Dual-channel pricing model in the supply chain for replacement by taking the disruption risk using game theory

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Abstract:

In this paper Dual-channel replacement pricing model considering brand value in a two - level supply chain alternative was presented with the possibility of disruption risk, in order to meet a percent of the retail order by the Producer with the disruption risk. The rate for each of the goods is a combination of the ideal product prices, the price of the rival product and the distance from brand and also customers for every commodity were divided into two groups, loyal customers and indifferent customers, that the demand for every commodity is a total demand by loyal customers and indifferent customers. The model of problem is offered in two exclusive and Non-exclusive market monopoly conditions and in the exclusive market each retailer sells his product manufacturer and in the Non-exclusive retailer can sell the product of the both manufacturer. In both cases, exclusive and non-exclusive market, the model of problem has been solved by using stackelberg model the leader retailer and cooperative model. At the end the problem was conducted by using random data for the solved parameters and sensitivity analysis on important parameters.

Keywords: pricing, supply chain, replacement product, brand value, disruption risk, game theory

1-Introduction and review of the literature:

Profitability and ensure the profitability is one of the main reasons for the creation of a supply chain so that, every chain of the supply chain instead of its profits looks for profit maximization of the entire chain . Hence, pricing of single commodity or more products in the system will be of paramount importance. Meanwhile it may happen conditions in the supply chain that is likely to endanger the profitability of supply chain, or in other words, the risk would happen for the supply chain, one of the most important risks that the supply chain may be facing is the disruption risk that may occurs because of some reasons including machinery of destruction, lack of labor, and political issues and etc. that resulted in danger of supply chain profitability, so pricing in this case is of paramount importance. Most articles published in the field of pricing on commodities were about replacing goods and with the increased use of a product using other commodity decreases. But the few number of articles have pointed to replaced goods with different brands, so that suppose we have two same products of the different brands For example, the two companies for producing and packaging tea that both manufacture the tea product and supply them with different brands to the market.

Customers who use these products are usually divided into two categories loyal customers and indifferent clients. Since the type of manufactured goods in both the company is the same loyal customers in any case use their own special brand, but for indifferent customers the distance dimension (customer position to the product supplying location) is important. On issues related to the real world, competing supply chains do not always make decisions at the same time and in many cases there is a relationship between them. On the other hand, the study of literature in SCM competition shows that Most research in this area focus on implementing game theory to obtain the optimal decisions , taking into account the different assumptions and strategies adopted by the supply chain . In general game theory presented in the supply chain due to the creation of the interaction between members of the supply chain. Supply chain members may have conflicting goals with each other. So that each loop supply chain looking for maximizing their profits and this action may cause the entire supply chain to reduce interest rates. Most models in the supply chain to seek cooperation between members of the supply chain so that the maximum benefit the entire supply chain and profits or losses in the supply chain between all chains of the supply chain will be obtained.

The literature review of works done in this paper is divided into two following parts:

1- Review articles related to replace goods pricing and complementarity and theoretical games in the supply chain.

2- Review articles related to the disruption risk in the supply chain.

Pricing is one of the issues that researchers studied in recent years and a variety of studies have been published in this field [1 - 6]. [7] Moorthy (1985) in a market of competition between various businesses , the results not only depend on the performance and the decision made by the same company but also concerns about the kinds of strategies which the other companies use for gaining the market .

[8] Taleizadeh and Noori - daryan (2015),proposed a three - level supply chain consists of several supplier , a

producer and a few retailer with the operation of rework in the integrated non-integrated structure to optimize chain interest rates under both cases to determine the optimal price policy and production and stackelberg model is used among supply chain members. Many studies offered for pricing alternative goods, which are as follows:

[9] Karakul and Chan (2008) studied analysis and management effect of substitute products in a combination form on pricing and supplying decisions , which their model is a single periodic model with two product: the old and new product and the new product will be replaced by the old product , if there is a shortage .

