\frac{D}{n_2}}{\left[h_{m,w} + S_{m,w} + (n_1 - 1)(h_s + S_s)\right]\frac{xn_2 D}{2n_1 P} + \left[\frac{S_r P + h_r P + \left(h_{m,f} + S_{m,f}\right)\left(2D + n_2(P - D) - P\right)}{2P}\right]}} \quad (9)$$

3.6.2 Proving the convex function at point Q

At this point it should be specify that the cost function is convex at point Q. In other words, the objective function has its minimum amount for the optimal value Q. Accordingly the second derivative of the function at the point Q should be positive.

$$\begin{aligned}\frac{\partial^2}{\partial Q^2}\mathbf{Z}_t &= -\left(A_s + n_1 A_{m,w} + A_{m,f} + A_r n_2\right)\frac{D}{n_2 Q^4} \\ &+ \frac{h_{m,w} xPn_1 Dt_s^2}{2n_2 Q^4} + \frac{h_r n_2}{2n_2} + \frac{h_r n_2 D^2 t_m^2}{2n_2 Q^2} > 0\end{aligned} \quad (10)$$

For all values the second derivative of the cost function at the point Q is positive. So the function at the point Q is convex.

3.6.3 Calculating the optimal values of n_1 and n_2

To calculate the optimal value of n under the previous policy the partial derivative of the objective function is calculated at n_1 and n_2 points and equalized with zero and the optimum amount of n_1 and n_2 is obtained.

$$n_1 = \sqrt{\frac{\frac{xD}{P}\left[h_{m,w}+S_{m,w}-h_s-S_s\right]}{2D\left[A_{m,w}+\frac{xh_w Pt_s^2}{2}\right]}} \quad (11)$$

$$n_2 = \sqrt{\frac{4D\left[A_s+A_{m,f}+A_{m,w}+\frac{xh_{m,w}Pt_s^2}{2}\right]}{4\left(\left(h_{m,w}+S_{m,w}\right)\frac{xD}{2P}+\left[\frac{\left(h_{m,f}+S_{m,f}\right)\left(P-D\right)}{2P}\right]\right)}} \quad (12)$$

3.6.4 Proving the convex function at points n_1 and n_2

Under the previous policy the second derivative of function at n_1 and $n_2$is calculated.

$$\frac{\partial^2}{\partial n_2{}^2}\mathbf{Z}_t = \frac{\left(A_s+n_1A_{m,w}+A_{m,f}\right)2D}{n_2{}^3Q}+\frac{h_{m,w}xPn_1Dt_s^2}{n_2{}^3Q^2} \geq 0 \quad (13)$$

$$\frac{\partial^2}{\partial n_1{}^2}\mathbf{Z}_t = \frac{\left(2xn_2QD\left(h_{m,w}+S_{m,w}-h_s-S_s\right)\right)}{n_1{}^3} \geq 0$$

$$\text{if} \quad h_{m,w}+S_{m,w} \geq h_s+S_s \quad (14)$$

The second derivative of the cost function is not positive for all values. So with constraint $h_{m,w}+S_{m,w} \geq h_s+S_s$ the second derivative becomes positive so that the function is convex at these points.

3.6.5 Calculating the optimal values of t_s and t_m

Due to the constraint $t_s=\tau_m \leq \frac{n_2Q}{Pn_1}$ and $0 \leq t_m=\tau_r \leq \frac{Q}{D}$ the value of t_s is equal with τ_r and accordingly the derivative is obtained and the optimized values are calculated:

At this stage, as before, the partial derivative of the objective function is calculated at t_s and t_m and equalized with zero to obtain the optimum amounts of t_s and t_m.

$$t_s = \frac{\left(h_{m,w}xD-\left(C_{m.w}-C_s\right)xDk_s\right)n_2Q}{h_{m,w}xPn_1D} \quad (15)$$

$$t_m = \frac{\left(\left(C_r-C_{m,f}\right)k_m-C_rk_r\right)Q}{h_rD}-\frac{Q}{D} \quad (16)$$

3.6.6 Proving the convex function at t_s and t_m

$$\frac{\partial^2}{\partial t_s{}^2}\mathbf{Z}_t = h_{m,w}xPn_1D \geq 0 \quad (17)$$

$$\frac{\partial^2}{\partial t_m{}^2}\mathbf{Z}_t = h_rD \geq 0 \quad (18)$$

The second derivative of the function is positive for all applicable values. So the function is convex at these points.

4. Numerical Example

After calculating the overall cost function and the optimal values of variables a series of parameters with different numbers is calculated and the optimal values of variables and the total cost of the chain members are calculated. Parameter values are specified in the table below.

