Mean Residual lifetime Analysis of Systems with Weibull Lifetime

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Abstract

The concept of "residual lifetime" is a useful tool to study the reliability engineering. In the present article, we consider a system with weibull distribution and truncated weibull distribution lifetime and assume that the system is working at time t. Under these conditions, the mean residual lifetime of the k-out-of-n system is obtained under different scenarios on the system. Several properties of the mean residual lifetime are derived. In addition, the discussed concepts are explained by working out real numerical example about airplane tires.

Keywords:

Reliability function, Mean residual lifetime, Hazard rate, Weibull distribution, Truncated weibull distribution.

Introduction

In reliability engineering, to study the aging behavior of a system, there have been defined a measure such as the mean residual lifetime (MRL). The mean residual lifetime is a useful tool to study the burn-in and maintenance policies. Suppose that the random variable continuous T denotes the lifetime of a system with reliability function $R(t) = 1 - F(t)$, where $F(t)$ denote the distribution function of T. The residual lifetime of the system is defined as

$$T_t = \{T - t | T \geq t\}. \qquad (1)$$

The conditional random variable T_t gives the residual lifetime of the system with lifetime T when the system is still alive at time t. The expected value of T_t, which we denote throughout this paper by $M(t)$, is studied by several authors. The mean residual lifetime (MRL) of system is defined as

$$M(t) = E(T - t | T > t) = \frac{\int_t^\infty R(x)dx}{R(t)}. \qquad (2)$$

For all t provided $R(t) > 0$. In fact the MRL is the expected value of the remaining lifetime $T - t$ given that T has survived at least t units of time. The mean residual lifetime is closely related to the hazard rate $h(t) = \frac{f(t)}{R(t)}$. In fact

$$h(t) = \frac{M'(t) + 1}{M(t)}. \qquad (3)$$

The MRL of a system is studied by several authors, see for example Kots and Shanbhag (1980) and Guess and Proschan (1988).
In this paper we discuss about the MRL of the systems for which the distribution function is one of the following models:

i) Weibull distribution
ii) Truncated weibull distribution

The weibull distribution has been used in many different applications and for solving a variety of problems from many different disciplines. Also, the truncated weibull distribution is widely utilized in industry, for example suppose counting of the failures of an item during the warranty period may be missed or items may be replaced after a certain time following the replacement policy, so that failures of the items are not considered.
The aim of this paper is to investigate more properties of the MRL of a system. At first, we concentrate on the MRL of a system with lifetime T which is distributed according to a weibull and truncated weibull distribution. We also consider a (n-k+1)-out-of-n system and assume that all or (n-r+1), r=1, 2, …, k, components of the system are working. Among the results in this section, we represent the formula for the MRL of the system in terms of the distribution function and show for a system with DFR components, the MRL of the system is not necessarily increasing in time. Finally, a real numerical example about airplane tires is provided.

The MRL of a system with weibull distribution

In reliability engineering, survival analyses and industry, the weibull distribution has been shown to be useful. There are many papers dealing with various aspects of weibull inference and modeling, as well as application. Also, the truncated weibull distribution is widely utilized in industry, for example suppose counting of the failures of an item during the warranty period may be missed or items may be replaced after a certain time following the replacement policy, so that failures of the items are not considered.
The probability density function of a weibull random variable is

$$f(t) = \begin{cases} \frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1} e^{-\left(\frac{t}{\beta}\right)^{\alpha}}, & t > 0, \\ 0, & t \le 0, \end{cases} \quad (4)$$

where $\alpha > 0$ is the shape parameter and $\beta > 0$ is the scale parameter of the distribution. A number of other probability distributions relate to The weibull distribution; in particular, it interpolates between the exponential distribution ($\alpha = 1$) and the Rayleigh distribution ($\alpha = 2$ and $\beta = \sqrt{2}\sigma$).
The Reliability function for the weibull distribution is

$$R(t) = e^{-\left(\frac{t}{\beta}\right)^{\alpha}}, \quad t \ge 0. \quad (5)$$

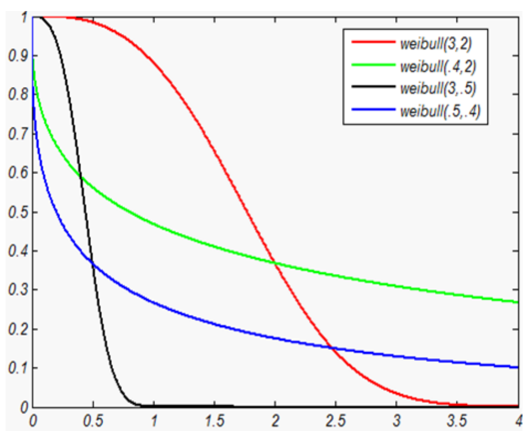

Figure1. The reliability of the system for weibull distribution

The MRL of the weibull distribution is given by

$$M(t) = e^{\left(\frac{t}{\beta}\right)^{\alpha}} \int_t^{\infty} e^{-\left(\frac{x}{\beta}\right)^{\alpha}} dx, \quad x > 0. \quad (6)$$