[10] Karakul and Chan 2010) presented, a single periodic model for substitute products in a combination form of the pricing of products and preparations for alternative products in which each product need more time for preparation and amount of demand for alternative products is random.[11] Chen et al (2013) presented pricing policy in the supply chain , with replacement goods in which producers directly or via the internet sell their products and the retailer sells an alternative product that produced by another producer . [12] Zhao et al (2012) presented alternative goods pricing issue with one producer and two retailer in which consumer demand and the cost of producing are uncertain with a centralized pricing model and three decentralized pricing models. Unlike the researches on pricing of alternative goods, a small number of studies have been done on the complementarity commodities which are as follows:

[13] Esmaeilzadeh and Taleizadeh (2016) offered the optimal price of two complementarity products in a two-level supply chain in two modes and the provided supply chain at every level, including one retailer and two manufacturers. In the first case they assumed the cost of producing complementarity goods are the same at any level as in the second case they assumed that, the production cost is different and depends on demand. [14] Arshadi Khamseh et al (2014) proposed a pricing model for alternative goods in a supply chain with two producer and one retailer, with four pricing models. In most of the articles related to the pricing of alternative product a linear function of demand is used that is a combination of that product's demand in zero price and sensitivity towards alternative goods and in a small number of articles the function of compliance demand was used .

15] Wong and Eyers (2010)and [16] Xia and Rajagopalan (2009) used the desirability function for the of customer demand that this function of desirable customer demand is a function of price of the product , time and distance from the client . [17] Xiao et al (2014) developed the game theory including one producer and one retailer in which the interaction between the time and the price were studied. The presented model was including Product customization in a production system which is based on product demand and the amount of demand depends on the time of preparation and the sales price. The supply chain may face with risk due to different factors. One of the key risk that threatens supply chain is the disruption risk that occurs for some reasons, including machinery destruction, lack of labor, and political issues that affect the profitability of the supply chain. [18] Xanthopoulos et al (2012) proposed the boy selling newspaper with two supply channels that in each channel there is a possibility of disruption risk between retail and in the case of the disruption risk only a percentage of the order by the distributor will be fulfilled. [19] Mohsenzadeh Ledari et al (2015) presented the boy selling newspaper in the multi - level supply chain with two providing channels there is a possibility of the disruption risk between retailer and the distributor in each of the supply channels and in the case of interruption risk, no percentages of the order will be met and the retailer will supply unmet amount of order from the manufacturer directly and specially.

[20]. Qi (2013) is a model in which the retailer has the possibility of providing the product from two suppliers and the first source presents the product with low cost and without guarantee (the probability of risk) and the second supplier presents product at a higher price, and full confidence (there is no possibility of risk) .

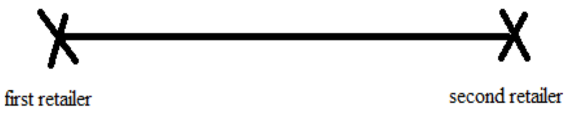
The rest of the article is organized as follows: the second section is about the problem definition and the third section deals with the definition of exclusive and non-exclusive markets and their symbols and determination of the prices. Numerical examples are given in fourth section and in fifth section results are presented.

2- The problem definition:

In this paper, a two - level supply chain consists of two producer and two retailer was presented so that, the two producers manufacture the same products and replaceable with different brands, and each of them seeking to win a greater share of the market to increase their profitability.

In this supply chain each producer has its own retailer so that retailer can only offer its producers in exclusive market but in non-exclusive market, retailer can offer both producers. As a result the strategy of producers is having the exclusive or non-exclusive market.