Table 2- Numerical values of parameters

D	P	x	A_s	$A_{m,w}$	$A_{m,f}$	A_r	C_s	$C_{m,w}$	$C_{m,f}$	C_r
2000	3000	1	200	200	200	200	20	30	50	70
h_s	S_s	$h_{m,w}$	$S_{m,w}$	$h_{m,f}$	$S_{m,f}$	h_r	S_r	k_s	k_m	k_r
3	3	3	8	10	10	15	7	1%	8%	4%

After placement of the parameters' values, the optimal values of the variables are obtained by replacing the parameters in the variable's formula and solving a system of 5 equations and five unknowns. After solving the system the optimal values of variables is as follows:

Table 3- Total cost of the chain with and without delay in the payment

	n_1	n_1	Q	t_s	τ_m	t_m	τ_r	supplier	manufacture	retailer
No delay	1	2	291	0	0	0	0	2128	106291	236188
With delay	1	2	305	0.2001	0.2001	0.1809	0.1809	2943	103927	231910

4.1 Parameter sensitivity analysis

At this stage, sensitivity analysis is done by replacing the different parameter values and based on different modes the cost of different modes is calculated.

In the following table the sensitivity of the overall cost function for changing the parameter values is presented. Parameters marked as ✓ mean that the cost function value is more sensitive to the values of these parameters. And parameters marked as × mean that the cost function value is less sensitive to the change of these parameters.

Table 4- Effect of parameters on total cost function

	D	P	A_s	h_s	S_s	K_s	$A_{m,w}$	$h_{m,w}$
No delay	✓	✓	✓	✓	✓	×	✓	✓
With delay	✓	✓	✓	✓	✓	×	✓	×

	$S_{m,w}$	A_{mf}	h_{mf}	S_{mf}	K_m	A_r	h_r	K_r
No delay	✓	✓	✓	✓	×	✓	✓	×
With delay	✓	✓	✓	✓	✓	✓	✓	×

4.1.1 Sensitivity analysis of return on investment

The overall cost level is calculated by substituting different values of k in function and as it can be observed the first mode has the lowest cost.

Table 5 – Comparing the total cost with permissible delay condition in payments for different values of k

	$k_r > k_m > k_s$	$k_m > k_r > k_s$	$k_s > k_m > k_r$	$k_s > k_r > k_m$	$k_s = k_m = k_r$
Total cost	338503	338710	338891	338880	338591

4.1.2 Sensitivity analysis of financial maintenance costs

In this section the overall cost is calculated by substituting different values of h in the function and as it can be observed the fourth mode has the lowest cost.

Table 6- Comparing the total cost with permissible delay condition in payments for different values of h

	$h_r > h_{mf} > h_s$	$h_{mf} > h_r > h_s$	$h_s > h_{mf} > h_r$	$h_s > h_r > h_{mf}$
Total cost	338710	339325	339053	340100

4.1.3 Sensitivity analysis of the return on investment and demand to production ratio

In final analysis of the total cost by substituting different values of k is calculated for different D/P ratios in the function. As it can be observed the first mode has the lowest cost.

Table 7- Comparing the total cost with permissible delay condition in payments for different values of D/P

$\frac{D}{P}$	Total cost
0.25	337749
0.5	338173
0.75	339037
1	340374

5. Conclusion

This study considers a multi-level supply chain (3 levels including supplier, manufacturer and buyers) based on Hill transport model (in the transport model the packages produced during the production will be sent to the buyer and manufacturer's inventory becomes zero after the first shipment) where the buyer and the manufacturer have a change of credit purchase and this model is considered as permissible period to settle the debt as a decision variable. The supplier allows the manufacturer to settle his debt after a period of permissible period which is determined by the supplier. The manufacturer gives the same opportunity to the buyer. In fact, the permissible delay in payments is used as a coordinating mechanism in the supply chain.

After creating the model, a series of numerical parameter values are replaced in the model and the total cost is calculated with and without delay and it is observed that when the permissible delay in payment is applied, the overall supply chain cost is reduced.

Finally, a sensitivity analysis is performed and the influence of each parameter on the overall cost is determined. Among the parameters that can have a large impact on the cost of the entire chain are k_s, $h_{m,w}$ and k_r that indicate supplier's return of capital, the manufacturer's financial cost of maintaining inventory of raw materials and buyer's return on investment. In the nest sensitivity analysis, the overall supply chain cost is calculated by five different values for the interest rate. And finally the total cost function for different values of D/P is calculated and observed that by increasing D/P chain cost increases.

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