and the hazard rate function can also be represented as follows

$$h(t) = \frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1}, \quad t > 0. \quad (7)$$

Theorem 1. In weibull distribution function if $\alpha > 1$ ($\alpha < 1$),hazard rate is increasing(decreasing) in t.

Proof: Note that

$$\frac{d}{dt}h(t) = \frac{d}{dt}\left(\frac{\alpha}{\beta}\left(\frac{t}{\beta}\right)^{\alpha-1}\right) = \frac{\alpha(\alpha-1)}{\beta^2}\left(\frac{t}{\beta}\right)^{\alpha-2} \quad (8)$$

It is obvious in above equation when $\alpha > 1, (\alpha < 1)$,

$$\frac{d}{dt}h(t) > 0, \left(\frac{d}{dt}h(t) < 0\right) \quad (9)$$

This result is shown in Figure 2.

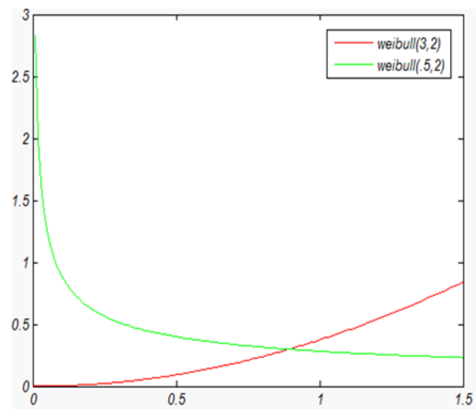

Figure2. Figure of The hazard rate function of the system

Marshall and Olkin (1997) introduced a three-parameter distribution called extended weibull distribution by adding parameter into a family of distributions. The truncated weibull distribution contains an additional parameter. It has the probability density function

$$f(t) = \begin{cases} \frac{\alpha}{\beta}\left(\frac{t-\theta}{\beta}\right)^{\alpha-1} e^{-\left(\frac{t-\theta}{\beta}\right)^{\alpha}} & t > \theta, \\ 0, & t \le \theta, \end{cases} \quad (10)$$

where $\alpha > 0$ is the shape parameter, $\beta > 0$ is the scale parameter and θ is the location parameter of the distribution. When $\theta = 0$, this reduces to the 2-parameter distribution. The Reliability function for the truncated weibull distribution is given by

$$R(t) = e^{-\left(\frac{t-\theta}{\beta}\right)^{\alpha}}, \quad t > \theta. \quad (11)$$

The mean residual lifetime of the truncated weibull distribution , $M(t)$, can be stated as

$$M(t) = e^{\left(\frac{t-\theta}{\beta}\right)^{\alpha}} \int_t^{\infty} e^{-\left(\frac{x-\theta}{\beta}\right)^{\alpha}} dx. \qquad (12)$$

and the hazard rate is obtained as

$$h(t) = \frac{\alpha}{\beta}\left(\frac{t-\theta}{\beta}\right)^{\alpha-1}, \quad t > \theta. \qquad (13)$$

The MRL of a k-out-of-n system

In recent years, researchers in reliability theory have shown intensified interest in the study of reliability properties of technical systems. The $(n-k+1)$ -out-of- n system structure is a very popular type of redundancy in technical systems. A $(n-k+1)$-out-of-n system is a system consisting of n components (usually the same) and functions if and only if at least $n-k+1$ out of n components are operating ($k \leq n$). Hence, such system fails if k or more of its components fail. Let $T_1, T_2, \dots, T_n$ denote the component lifetimes of the system and assume that $T_{1:n}, T_{2:n}, \dots, T_{n:n}$ represent the ordered lifetimes of the components. Then it is easy to argue that the lifetime of the system is $T_{k:n}$ where $T_{k:n}$ denotes the k, the order statistics corresponding to T_i 's, i =1,2,…,n .
We first assume that at time t, all components of the system are working and obtaining the functional form of the mean of $T_t^{r,k,n}$, when T_i's as distributed weibull. This is in fact the MRL of the system, denoted by $M_n^k(t)$, under the condition that all components of the system are operating at time t, i.e. $T_{1:n}>t$. The residual lifetime of the system, under the condition that all components of the system are working at time t, is

