Simultaneous design of robust inter and intra-cell layouts in uncertain multi-period situations of manufacturing systems

Ghorbanali Moslemipour
Department of Industrial Engineering, Payame Noor University, I.R. of Iran

E-mail:gh.moslemipour@golestan.pnu.ac.ir

Abstract *Facility layout problem has a considerable effect on manufacturing cost; hence, it can be viewed as a crucial subject in the design of manufacturing systems operating at dynamic and stochastic situations in particular. This paper proposes a new quadratic assignment-based mathematical model for concurrent design of robust inter and intra-cell layouts in a multi-period and uncertain environment of manufacturing systems. The product demands are presumed to be independent normally distributed random variables with known expectation that change from period to period at random. In the proposed model, time value of money is also considered. To validate the proposed model, a randomly generated test problem is solved by using simulated annealing (SA) algorithm programmed in Matlab. Finally, the results are analyzed from statistical and sensitivity points of veiw.*

Keywords: Simulated annealing; Stochastic dynamic facility layout problem; Robust inter and intra-cell layout

1. Introduction

One of the most critical stages in the design of manufacturing system is the facility layout problem (FLP). In manufacturing systems, a facility can be a work station such as a machine or a group of machines named cell. Material handling cost (MHC) is 20 to 50 percent of total manufacturing costs and it can be reduced by at least 10 to 30 percent by designing an optimal layout (Tompkins, White, Bozer, & Tanchoco, 2003). According to the nature of product demands and time planning horizon, the FLP can be classified into four problems as follows: (i) static facility layout problem (SFLP) with deterministic and constant flow of materials over a single time period, (ii) dynamic facility layout problem (DFLP) with different deterministic flow of materials in each period, (iii) stochastic static facility layout problem (SSFLP) with stochastic flow of materials over a single time period, (iv) stochastic dynamic facility layout problem (SDFLP) where product demands are random variable so that their parameters change from period to period. Design of a robust layout is an approach to deal with stochastic layout problems, particularly the SDFLP. A robust layout, which is not necessarily an optimal layout for a particular time period is designed as the best layout over the entire time planning horizon. The robust approach has the advantage of lack of rearrangement cost and the disadvantage of not having an optimal layout for each period. The SDFLP is the most realistic and complicated form of the layout problems so that the first three aforementioned problems can be regarded as a special case of it. Thus, this paper proposes a new quadratic assignement problem (QAP)-based mathematical model to concurrent design of a robust inter and intra-cell layout throughout the multi-period time planning horizon of the SDFLP. The novelty of this paper is considering independent product demands as normally distributed stochastic variables with known expected value that change from period to period at random and time value of money. In addition, concurrent design of a robust inter and intra-cell layout as a flexible layout for the SDFLP. It is essential to mention that many real world data naturally follow a normal distribution (Kulturel-Konak, Smith, & Norman, 2004). Product demands have also been considered as normally distributed random variables in layout design problem by the following authors (Azadeh, Haghighi, & Asadzadeh, 2014; Ripon, Glette, Hovin, & Torresen, 2011; Tavakkoli-Moghaddam, Javadian, Javadi, & Safaei, 2007; Vitayasak, Pongcharoen, & Chris Hicks, 2016; Zhao & Wallace, 2014; Zhao & Wallace, 2015).

The QAP is a nonlinear nondeterministic polynomial (NP)-complete combinatorial optimization problem (COP) (Sahni & Gonzalez, 1976). Besides, the computational time for solving the QAP is exponentially proportional to the size of the problem (Foulds, 1983). Therefore, intelligent approaches should be used to solve the large-sized problem rather than the exact methods. Simulated annealing (SA) intelligent approach is one of the promising tools for solving COPs such as the FLP (Alvarenga, Gomes, & Mestria, 2000). SA algorithm is a simulation of physical annealing process of solids in statistical mechanics, which starts with a known or randomly generated initial solution and a high initial value of temperature. It is formed by two loops namely, the inner loop to search for a neighbouring solution, and the outer loop for decrease the temperature to reduce the probability of accepting the non-improving neighbouring solutions. The better performance of SA in comparison with GA and tabu search (TS) was concluded in solving a dynamic cell formation problem (Tavakkoli-Moghaddam, Aryanezhad, Safaei, & Azaron, 2005). SA has good convergence property and it is more robustness and flexible in comparison with other local search methods. This algorithm not only can solve the stochastic and single period inter and intra-cell layout problem as good as the lingo software in quality solution standpoint, but also it can solve the larger problems in a reasonable computation time (Tavakkoli-Moghaddam, Javadi, & Mirghorbani, 2006). In this paper, the SA approach is used for solving the problem because of the above-mentioned reasons and the complexity of the proposed model.