In exclusive market because the retailer simply offers product of its own producer, supplier also considers a percent of discount on the sales price for retailer, but in non-exclusive market because the retailer can offer both producers, supplier considers no discount for retailer. The customers who refer to retailer in order to buy goods, are divided into two groups: loyal customers and indifferent customers. Those who are loyal customers, only use of a particular brand ,but a special brand is not important for indifferent customers and the amount of demand for a product with particular brand is equal to the total demand of loyal and indifferent customers. In this supply chain, manufacturers produce the same goods with different brands which sell in the exclusive markets to both retailers, in this case may some percentages of retailer (s) demand from Distributor (s) not be met due to political issues, destruction of machinery, natural disasters and … this possibility is called the disruption risk and in case of happening, a percentage of retailer's demand is met by producer. As mentioned earlier, manufactured goods from two producers can be replaced and the consumers can use any of goods. In this model for every commodity demand, goods Utility function has been used and demand for every commodity is concerned to the distance from the ideal price, the price of rival's commodity and customer distance to the place of supplying product and demand for every commodity equal to the total demand of loyal and indifferent customers for that product.



$$U_j = r - \alpha_{jk}.r_{jk} - t.x_j$$

Figure 1. Place of two retailers

The aim of this model is determination of optimal price for goods sales by producing for retailer and also determination of the optimal sales price by retailer for the final customer that game theory is used in the model to pricing and interaction between members of the supply chain. Model has been solved in two modes of stackelberg in exclusive and non-exclusive market, in which retailer is leader and distributor follower and in a cooperative manner that the entire chain works as a single system.

2-1-Model assumptions

1. supplying two alternative products, taking into account the brand value and its importance
2 .The risk of possible between retail and distributor
3. model presented in two cases of the exclusive market and non - exclusive market
4. Use of game theory for pricing and interaction between members of the supply chain model in 2 cases of stackelberg model and cooperation.
5. Customer segmentation into two groups of loyal customers and apathetic customers and using the function of the demand utility of for determining the demand's rate.
6. Lack of shortage and the lead time is considered zero.

3-The problem definition:

In this part, we provide mathematical model for the problem in two cases of exclusive and non-exclusive market and then we propose objective functions Concavity related to each of the supply chain using Hessian matrix.

Sets:

j : The sets of manufactured goods.

k : Retailers' sets

i : Producers' sets

Parameters:

r : Ideal price for product.

α_{jk} : percent of the j th goods to the k th retailer .

c_j : The cost of producing j th goods

U_j : The utility function related to the jth product's demand

t : The rate of customer Sensitivity to brand

x_i : Customer situation (customers' distance from brand)

p_i : The probability of disruption risk in the i th distributor

y_i : The percent which is met by producer in disruption risk

λ_i : The discount rate by the i th producer

Decision variable:

r_{jk} : The jth commodity price in kth retailer.

w_{ji} : The Cost of goods sold by the jth producer

d_j : The demand for jth goods

3-1- The model of problem in exclusive market

On the exclusive market, it is assumed that each producer has its own and each retailer only sells the goods related to its producer to the final customer and supplier for per unit of the goods which gives to its special retailer considers a percent of discount for it.

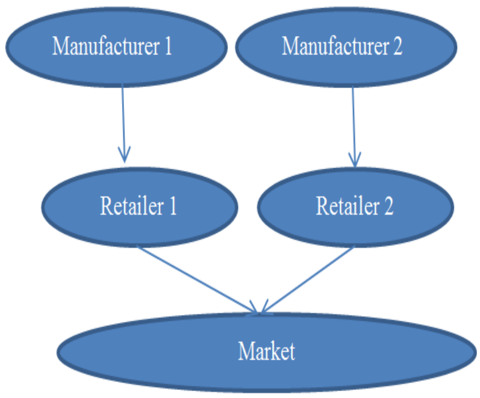


Figure 2.supplying goods with special brand by retailer of the same producer

3-1- 1 stackelberg model

In this model, it is assumed that the retailer act as a leader and producer as a follower that means the price of producer is provided after determining the price of retailer.