$$T_t^{1,k,n} = (T_{k:n} - t \mid T_{1:n} > t). \qquad (14)$$

Hence the MRL function of the system, can be obtained as follows:

$$M_n^k(t) = E\,(T_{k:n} - t \mid T_{1:n} > t)$$

$$= \sum_{s=0}^{k-1}\binom{n}{s}\int_0^{\infty}\left(\frac{\bar{F}(x+t)}{\bar{F}(t)}\right)^{n-s}\left(1-\frac{\bar{F}(x+t)}{\bar{F}(t)}\right)^{s} dx$$

$$= \sum_{s=0}^{k-1}\binom{n}{s}\int_0^{\infty}\left(\frac{\bar{F}(x+t)}{\bar{F}(t)}\right)^{n-s}\sum_{j=0}^{s}\binom{s}{j}(-1)^j\left(\frac{\bar{F}(x+t)}{\bar{F}(t)}\right)^{j}dx$$

$$= \sum_{s=0}^{k-1}\sum_{j=0}^{s}\binom{n}{s}\binom{s}{j}(-1)^j\int_0^{\infty}\left(\frac{\bar{F}(x+t)}{\bar{F}(t)}\right)^{n-s+j} dx$$

$$= \sum_{s=0}^{k-1}\sum_{j=0}^{s}\binom{n}{s}\binom{s}{j}(-1)^j\int_0^{\infty}\left(e^{(\frac{t}{\beta})^{\alpha}-(\frac{t+x}{\beta})^{\alpha}}\right)^{n-s+j} dx. \qquad (15)$$

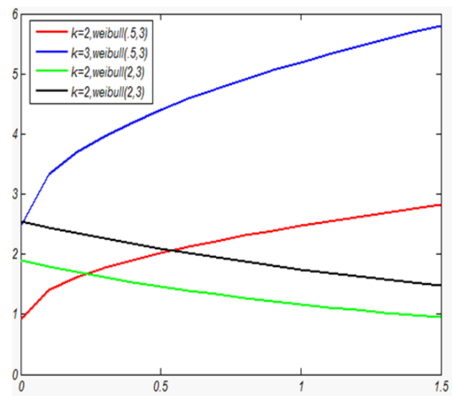

Figure3. Figure of the MLR of the system

The distribution function of the order statistics $T_{k:n}$ can be represented in terms of incomplete beta function as follows:

$$P\,(T_{k:n} < x) = \sum_{j=k}^{n}\binom{n}{j}F(x)^j(1-F(x))^{n-j}$$

$$= \frac{1}{B(k, n-k+1)}\int_0^{F(x)} u^{r-1}(1-u)^{n-r}du.$$

So we can write

$$M_n^k(t) = \int_0^{\infty}\frac{1}{B(k, n-k+1)}\int_{1-\frac{\bar{F}(x+t)}{\bar{F}(t)}}^{1} u^{k-1}(1-u)^{n-k}du\,dx$$

$$= \int_0^{\infty}\frac{1}{B(k, n-k+1)}\int_{1-e^{(\frac{t}{\beta})^{\alpha}-(\frac{t+x}{\beta})^{\alpha}}}^{1} u^{k-1}(1-u)^{n-k}du\,dx.$$

In following theorem, we prove that if the components of the system for weibull distribution have increased (decreased) failure rate, $M_n^k(t)$, is a decreasing(increasing) function of t.

Theorm 2. If the components of the $(n-k+1)$-out-of-n system for weibull distribution have an increasing (decreasing) hazard rate, then $M_n^k(t)$ is decreasing (increasing) in t.