2. Literature review

Rosenblatt and Lee (1987) and Kouvelis et al. (1992) defined the robustness of a layout as the number of times that the layout falls within a pre-specified percentage of the optimal solution for different sets of product demand scenarios. Using the robust approach, a single robust layout is designed for the whole time planning horizon so that total material handling cost is minimized (Kouvelis & Kiran, 1991). Montreuil et al. (1993) proposed a robust layout named *holographic* or *holonic* layout where different types of machines are spread over the shop floor to cope with uncertainties in a manufacturing system. For more information about holonic layout the following papers can be referred (Hsieh, 2009a, , 2009b; Hsieh & Chiang, 2011). The robust layout design approach is a good method to prevent the shifting cost (Hassan, 1994). Benjaafar and Sheikhzadeh (2000) proposed a robust layout by duplicating the same facilities in the FLP to deal with uncertainties in product demands. The robustness can be an intrinsic property of a layout for example, by replication of the main facilities at the strategic places within the shop floor, which will guarantee a reasonable efficiency for the MHS during the different production periods (Benjaafar, Heragu, & Irani, 2002). Braglia et al. (2003) designed the most robust layout for a single row FLP by assuming the product demands as normally distributed random variables. Kulturel-Konak et al. (2004) considered the most robust layout with minimum region under the total MHC curve over a pre-determined range of uncertainty. Enea et al. (2005) proposed a fuzzy model to design a robust layout for the stochastic FLP with multiple product demand scenarios. Braglia et al. (2005) proved that in the stochastic FLP, the most robust layout is obtained by using the matrix of average flows between facilities. Norman and Smith (2006) proposed a mathematical model to design the most robust layout by considering a large number of independent product demands as random variables with known expected value and variance. Tavakkoli-Moghaddam et al. (2007) proposed a new mathematical model to concurrent design of the optimal machine and cell layouts in a single time planning horizon of a cellular manufacturing system by considering the stochastic product demands with known normal probability density function (PDF). Irappa-Basappa and Madhusudanan-Pillai (2008) designed a robust machine layout for the DFLP using the quadratic assignment formulation by considering machine sequence and part handling factor, which represents changes in the attributes of parts from process to process. Balakrishnan and Cheng (2009) considered fixed and rolling planning horizon in the DFLP. They concluded that the algorithms with good performance under fixed planning horizon are not necessarily good under effectiveness of rolling planning horizon. Madhusudanan-Pillai et al. (2011) proposed a SA algorithm to solve their robust layout design model in dynamic environment. Moslemipour and Lee (2012) proposed a new nonlinear mathematical model for designing a dynamic layout in uncertain environment of the FMS where the independent product demands are normally distributed random variables with known PDF, which changes from period to period. Lee and Moslemipour (2012) proposed a new mathematical model for a multiple periods inter-cell layout problem in which the flow of materials is assumed to be a random variable with known expected value. Lee and Moslemipour (2012) proposed a new mathematical model for designing a machine layout having maximum stability for the whole time planning horizon of the stochastic dynamic facility layout problem by using quadratic assignment formulation. This layout has the maximum ability to display a small sensitivity to demand changeability.

Forghani, Mohammadi and Ghezavati (2013) proposed a new robust method to deal with cell formation and layout design problem by considering stochastic demands. Neghabi et al. (2014) developed a novel mathematical model named RABSMODEL along with a two-stage algorithm to design a robust layout in which facilities have flexible dimensions. Tavakkoli-Moghaddam, Sakhaii, Vatani (2014) proposed a robust optimization method to design a dynamic CMS by incorporating production planning so that processing time of parts is assumed to be stochastic. Hosseini, Al Khaled & Vadlamani (2014) developed a robust and simple hybrid approach founded on incorporating three meta-heuristic methods including imperialist competitive algorithms, variable neighborhood search, and SA to cope with a DFLP, competently.

3. Concurrent design of a robust inter and intra-cell layout

In this section, the new mathematical model is formulated by considering the following assumptions:

i. Equal-sized machines/cells are assigned to the same number of known machines/cells locations.
ii. The discrete representation of the SDFLP is considered.
iii. Demands of parts are independent normally distributed random variables with known expected value that change from period to period at random.
iv. Demands of a particular part in different time periods are independent of each other.
v. Time value of money is considered.
vi. The parts are moved in batches between facilities.
vii. The data on machine sequence, interest rate, part movement cost, batch size, distance between machine locations, and distance between cell locations, are known.
viii. There is no constraint for dimensions and shapes of the shop floor.
ix. Cell formation is accomplished in advance so that each cell is formed by a certain number of known machines used for doing known operations on parts.