Profit retailer1:

$$\pi_{R1} = (1-p_1).(r_{11}-(1-\lambda_1).w_{11}).d_1 + p_1.(r_{11}-(1-\lambda_1).w_{11}).y_1.d_1$$

Profit retailer 2:

$$\pi_{R2} = (1-p_2).(r_{22}-(1-\lambda_2).w_{22}).d_2 + p_2.(r_{22}-(1-\lambda_2).w_{22}).y_2$$

Profit manufacturer 1:

$$\pi_{M1} = (1-p_1).((1-\lambda_1).w_{11}-c_1).d_1 + p_1.((1-\lambda_1).w_{11}-c_1).y_1.d_1$$

Profit manufacturer 2:

$$\pi_{M2} = (1-p_2).((1-\lambda_2).w_{22}-c_2).d_2 + p_2.((1-\lambda_2).w_{22}-c_2).y_2$$

The demand for each of the goods equals to the total demand of loyal and indifferent customers that the customer demand for it, is equal to zero compliance function demand for it, and demand of indifferent customers is equal to the equality of two compliance functions together so that expression $x_1 + x_2 = d$ should be considered.

Function of the utility of the demand for the product 1:

$u_1 = r - r_{11} - t.x_1$

Function of the utility of the demand for the product 2:

$u_2 = r - r_{22} - t.x_2$

Loyal customer demand for the first product:

$$r - r_{11} - t.x_1 = 0 \rightarrow x_1 = \frac{r - r_{11}}{t}$$

Indifferent customer demand for the first product:

$$r - r_{11} - t.x_1 = r - r_{22} - t.(d - x_1) \rightarrow x_1 = \frac{r_{22} - r_{11} + t.d}{2t}$$

The demand of first product is equal to:

$$d_1 = \frac{r_{22} - r_{11} + t.d}{2t} + \frac{r - r_{11}}{t}$$

Loyal customer demand for the second product:

$$r - r_{22} - t.x_2 = 0 \rightarrow x_2 = \frac{r - r_{22}}{t}$$

Demand of indifferent customer for the second product:

$$r - r_{11} - t.(d - x_2) = r - r_{22} - t.x_2 \rightarrow x_2 = \frac{r_{11} - r_{22} + t.d}{2t}$$

Total demand of second product is:

$$d_2 = \frac{r_{11} - r_{22} + t.d}{2t} + \frac{r - r_{22}}{t}$$

To show the concavity of objective function of retailer's profit, Hessian matrix is used:

$$H_{retailer1} = \begin{bmatrix} \frac{\partial^2 \pi_{R1}}{\partial {r_{11}}^2} & \frac{\partial^2 \pi_{R1}}{\partial r_{11} \partial r_{21}} \\ \frac{\partial^2 \pi_{R1}}{\partial r_{21} \partial r_{11}} & \frac{\partial^2 \pi_{R1}}{\partial {r_{21}}^2} \end{bmatrix} \Rightarrow H_{retailer1} = \begin{bmatrix} -\frac{3(1-p_1)}{t} - \frac{3p_1.y_1}{t} & 0 \\ 0 & 0 \end{bmatrix}$$

$$H_{retailer2} = \begin{bmatrix} \frac{\partial^2 \pi_{R2}}{\partial {r_{21}}^2} & \frac{\partial^2 \pi_{R2}}{\partial r_{21} \partial r_{22}} \\ \frac{\partial^2 \pi_{R2}}{\partial r_{22} \partial r_{21}} & \frac{\partial^2 \pi_{R2}}{\partial {r_{22}}^2} \end{bmatrix} \Rightarrow H_{retailer2} = \begin{bmatrix} 0 & 0 \\ 0 & -\frac{3(1-p_2)}{t} - \frac{3p_2.y_2}{t} \end{bmatrix}$$

As it is clear from the Hessian matrix, profit function of first retailer is concave and also the second function is concave .in the same way as the profit functions of producer 1 and producer 2 are in terms of the same variable, we can prove that both of them are concave.