Proof: Note that

$$\frac{d}{dt}M_n^k(t)$$
$$=\frac{d}{dt}\left(\int_0^{\infty}\frac{1}{B(k,n-k+1)}\int_{1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}}^{1}u^{k-1}(1-u)^{n-k}du\,dx\right)$$
$$=\int_0^{\infty}\frac{1}{B(k,n-k+1)}\frac{d}{dt}\int_{1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}}^{1}u^{k-1}(1-u)^{n-k}du\,dx.$$

Since

$$\frac{d}{dt}\int_{1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}}^{1}u^{k-1}(1-u)^{n-k}du$$
$$=\frac{\alpha}{\beta}e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}\left(\left(\frac{t}{\beta}\right)^{\alpha-1}-\left(\frac{t+x}{\beta}\right)^{\alpha-1}\right)\left(1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}\right)^{k-1}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}\right)^{n-k},$$

for $\alpha>1(\alpha<1)$, $\frac{d}{dt}H_n^k(t)$ is negative(positive) thus $M_n^k(t)$ is decreasing(increasing). (Note that in weibull distribution (α,β) for $\alpha>1(\alpha<1)$, h(t) is increasing. (decreasing)).
In what follows, we focus on the MRL of a $(n-k+1)$ −out-of-n system when at time t at least $(n-r+1)$ components of the system are working, the residual lifetime of the system can be defined as follows:

$$T_t^{r,k,n}=(T_{k:n}-t\,|\,T_{r:n}>t),\quad r=1,\dots,k,\ k=1,\dots,n. \tag{16}$$

Suppose $T_1,\dots,T_n$ denote the lifetime of n independent components which are connected in a $(n-k+1)$-out-of-n system and T_i be distributed as weibull (α,β) where α and β denote the shape, and the scale parameters of the distribution, respectively. Let, at time t, at least $n-r+1,(r\le k)$ components of the system are working under this condition, the reliability function $T_t^{r,k,n}$ is:

$$P(T_{k:n}-t>x\,|\,T_{r:n}>t)=\frac{P(T_{k:n}-t>x,T_{r:n}>t)}{p(T_{r:n}>t)}$$
$$\sum_{i=0}^{r-1}p_i(t)\sum_{u=0}^{k-i-1}\binom{n-i}{u}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}\right)^{n-i-u}\left(1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{t+x}{\beta}\right)^{\alpha}}\right)^{u}. \tag{17}$$

Where

$$p_i(t)=P(Z_i=i|Z_i\le r-1)=\frac{\binom{n}{i}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}}-1\right)^{i}}{\sum_{j=0}^{r-1}\binom{n}{j}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}}-1\right)^{j}}, \tag{18}$$

Z_i is a binomial random variable with parameters $\left(n,1-e^{\left(\frac{t}{\beta}\right)^{\alpha}}\right)$.

The MRL of the system can be expressed as:

$$M_n^{r,k}(t)=E(T_{k:n}-t\,|T_{r:n}>t)$$
$$=\int_0^{\infty}P(T_{k:n}>t+x|T_{r:n}>t)dx$$
$$=\int_t^{\infty}\frac{\sum_{i=0}^{r-1}\binom{n}{i}F^i(t)\bar{F}^{n-i}(t)\sum_{u=0}^{k-i-1}\binom{n-i}{u}\varphi_t^{\,n-i-u}(x)\left(1-\varphi_t(x)\right)^{u}}{\sum_{i=0}^{r-1}\binom{n}{i}F^i(t)\bar{F}^{n-i}(t)}dx$$
$$=\frac{\sum_{i=0}^{r-1}\binom{n}{i}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}}-1\right)^{i}\sum_{j=0}^{k-i-1}\binom{n-i}{j}\int_t^{\infty}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{x}{\beta}\right)^{\alpha}}\right)^{n-i-j}\left(1-e^{\left(\frac{t}{\beta}\right)^{\alpha}-\left(\frac{x}{\beta}\right)^{\alpha}}\right)^{j}dx}{\sum_{i=0}^{r-1}\binom{n}{i}\left(e^{\left(\frac{t}{\beta}\right)^{\alpha}}-1\right)^{i}},$$

where

$$\varphi_t(x)=\frac{\bar{F}(x)}{\bar{F}(t)}. \tag{19}$$

We know when T is IFR(DFR) then the MRL is decreasing (increasing) at time but in this section, the components of the system are distributed as weibull with scale parameter $\beta=1$, and shape parameter $\alpha=0.5$ (which is a DFR distribution in this case), the $M_n^{r,k}(t)$ is not increasing for all values of n,k and r. As Fig. 1 indicates, the MRL $M_7^{5,5}(t)$ first decreases for a period of time, and then starts to increase.