Table 1 shows the notations used in this model.

Table 1: Notations

Notation	Description
K	Number of parts
M	Total number of machines / machine locations.
T	Number of periods under consideration
M_c	Number of machines / machine locations within cell c
C	Number of cells / cell locations
k	Index for parts ($k = 1, 2,. . . , K$)
t	Index for period ($t = 1, 2,..., T$)
i, j	Index for machines ($i, j = 1, 2,. . . , M$); $i \neq j$
l, q	Index for machine locations ($l, q = 1, 2,. . . , M$); $l \neq q$
c, w	Index for cells ($c, w = 1, 2,. . . , C$); $c \neq w$
u, v	Index for cell locations ($u, v = 1, 2,. . . , C$); $u \neq v$
N_{ki}	Operation number for the operation done on part k by machine i
f_{tijk}	Materials flow for part k between machines i and j in period t
f_{tij}	Materials flow for all parts between machines i and j in period t
f_{tcw}	Materials flow between cells c and w in period t
D_{tk}	Demand for part k in period t
B_k	Transfer batch size for part k
C_k	Present value of the movement cost per batch for part k
C_{tk}	Cost of movements for part k in period t
a_{tilq}	Fixed cost of shifting machine i from location l to location q in period t
a_{tcuv}	Fixed cost of shifting cell c from location u to location v in period t
d_{lq}	Distance between machine locations l and q
d_{uv}	Distance between cell locations u and v
x_{til}	Decision variable for intra-cell layout problem
x_{tcu}	Decision variable for inter-cell layout problem
$C(\pi)$	Total MHC for layout π
$E(\ )$	Expected value of a parameter
I_r	Interest rate
T_c	Total MHC and rearrangement costs for cell c
b_{ic}	Binary variable indicating the assignment of machine i to cell c
OFV	Objective Function Value (total cost of intra and inter-cell layouts)

In general, the FLP having discrete representation and equal-sized facilities assigned to the same number of locations is usually formulated as the QAP model. In discrete representation the shop floor is divided into a number of equal-sized facility locations.
In the robust layout design method, a dynamic (multi-period) layout problem is converted to a static (single period) problem. Therefore, the following 0-1 integer programming form of the QAP formulation suggested by Koopmans and Beckman (1957) is used to develop a model for the SDFLP using the robust approach:

$$Minimize \sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M} f_{ij} d_{lq} x_{il} x_{jq} \qquad (1)$$

Subject to:

$$\sum_{i=1}^{M} x_{il} = 1 \qquad ;\forall l \qquad (2)$$

$$\sum_{l=1}^{M} x_{il} = 1 \qquad ;\forall i \qquad (3)$$

$$x_{il} = \begin{cases} 1 & \text{if facility } i \text{ is assigned to location } l \\ 0 & \text{otherwise} \end{cases} \qquad (4)$$

The objective function Eq. (1) is a second-degree (quadratic) function of the decision variables. In this equation, f_{ij} denotes the flow of materials between facilities i and j. The distance between locations l and q is denoted by d_{lq}. In fact, the objective function represents the total MHC, which is calculated as the summation of the product of materials flow between facilities and distance between the locations of these facilities. The linear constraints (2) and (3) ensure that each location must be contained only one facility and each facility must be assigned to exactly one location respectively. Equation (4) represents the 0-1 integer decision variables x_{il} that are the solution of the problem so that they determine the location of each facility.

According to the assumption (i), the QAP model given in Equations (1) to (4) is used to formulate the new mathematical model for the SDFLP. The data on machine sequence, transfer batch size, the present value of part movement cost per batch, distance between machine locations, distance between cell locations, and the expected value, variance, and covariance of part demands in each periods are the *inputs* of the model. The *output* of the model is total MHC that must be minimized to design the best layout of machines within each cell (intra-cell layout) and the best layout of cells on the shop floor (inter-cell layout) for the entire time planning horizon.