To determine the prices of products in retail's chain, we use the following relations:

$$\begin{cases} \frac{\partial \pi_{R1}}{\partial r_{11}} = 0 \\ \frac{\partial \pi_{R2}}{\partial r_{22}} = 0 \end{cases}$$

Replacing these values into demand function and producer's profit and using following relationships the optimal value of producer's selling price will be obtained:

$$\begin{cases} \frac{\partial \pi_{M1}}{\partial w_{11}} = 0 \\ \frac{\partial \pi_{M2}}{\partial w_{22}} = 0 \end{cases}$$

3-1-2- cooperation model

In this case, all members of the supply chain act as a single system for profit maximization of the entire supply chain. Total profit of supply chain:

$$\pi_{sc} = (r_{11} - c_1).d_1.((1-p_1) + p_1.y_1) +$$
$$(r_{22} - c_2).d_2.((1-p_2) + y_2.p_2)$$

Hessian matrix is used to show concavity of objective function form the supply chain profit of:

$$H_{sc} = \begin{bmatrix} \frac{\partial^2 \pi_{R1}}{\partial {r_{11}}^2} & \frac{\partial^2 \pi_{R1}}{\partial r_{11} \partial r_{21}} \\ \frac{\partial^2 \pi_{R1}}{\partial r_{21} \partial r_{11}} & \frac{\partial^2 \pi_{R1}}{\partial {r_{21}}^2} \end{bmatrix} \Rightarrow$$

$$H_{sc} = \begin{bmatrix} -\frac{3(1-p1+p1y1)}{t} & \frac{1}{2}.\frac{1-p1+p1y1}{t} + \frac{1}{2}.\frac{1-p2+p2y2}{t} \\ \frac{1}{2}.\frac{1-p1+p1y1}{t} + \frac{1}{2}.\frac{1-p2+p2y2}{t} & -\frac{3(1-p2+p2y2)}{t} \end{bmatrix}$$

The determinants of minor 1: equals to $-\frac{3(1-p1+p1y1)}{t}$ which has a negative value.

Determinant of second minor:

$$\frac{-1}{t^2}(-32 + 32p_2 - 32y_2p_2 + 32p_1 - 34p_1p_2 + 34p_1y_2p_2 - 32p_1y_1$$
$$+34p_1y_1p_2 - 34p_1y_1y_2p_2 + p_1^2 - 2p_1^2y1 + p_1^2y_1^2 + p_2^2 - 2y_2p_2^2 + y_2^2p_2^2)$$

It has a positive value, so the objective function is concave.

To obtain optimum values of r_{11}, r_{22}, following relations should be used:

$$\begin{cases} \frac{\partial \pi_{sc}}{\partial r_{11}} = 0 \\ \frac{\partial \pi_{sc}}{\partial r_{22}} = 0 \end{cases}$$

2. The non-exclusive market:

In the non-exclusive market, it is assumed that any retailers in addition to selling goods related to its producer can deliver product of rival producer to the final customer. In this case, no discount from the manufacturer to its own retailer will be considered.

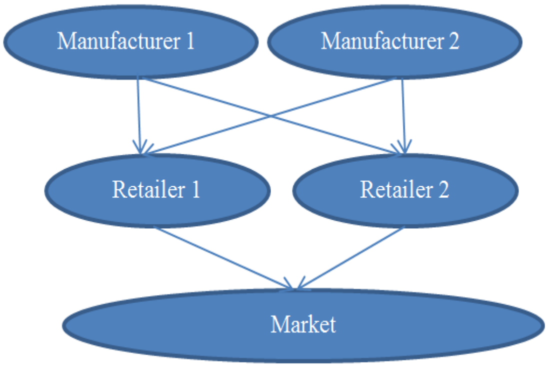


Figure 3. Supplying goods with any brands by both retailers

3-2- 1 The stackelberg model.

In this model, it is assumed that the retailer act as a leader and producer as a follower that means the price of producer is provided after determining the price of retailer.