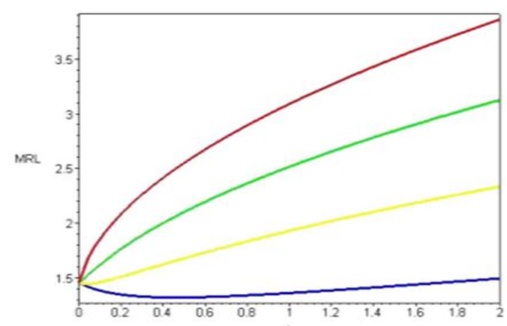

Figure4. The MLR of the system for the weibull (0.5,1) distribution with n=7, k=5, and r=2,3,4,5, from the top respectively

Numerical Example : the MRL analysis of airplane tiers

Tiers of airplane are subject to a number of wear out processes, i.e. uniform wear, accelerated wear at certain spots, etc. In the airplane when the tiers are in contact with the runway on landing. The conditions of wear are far more server than the corresponding conditions in automobiles on the highways, but in the airplanes the loads are not so uniform, there is a variety

of shock loads, or a sever load spectrum is generated which can cause accelerated wear. Tire lifetime is introduced by the wear limited set by controlling aviation agencies. The tier is considered to have failure when the tire is damaged due to wear out processes reaches this critical limit. The time to reach this critical manifestation of wear can be obtained either by associated flight time.
The tires lifetime is a random value, rather a random quantity t, whose values are characterized by a lower case letter t, and bounded by $t_0 < t < \alpha$, where t_0, is the minimum guaranteed lifetime.
We know the form of the weibull distribution function is

$$F(t) = 1 - \exp\left\{-\left(\frac{t-t_0}{\lambda-t_0}\right)^{k-1}\right\}, \quad t > t_0. \qquad (20)$$

In the sequel, some approaches commonly used in fitting the weibull model to the failure data are as follows

1. Using the weibull probability paper, we assume $t_i's$ are specified and plot the estimated values of $\hat{F}(t) = \hat{F}(t_i) = \frac{i}{N+1}$ against t_i and determined k and λ from these plots.
2. Using the maximum likelihood estimated of the parameters and utilizing the Kolmogrov-Smirinov test check the validity of weibull model.
3. In the paper we will show how a spreadsheet-based analysis is so straightforward and it is obvious that there is no need to plot on probability paper, etc. The method to fit the data on a weibull plot is explained below. From equation (20) we have

$$\ln[1 - F(t)] = -\left(\frac{t-t_0}{\lambda-t_0}\right)^k,$$

and

$$\ln\left\{ln\left(\frac{1}{1-F(t)}\right)\right\} = k\ln(t-t_0) - k\ln(\lambda-t_0). \qquad (21)$$

Equation (3) has the from of $Y = aX + b$, where
Y=ln$\left\{ln\left(\frac{1}{1-F(t)}\right)\right\}$,
X=ln$(t-t_0)$,
$a = k$,
$b = -k\ln(\lambda-t_0)$.

For a given time to failure data organized in an ordered fashion, the distribution function F(t) can be substituted by its estimate F(t_i) using median rank function:

$$\hat{F}(t_i) = \frac{i}{N+1}, \qquad 0 \le i \le N.$$

Now we can fit Equation (21) to the experimental data $\hat{F}(t_i)$ versus $t_i - t_0$ for i=0,1,…,N. By performing, the linear regression analysis using linearly transformed equation (20), the parameters k and $\boldsymbol{\lambda}$ can be determined. This method implies that t_0 is known. The value of t_1 , or that $t_0 = st_1$. Where $0.85 < s < 1$.
A starting point can be taken as $t_0 = 0.9t_1$: if the straight line fit is poor, then this value can be adjusted between $0.85t_0 - 0.99t_0$. Until a good fit is obtained. This can easily be accomplished in a spreadsheet format. Where this trial-and-error procedure is quite convenient compared to when the data are manually plotted on a weibull probability paper.
In this part we will study a group of data obtained from a local aviation facility. This data represent time to failure of tires. The columns in table 1 represent the format of analysis as outlined below:

1. Flying time to *i*-th type failure t_i
2. $t_i - t_0$, where $t_0 = 70\ hours$.
3. Failure number of the type *i*, *i=1,2,…,N* (in this case *N*=28).
4. $\hat{F}(t_i) = \frac{i}{N+1}$.
5. $\hat{R}(t_i) = 1 - \hat{F}(t_i)$.
6. $\ln(t_i - t_0) = X_i$.
7. $\ln\left\{ln\left(\frac{1}{[1-\hat{F}(t_i)]}\right)\right\} = Y_i$.
8. Values of Y_i estimated from best fittied for various specified values of X_i.
9. $\hat{M}(t_i) = \frac{\int_{t_i}^{\infty}\hat{R}(x)dx}{\hat{R}(t_i)}$