In the inter and intra-cell layout problem, M machines are placed into C cells so that $\bigcup_{c=1}^{C} c = M$ and $\bigcap_{c=1}^{C} c = \varnothing$. Cell c includes M_c machines by considering Constraints (5) and (6).

$$\sum_{i=1}^{M} b_{ic} = M_c \, ; \quad c = 1, 2, \ldots, C \qquad (5)$$

$$b_{ic} = \begin{cases} 1 & \text{if machine } i \text{ is assigned to cell } c \\ 0 & \text{otherwise} \end{cases} \qquad (6)$$

Eq. (7) shows the formula for calculating f_{tijk}. In this equation, the condition $|N_{ki} - N_{kj}| = 1$ refers to two consecutive operations, which are done on part k by machines i and j. The formula for calculating f_{ijk} is given in Eq. (8), which is reformed as Eq. (9) in combination with Eq. (7). Actually, according to Eq. (8), the average flow of part k of different time periods f_{ijk} is considered as the flow of this part over the entire time planning horizon. The total flow between machines i and j resulting from all parts (f_{ij}) is calculated as Eq. (10), which is written as Eq. (11), after combining with Eq. (9). Finally, the Eq. (11) is rearranged as Eq. (12). In Eq. (12), D_{tk} is a normally distributed random variable with the expected value $E(D_{tk})$, therefore, f_{ij} is a normally distributed random variable with the expected value given in Eq. (13).

$$f_{tijk} = \begin{cases} \frac{D_{tk}}{B_k} C_{tk} & \text{if } |N_{ki} - N_{kj}| = 1 \\ 0 & \text{otherwise} \end{cases} \qquad (7)$$

$$f_{ijk} = \frac{\sum_{t=1}^{T} f_{tijk}}{T} \qquad (8)$$

$$f_{ijk} = \frac{1}{T} \sum_{t=1}^{T} \frac{C_{tk}}{B_k} D_{tk} \qquad (9)$$

$$f_{ij} = \sum_{k=1}^{K} f_{ijk} \qquad (10)$$

$$f_{ij} = \sum_{k=1}^{K} \sum_{t=1}^{T} \frac{C_{tk}}{T.B_k} D_{tk} \qquad (11)$$

$$f_{ij} = \sum_{t=1}^{T} \sum_{k=1}^{K} \frac{C_{tk}}{T.B_k} D_{tk} \qquad (12)$$

$$E(f_{ij})=\sum_{t=1}^{T}\sum_{k=1}^{K}\frac{C_{tk}}{T.B_k}E(D_{tk}) \quad (13)$$

Since we consider time value of money, the movement cost for part k in period t can be calculated using the formula given in Eq. (14). In this equation, c_k is the present value of the movement cost for part k and I_r is the *interest rate* for each period.

$$c_{tk}=c_k(1+I_r)^t \quad (14)$$

According to Eq. (1), the MHC for the given layout π (*i.e.* $C(\pi)$) is defined as Eq. (15). Using Eq. (15) being f_{ij} as a random variable with normal distribution leads to being $C(\pi)$ as a normally distributed random variable with the expected value given in Eq. (16). Combining equations (13), (14), and (16) leads to the Eq. (17).

$$C(\pi)=\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M}f_{ij}d_{lq}x_{il}x_{jq} \quad (15)$$

$$E\big(C(\pi)\big)=\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M}E\big(f_{ij}\big)d_{lq}x_{il}x_{jq} \quad (16)$$

$$E\big(C(\pi)\big)=\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{l=1}^{M}\sum_{q=1}^{M}\sum_{t=1}^{T}\sum_{k=1}^{K}\left(\frac{c_k(1+I_r)^t}{T.B_k}\right)E\big(D_{tk}\big)d_{lq}x_{il}x_{jq} \quad (17)$$

3.1 Intra-cell layout design:

To obtain the optimal layout of machines, which are located in cell c, for each period, the objective function T_c can be written as Eq. (18) by using Eq. (17).

$$T_c=\sum_{i=1}^{M_c}\sum_{j=1}^{M_c}\sum_{l=1}^{M_c}\sum_{q=1}^{M_c}\sum_{k=1}^{K}\sum_{t=1}^{T}\left(\frac{c_k(1+I_r)^t}{T.B_k}\right)E\big(D_{tk}\big)d_{lq}x_{il}x_{jq} \quad (18)$$

Since there are C cells in this problem, the total cost of the intra-cell layouts can be calculated as the summation of every intra-cell layout cost by using the Eq. (19). By doing so, the mathematical model to design of the intra-cell layouts can be written as follows:

$$Minimize\sum_{c=1}^{C}T_c \quad (19)$$

Subject to: *Equations* (2), (3), *and* (4).