Profit retailer1:

$$\pi_{R1} = (1-p_1).((r_{11}-w_{11}).\alpha_{11}.d_1) + (1-p_2).((r_{21}-w_{21}).\alpha_{21}.d_2) + p_1.((r_{11}-w_{11}).\alpha_{11}.y_1.d_1) + p_2.((r_{21}-w_{21})\alpha_{21}.y_2.d_2)$$

Profit retailer 2:

$$\pi_{R2} = (1-p_2).((r_{22}-w_{22}).\alpha_{22}.d_2) + (1-p_1).((r_{12}-w_{12}).\alpha_{12}.d_1) + p_2.((r_{22}-w_{22}).\alpha_{22}.y_2.d_2) + p_1.((r_{12}-w_{12}).\alpha_{12}.y_1.d_1)$$

Profit manufacturer 1:

$$\pi_{M1} = (1-p_1).((w_{11}-c_1).\alpha_{11}.d_1 + (w_{12}-c_1).\alpha_{12}.d_1) + p_1.((w_{11}-c_1).\alpha_{11}.y_1.d_1 + (w_{12}-c_1).\alpha_{12}.y_1.d_1)$$

Profit manufacturer 2:

$$\pi_{M2} = (1-p_2).((w_{22}-c_2).\alpha_{22}.d_2 + (w_{21}-c_2).\alpha_{21}.d_2) + p_2.((w_{22}-c_2).\alpha_{22}.y_2.d_2 + (w_{21}-c_2).\alpha_{21}.y_2.d_2)$$

The demand for each of the goods is calculated as did in the exclusive market, thus we have:

$$u_1 = r - \alpha_{11}.r_{11} - \alpha_{12}.r_{12} - t.x_1$$

$$u_2 = r - \alpha_{22}.r_{22} - \alpha_{21}.r_{21} - t.x_2$$

$$x_1 = \frac{r - \alpha_{11}.r_{11} - \alpha_{12}.r_{12}}{t},\; x_1 = \frac{\alpha_{21}.r_{21} + \alpha_{22}.r_{22} + t.d - \alpha_{11}.r_{11} - \alpha_{12}.r_{12}}{2t}$$

$$x_2 = \frac{r - \alpha_{22}.r_{22} - \alpha_{21}.r_{21}}{t},\; x_2 = \frac{\alpha_{11}.r_{11} + \alpha_{12}.r_{12} - \alpha_{21}.r_{21} - \alpha_{22}.r_{22} + t.d}{2t}$$

$$d_1 = \frac{r - \alpha_{11}.r_{11} - \alpha_{12}.r_{12}}{t} + \frac{\alpha_{21}.r_{21} + \alpha_{22}.r_{22} + t.d - \alpha_{11}.r_{11} - \alpha_{12}.r_{12}}{2t}$$

$$d_2 = \frac{r - \alpha_{22}.r_{22} - \alpha_{21}.r_{21}}{t} + \frac{\alpha_{11}.r_{11} + \alpha_{12}.r_{12} - \alpha_{21}.r_{21} - \alpha_{22}.r_{22} + t.d}{2t}$$

Hessian matrix is used to show the concavity of objective function of the first retailer's profit:

$$H_{retailer1} = \begin{bmatrix} \frac{\partial^2 \pi_{R1}}{\partial r_{11}^2} & \frac{\partial^2 \pi_{R1}}{\partial r_{11}\partial r_{21}} \\ \frac{\partial^2 \pi_{R1}}{\partial r_{21}\partial r_{11}} & \frac{\partial^2 \pi_{R1}}{\partial r_{21}^2} \end{bmatrix} \Rightarrow$$