Table 1. Analysis of the tire of airplane failure data(hours)

(1) i	(2) t_i	(3) $t_i - t_0$	(4) $\hat{R}$=1-$\hat{F}$	(5) $\hat{F} = \frac{i}{N+1}$	(6) $\ln(t_i - t_0) = x_i$
1	73	3	0.96	0.034	1.09
2	78	8	0.93	0.068	2.07
3	86	16	0.89	0.10	2.77
4	91	21	0.86	0.13	3.04
5	99	29	0.82	0.17	3.36
6	105	35	0.79	0.20	3.55
7	111	41	0.75	0.24	3.71
8	115	45	0.72	0.27	3.80
9	123	53	0.68	0.31	3.97
10	132	62	0.65	0.34	4.12
11	138	68	0.62	0.37	4.21
12	140	70	0.58	0.41	4.24
13	146	76	0.55	0.44	4.33
14	151	81	0.51	0.48	4.39
15	157	87	0.48	0.51	4.46
16	163	93	0.44	0.55	4.53
17	169	99	0.41	0.58	4.59
18	174	104	0.37	0.62	4.64
19	178	108	0.34	0.65	4.68
20	189	119	0.31	0.68	4.77
21	196	126	0.27	0.72	4.83
22	199	129	0.24	0.75	4.85
23	203	133	0.20	0.79	4.89

24	208	138	0.17	0.82	4.92
25	215	145	0.13	0.86	4.97
26	228	158	0.10	0.89	5.06
27	235	165	0.06	0.93	5.10
28	241	171	0.03	0.96	5.14

(7) $\ln\left(ln\left[\frac{1}{\hat{R}}\right]\right) = y_i$	(8) $Regression, \widehat{y_i}$	(9) $M(t_i) = \frac{\int_{t_i}^{\infty} \hat{R}(x)dx}{\hat{R}(t_i)}$
-3.34	-3.99	14.5
-2.63	-2.88	14
-2.21	-2.09	13.5
-1.90	-1.78	13
-1.66	-1.42	12.5
-1.45	-1.20	12
-1.28	-1.03	11.5
-1.13	-0.92	11
-0.99	-0.73	10.5
-0.86	-0.56	10
-0.74	-0.45	9.5
-0.62	-0.42	9
-0.51	-0.33	8.5
-0.41	-0.25	8
-0.31	-0.17	7.5
-0.22	-0.10	7
-0.12	-0.03	6.5
-0.03	0.02	6
0.06	0.06	5.5
0.15	0.17	5
0.25	0.24	4.5
0.35	0.26	4
0.45	0.30	3.5
0.56	0.34	3
0.68	0.40	2.5
0.81	0.49	2
0.98	0.54	1.5

Table 2. Regression output

Constant	**-5.24**
Standard error of Y estimated	.2726
R^2	0.94
Number of observations	28
Degrees of freedom	26
X coefficient	1.13
k	1.13
λ	171.701
t_0	70
Mean	167.1861

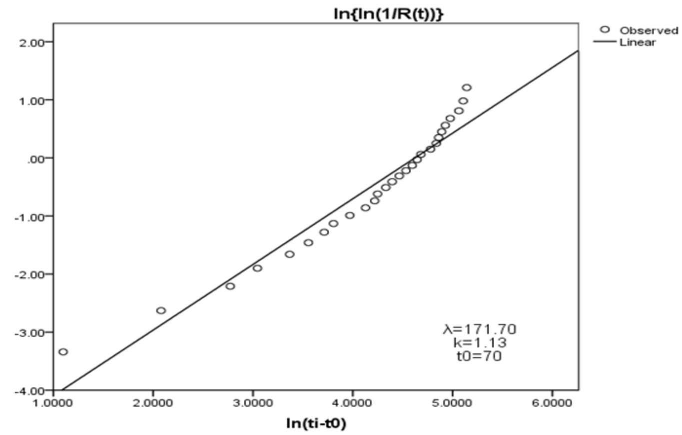

Figure5. The figure of the regression of data

Data of column 7 versus column 6 is plotted in below Figure5, note that the best fitted line is shown on this figure.
The line was obtained by linear regression of column 7 and column 6 from regression output in Table1.
The data of column 9 in Table1, represents the MRL of airplane tire at time *t*.
The table 2, shows the weibull parameters. Thus the reliability model for this data is:

$$R(t) = \exp\left\{-\left(\frac{t-70}{101.701}\right)^{1.13}\right\}, \qquad t > 70. \tag{22}$$

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