3.2 Inter-cell layout design:

In this section, the new mathematical model to determine the relative location of each cell in the shop floor, which is defined as inter-cell layout design, is proposed. The total flow of parts between cells c and w can be calculated by using Eq. (20). In Eq. (20), since f_{ij} is a random variable, f_{cw} is also a random variable with the expected value given in Eq. (21). In Eq. (21), the variables of $E(f_{ij})$ and b_{ic} are defined in equations (13) and (6) respectively.

$$f_{cw}=\sum_{i=1}^{M}\sum_{j=1}^{M}f_{ij}b_{ic}b_{jw} \quad (20)$$

$$E(f_{cw})=\sum_{i=1}^{M}\sum_{j=1}^{M}E(f_{ij})b_{ic}b_{jw} \quad (21)$$

In the inter-cell layout design process, the cells are regarded as facilities. Therefore, using the equations (2), (3), (4), (17), and (21), the mathematical model for the inter-cell layout design can be written as follows:

Minimization of

$$\sum_{c=1}^{C}\sum_{w=1}^{C}\sum_{i=1}^{M}\sum_{j=1}^{M}\sum_{k=1}^{K}\sum_{t=1}^{T}\left(\frac{c_k b_{ic} b_{jw}(1+I_r)^t}{T.B_k}\right)E\big(D_{tk}\big)d_{uv}x_{cu}x_{wv} \quad (22)$$

Subject to:

$$\sum_{c=1}^{C}x_{cu}=1 \qquad ;\forall u \quad (23)$$

$$\sum_{u=1}^{C}x_{cu}=1 \qquad ;\forall c \quad (24)$$

$$x_{cu}=\begin{cases}1 & \text{if cell } c \text{ is assigned to location } u\\ 0 & \text{otherwise}\end{cases} \quad (25)$$

3.3 Intra and inter-cell layout design

Finally, the new mathematical model for concurrently design of inter-cell and intra-cell layout in multi-period and uncertain environment of the manufacturing system can be written as follows:

Minimization OFV={*Intra-cell cost* (Eq.(19)+*Inter-cell cost* (Eq.(22)}

(26)

Subject to:

Equations (2), (3), (4), (6), (23), (24), *and* (25).

4. Computation results and analysis

To validate the proposed model, a randomly generated test problem as numerical example is solved by the SA algorithm programmed in Matlab.The test problem includes ten parts, twelve machines grouped into three cells, and ten time periods. The three groups of machines, including (1,2,3,4), (5,6,7,8), and (9,10,11,12) constitute the cells 1, 2, and 3 respectively. The input data for parts, including the data on machine sequence, batch size, and movement cost, are given in Table 2. In this table, for example, the machine sequence for part 9 is 2→3→8. It means that the first, second, and third operations on part 9 are done by machines 2, 3, and 8 respectively. The distance between machine locations and the distance between cell locations are given in Tables 3 and 4 respectively. The randomly generated mean value of part demands for each period is given in Table 5. The initial solution required by the SA algorithm to solve the test problem is given in Table 6. This solution consists of the initial machine layout within each cell (intra-cell layout) and the initial layout of cells on the shop floor (inter-cell layout). Here, the solution of the dynamic inter and intra-cell layout problem is given as a row matrix where each column represents a location, and each element represents a machine/cell number.

Table 2: Input Data

Parts (k)	Machine sequence	Batch size (B_k)	movement cost (C_k)
1	2→3→6→11→9	50	5
2	1→4→3→9	50	5
3	3→4→7→8→10	50	5
4	3→5→8→2	50	5
5	8→3→10→12	50	5
6	5→6→7→9→11	50	5
7	4→8→6→10→11	50	5
8	11→10→1→6	50	5
9	2→3→8	50	5
10	6→3→12→10	50	5

Table 3: Distance between machine locations

From \ To	1	2	3	4	5	6	7	8	9	10	11	12
1	0	10	20	30	40	50	70	60	50	40	30	20
2	10	0	10	20	30	40	60	50	40	30	20	30
3	20	10	0	10	20	30	50	40	30	20	30	40
4	30	20	10	0	10	20	40	30	20	30	40	50
5	40	30	20	10	0	10	30	20	30	40	50	60
6	50	40	30	20	10	0	20	30	40	50	60	70
7	70	60	50	40	30	20	0	10	20	30	40	50
8	60	50	40	30	20	30	10	0	10	20	30	40
9	50	40	30	20	30	40	20	10	0	10	20	30
10	40	30	20	30	40	50	30	20	10	0	10	20
11	30	20	30	40	50	60	40	30	20	10	0	10
12	20	30	40	50	60	70	50	40	30	20	10	0