$$H_{retailer1} = \begin{bmatrix} \frac{-3(1-p_1)\alpha_{11}^2}{t} - \frac{3p1\alpha_{11}^2 y_1}{t} & \frac{1}{2}\frac{(1-p_1)\alpha_{11}\alpha_{21}}{2} + \frac{1}{2}\frac{(1-p_2)\alpha_{21}\alpha_{11}}{t} + \frac{1}{2}\frac{p_1\alpha_{11}y_1\alpha_{21}}{t} + \frac{1}{2}\frac{p_2\alpha_{21}y_1\alpha_{11}}{t} \\ \frac{1}{2}\frac{(1-p_1)\alpha_{11}\alpha_{21}}{t} + \frac{1}{2}\frac{(1-p_2)\alpha_{21}\alpha_{11}}{t} + \frac{1}{2}\frac{p_1\alpha_{11}y_1\alpha_{21}}{t} + \frac{1}{2}\frac{p_2\alpha_{21}y_2\alpha_{11}}{t} & \frac{-3(1-p_2)\alpha_{21}^2}{t} - \frac{3p_2\alpha_{21}^2 y_2}{t} \end{bmatrix}$$

The first Minor determinant equals to $\frac{-3(1-p_1)\alpha_{11}^2}{t} - \frac{3p1\alpha_{11}^2 y_1}{t}$ that has a negative value and the second minor determinant equals to:

$$\frac{-1}{4t^2}(\alpha_{11}^2\alpha_{21}^2(-32 + 32p_2 - 32p_2y_2 + 32p_1 - 34p_1p_2 + 34p_1p_2y_2 - 32p_1y_1 + 34p_1y$$

It has a positive value, so the Hessian matrix is concave.

Hessian matrix is used to show the concavity of profit objective function from second retailer:

$$H_{retailer2} = \begin{bmatrix} \frac{\partial^2 \pi_{R2}}{\partial r_{21}^2} & \frac{\partial^2 \pi_{R2}}{\partial r_{21}\partial r_{22}} \\ \frac{\partial^2 \pi_{R2}}{\partial r_{22}\partial r_{21}} & \frac{\partial^2 \pi_{R2}}{\partial r_{22}^2} \end{bmatrix} \Rightarrow$$

$$H_{retailer2} = \begin{bmatrix} \frac{-3(1-p_1)\alpha_{12}^2}{t} - \frac{3p_1\alpha_{12}^2 y_1}{t} & \frac{1}{2}\frac{(1-p_2)\alpha_{22}\alpha_{12}}{t} + \frac{1}{2}\frac{(1-p_1)\alpha_{12}\alpha_{22}}{t} + \frac{1}{2}\frac{p_2\alpha_{22}y_2\alpha_{12}}{t} + \frac{1}{2}\frac{p_1\alpha_{12}y_1\alpha_{22}}{t} \\ \frac{1}{2}\frac{(1-p_2)\alpha_{22}\alpha_{12}}{t} + \frac{1}{2}\frac{(1-p_1)\alpha_{12}\alpha_{22}}{t} + \frac{1}{2}\frac{p_2\alpha_{22}y_2\alpha_{12}}{t} + \frac{1}{2}\frac{p_1\alpha_{12}y_1\alpha_{22}}{t} & \frac{-3(1-p_2)\alpha_{22}^2}{t} - \frac{3p_2\alpha_{22}^2 y_2}{t} \end{bmatrix}$$

The first Minor determinant equals to $\frac{-3(1-p_1)\alpha_{12}^2}{t} - \frac{3p_1\alpha_{12}^2 y_1}{t}$ that has a negative value and the second minor determinant equals to:

$$\frac{-1}{4t^2}(\alpha_{12}^2\alpha_{22}^2(-32 + 32p_2 - 32p_2y_2 + 32p_1 - 34p_1p_2 + 34p_1p_2y_2 - 32p_1y_1 + 34p_1y_1p_2$$
$$-34p_1y_1p_2y_2 + p_2^2 - 2p_2^2y_2 + p_1^2 - 2p_1^2y_1 + p_2^2y_2^2 + p_1^2y_1^2))$$