Table 4: Distance between cell locations

From \ To	1	2	3
1	0	100	200
2	100	0	200
3	200	100	0

Table 5: Mean value of part demands

Period \ Part	1	2	3	4	5	6	7	8	9	10
1	6220	2565	7623	2067	8965	8736	6823	6088	6907	4093
2	5656	8863	9120	4347	2358	9998	8104	9696	7493	5496
3	3764	6636	3543	2646	2720	7804	6861	3116	4458	1606
4	6503	9101	4554	9746	7540	3677	4910	9253	5141	7172
5	6503	7589	5948	8496	8085	2066	8772	8257	6664	5258
6	5468	7614	7543	3220	3502	1784	2627	1487	8362	4417
7	6510	2045	8514	1847	7706	3538	2278	6682	9602	2105
8	1060	1231	6809	6784	6153	8842	2833	1104	5478	2384
9	7409	2802	3857	4097	9746	2321	1311	3767	7343	8405
10	2132	4250	1380	1386	5346	4873	6757	6488	6680	4007

Table 6: Initial solution (Robust layout)

	Intra-cell layout			Inter-cell layout
	Cell 1	Cell 2	Cell 3	
Location	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3
Facility	1 2 3 4	5 6 7 8	9 10 11 12	1 2 3

The best solution of the robust inter and intra-cell layout problem obtained by solving the test problem using SA algorithm is displayed in Table 7. In fact, using these solutions, the total material handling cost of inter and intra-cell layouts (*OFV*) defined by Eq. (26) is minimized. These results include the best layout of machines within each cell and the best layout of cells on the shop floor along with their corresponding cost function value, the total value of intra-cell cost, the total cost of intra and inter-cell layout (*OFV*), and elapsed computation time. The objective function values obtained by running the SA algorithm ten times are evaluated statistically. The results obtained from the statistical evaluation, including the worst, best, and standard deviation (Std. Dev.) of the objective function values are given in Table 8. The statistical evaluation shows that the *OFV*s are pretty close to each other. As a result, SA algorithm is a robust and promising tool to solve the proposed model with good solution quality and reasonable computaion time.

Table 7: The best solution (Robust layout)

	Intra-cell layout			Inter-cell layout
	Cell 1	Cell 2	Cell 3	
Location	1 2 3 4	1 2 3 4	1 2 3 4	1 2 3
Facility	2 3 4 1	7 8 5 6	12 10 11 9	2 1 3
Cost	47846	76785	54430	Inter-cell cost = 2191058
	Intra-cell cost = 179061			
OFV_{rc} = 2370119			Elapsed time = 1.258761 (seconds)	

Table 8: Statistical evaluation

Objective Function Value (*OFV*) - (10 trials)			
Worst	Mean	Best	Std. Dev.
2375400	2372231.4	2370119	2727.10

4.2 Sensitivity Analysis

To validate the proposed model, the sensitivity of the output (objective function) with respect to the expectation of materials flow as the input parameter of the proposed model is investigated. The sensitivity analysis is done in such a way that the expectation matrix E is changed as ($E' = E + r * E$), where r is a real number. It is necessary to mention that considering $r = 0$ the matrix E' is equal to E, which is corresponding to the best solution given in Table 7. Table 9 displays the values of the objective function with respect to different expectation matrix for the test problem. Actually, changes in the coefficient r lead to changes in the expectation matrix. The results indicate that the objective function value (*OFV*) enhances proportional to increase of the expectation of materials flow as it is expected. The diagram of the objective function versus the expectation of the materials flow is depicted in Figure 1, which illustrates this conclusion.

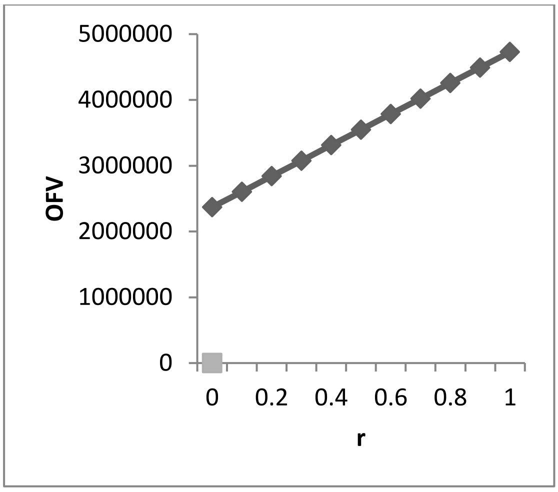

Figure 1: *OFV* with respect to E'

Table 9: *OFV* with respect to the expectation of materials flow

r	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
OFV	2370119	2605884	2841649	3077400	3313179	3548944	3784709	4020474	4256239	4492004	4727769

5. Conclusion

This paper proposed a new nonlinear QAP-based mathematical model for concurrent design of a robust inter and intra-cell layout in uncertain and multi-period environment of manufacturing systems where the independent product demands are normally distributed random variables with known expected value, which changes from period to period. This model was validated by solving a randomly generated test problems. The obtained conclusions can be summarized as follows: (i) according to statistical evaluation, the *OFV*s are pretty close to each other. As a result, SA algorithm is a robust and promising tool to solve the proposed model with good solution quality and reasonable computaion time. (ii) sensitivity analysis indicates that the objective function value (*OFV*) enhances proportional to increase of the expectation of materials flow as it is expected. Therefore, the model validation is approved. Finally, the following works can be taken into consideration in the future researches:

1. Concurrent design of a dynamic inter and intra-cell layout so that the best layout of each period is found.

2. Considering variance of product demand in each period in addition to the expected value.

3. Proposing a more effective hybrid meta-heuristic approach by combining the SA with other intelligent approaches to improve the quality of the solution and the computation time.

4. Considering some constraints such as unequal-sized machines/cells, adding and removing machines in different periods, closeness ratio, aisles, routing flexibility, and budget constraint for total cost.

References

Alvarenga, A. G., Gomes, N. J., & Mestria, M. "Metaheuristic methods for a class of the facility layout problems," *Journal of Intelligent Manufacturing,* Vol.11, 2000, pp. 421-430.

Azadeh, S., Haghighi, S. M., & Asadzadeh, S. M. "A novel algorithm for layout optimization of injection process with random demands and sequence dependent setup times," *Journal of Manufacturing Systems*, *33*, 2014, pp.287–302.

Balakrishnan, J., & Cheng, C. H. "The dynamic plant layout problem: Incorporating rolling horizons and forecast uncertainty," *Omega*, Vol. 37, 2009,pp.165 – 177.

Benjaafar, S., Heragu, S. S., & Irani, S. A. "Next generation factory layouts: research challenges and recent progress," *Interfaces,* Vol.32, No.(6), 2002, pp.58 - 77.

Benjaafar, S., & Sheikhzadeh, M. " Design of flexible layouts," *IIE Transactions,* Vol.32, 2000, pp. 309-322.

Braglia, M., Simone, Z., & Zavanella, L. " Layout design in dynamic environments: Strategies and quantitative indices" *International Journal of Production Research,* Vol.41, No.(5), 2003, pp. 995–1016.

Braglia, M., Simone, Z., & Zavanella, L. "Layout design in dynamic environments: Analytical issues" *International Transactions in Operational Research,* Vol. 12, 2005, pp.1–19.

Enea, M., Galante, G., & Panascia, E. "The facility layout problem approached using a fuzzy model and a genetic search" *Journal of Intelligent Manufacturing,* Vol. 16, 2005, pp.303–316.

Forghani, K., Mohammadi, M., & Ghezavati, V. " Designing robust layout in cellular manufacturing systems with uncertain demands," *International Journal of Industrial Engineering Computations,* Vol.4, No.(2), 2013, pp.215-226.

Foulds, L. R. "Techniques for Facilities Layout: Deciding which Pairs of Activities should be Adjacent," *Management Science*, Vol.29, No.(12), 1983, pp.1414-1426.

Hassan, M. M. D. "Machine layout problem in modern manufacturing facilities," *International Journal of Production Research*, Vol.32, No.(11), 1994, pp.2559–2584.

Hosseini, S., Khaled, A. A., & Vadlamani, S. "Hybrid imperialist competitive algorithm, variable neighborhood search, and simulated annealing for dynamic facility layout problem," *Neural Comput & Applic*, Vol. 25, 2014, pp.871–1885, DOI 1810.1007/s00521-00014-01678-x.

Hsieh, F. S. "Collaborative reconfiguration mechanism for holonic manufacturing systems," *Automatica*, Vol.45,No.(11), 2009a ,pp.2563-2569.

Hsieh, F. S. "Dynamic composition of holonic processes to satisfy timing constraints with minimal costs," *Engineering Applications of Artificial Intelligence*, Vol.22, No.(7), 2009b ,pp.1117-1126.

Hsieh, F. S., & Chiang, C. Y. "Collaborative composition of processes in holonic manufacturing systems," *Computers in Industry*, Vol. 62, No. (1), 2011,pp.51-64.

Irappa, B. H., & Madhusudanan, P. V. " Development of a heuristic for layout formation and design of robust layout under dynamic demand," *Proceedings of the International Conference on Digital Factory, ICDF 2008, August 11-13*, pp.1398 -1405.

Koopmans, T. C., & Beckman, M. "Assignment problems and the location of economic activities," *Econometric*, Vol. 25, 1957, pp.53-76.

Kouvelis, P., & Kiran, A. S. "Single and multiple period layout models for automated manufacturing systems," *European Journal of Operational Research*, Vol. 52, 1991, pp.300–314.

Kouvelis, P., Kuawarwala, A. A., & Gutierrez, G. J. "Algorithms for robust single and multiple period layout planning for manufacturing systems," *European Journal of Operational Research*, Vol. 63, 1992, pp.287–303.

Kulturel-Konak, S., Smith, A. E., & Norman, B. A. "Layout optimization considering production uncertainty and routing flexibility," *International Journal of Production Research*, Vol.42, No.(21),pp. 4475–4493.

Lee, T. S., & Moslemipour, G. "A dynamic and stochastic inter-cell layout design in a cellular manufacturing system," *Advanced Materials Research, special issue: Manufacturing Science and Technology*, Vol. *383-390*, 2012, pp. 1039-1046.

Lee, T. S., & Moslemipour, G. "Intelligent Design of a Flexible Cell Layout with Maximum Stability in a Stochastic Dynamic Situation," *Communication in Computer and Information Science*, 2012.

Madhusudanan-Pillai, V., Irappa-Basappa, H., & Krishna, k. K. (2011). Design of Robust Layout for Dynamic Plant Layout Problems. *Computers & Industrial Engineering*, Vol.61, 2011, pp.813-823.

Montreuil, B., LeFrancois, P., Marcotte, S., & Venkatadri, U. "Holographic layout of manufacturing systems operating in chaotic environments," *Technical Report 93-53, Document de Recherche GRGL, Faculte des Sciences de l "Administration, Universite Laval, Quebec*, 1993.

Moslemipour, G., & Lee, T. S. "Intelligent design of a dynamic machine layout in uncertain environment of flexible manufacturing systems," *J Intell Manuf*, Vol.23, No.(5), 2012, pp. 1849-1860.

Neghabi, H., Eshghi, K., & Salmani, M. H. "A new model for robust facility layout problem," *Information Sciences*, Vol.278, , 2014, pp. 498–509.

Norman, B. A., & Smith, A. E. "A continuous approach to considering uncertainty in facility design," *Computers and Operations Research*, Vol.33, 2006, pp.1760–1775.

Ripon, K. S. N., Glette, K., Hovin, M., & Torresen, J. "Dynamic facility layout problem under uncertainty: a Pareto-optimality based multi-objective evolutionary approach," *Cent. Eur. J. Comp. Sci*, Vol.1, No.(4), 2011, pp.375-386.

Rosenblatt, M. J., & Lee, H. L. "A robustness approach to facilities design," *International Journal of Production Research*, Vol.25, No.(4), 1987, pp.479–486.

Sahni, S., & Gonzalez, T. (1976). P-complete approximation problem. *Journal of ACM, 23*(3), 555-565.

Tavakkoli-Moghaddam, R., Aryanezhad, M. B., Safaei, N., & Azaron, A. "Solving a dynamic cell formation problem using metaheuristics," *Applied Mathematics and Computation*, Vol.170, 2005, pp.761-780.

Tavakkoli-Moghaddam, R., Javadi, B., Jolai, F , & Mirghorbani, S. M. "An efficient algorithm to inter and intra-cell layout problems in cellular manufacturing systems with stochastic demands," *Int. J. of Engineering, Transactions A: Basic*, Vol.19,No.(1), 2006, pp.67-78.

Tavakkoli-Moghaddam, R., Javadian, N., Javadi, B., & Safaei, N. "Design of a facility layout problem in cellular manufacturing systems with stochastic demands," *Applied Mathematics and Computation*, Vol.184, 2007,pp.721–728.

Tavakkoli-Moghaddam, R., Sakhaii, M., & Vatani, B. "A Robust Model for a Dynamic Cellular Manufacturing System with Production Planning," *IJE TRANSACTIONS A: Basics*, *27*(4), 2014, pp. 587-598.

Tompkins, J. A., White, J. A., Bozer, Y. A., & Tanchoco, J. M. A. "*Facilities planning*," New York: Wiley, 2003.

Vitayasak, S., Pongcharoen, P., & Chris Hicks, C. "A tool for solving stochastic dynamic facility layout problems with stochastic demand using either a Genetic Algorithm or modified Backtracking Search Algorithm," *Int. J. Production Economics*, 2016. http://dx.doi.org/10.1016/j.ijpe.2016.03.019.

Zhao, Y., & Wallace, S. W. "Integrated Facility Layout Design and Flow Assignment Problem Under Uncertainty," *INFORMS Journal on Computing*, Vol.26, No.(4), 2014, pp. 798–808.

Zhao, Y., & Wallace, S. W. "A heuristic for the single-product capacitated facility layout problem with random demand," *EURO J Transp Logist*, Vol. 4, 2015, pp.379–398, DOI 310.1007/s13676-13014-10052-13676.