It has a positive value, so the Hessian matrix is concave. And it can be shown that the profit functions of manufacturers are also concave. We use the following relation to determine the prices of products in retailer's chain:

$$\begin{cases} \frac{\partial \pi_{R1}}{\partial r_{11}} = 0 \\ \frac{\partial \pi_{R1}}{\partial r_{21}} = 0 \\ \frac{\partial \pi_{R2}}{\partial r_{12}} = 0 \\ \frac{\partial \pi_{R2}}{\partial r_{22}} = 0 \end{cases}$$

Replacing these values into demand function and producer's profit and using following relationships the optimal value of producer's selling price will be obtained:

$$\begin{cases} \frac{\partial \pi_{M1}}{\partial w_{11}} = 0 \\ \frac{\partial \pi_{M1}}{\partial w_{12}} = 0 \\ \frac{\partial \pi_{M2}}{\partial w_{22}} = 0 \\ \frac{\partial \pi_{M2}}{\partial w_{21}} = 0 \end{cases}$$

2-2-3- cooperation model

In this case, all members of the supply chain act as a single system for profit maximization of the entire supply chain. Total profit of supply chain:

$$\pi_{sc} = (1-p_1).((r_{11}-c_1).\alpha_{11}.d_1 + (r_{12}-c_1).\alpha_{12}.d_1) + (1-p_2).((r_{21}-c_2).\alpha_{21}.d_2 + (r_{22}-c_2).\alpha_{22}.d_2) + p_1.((r_{11}-c_1).\alpha_{11}.y_1.d_1 + (r_{12}-c_1).\alpha_{12}.y_1.d_1)) + p_2.((r_{21}-c_2).\alpha_{21}.y_2.d_2 + (r_{22}-c_2).\alpha_{22}.y_2.d_2)$$

$$H_{sc} = \begin{bmatrix} \frac{\partial^2 \pi_{sc}}{\partial r_{11}^2} & \frac{\partial^2 \pi_{sc}}{\partial r_{11}\partial r_{21}} & \frac{\partial^2 \pi_{sc}}{\partial r_{11}\partial r_{12}} & \frac{\partial^2 \pi_{sc}}{\partial r_{11}\partial r_{22}} \\ \frac{\partial^2 \pi_{sc}}{\partial r_{21}\partial r_{11}} & \frac{\partial^2 \pi_{sc}}{\partial r_{21}^2} & \frac{\partial^2 \pi_{sc}}{\partial r_{21}\partial r_{12}} & \frac{\partial^2 \pi_{sc}}{\partial r_{21}\partial r_{22}} \\ \frac{\partial^2 \pi_{sc}}{\partial r_{12}\partial r_{11}} & \frac{\partial^2 \pi_{sc}}{\partial r_{12}\partial r_{21}} & \frac{\partial^2 \pi_{sc}}{\partial r_{12}^2} & \frac{\partial^2 \pi_{sc}}{\partial r_{12}\partial r_{22}} \\ \frac{\partial^2 \pi_{sc}}{\partial r_{22}\partial r_{11}} & \frac{\partial^2 \pi_{sc}}{\partial r_{22}\partial r_{21}} & \frac{\partial^2 \pi_{sc}}{\partial r_{22}\partial r_{12}} & \frac{\partial^2 \pi_{sc}}{\partial r_{22}^2} \end{bmatrix}$$

$$H_{sc} = \begin{bmatrix} -\frac{3(1-p_1)\alpha_{11}^2}{t} - \frac{3p_1\alpha_{11}^2 y_1}{t} & \frac{(1-p_1)\alpha_{11}\alpha_{21}}{2t} + \frac{(1-p_2)\alpha_{11}\alpha_{21}}{2t} + \frac{p_1\alpha_{11}y_1\alpha_{21}}{2t} + \frac{p_2\alpha_{21}y_2\alpha_{11}}{2t} & -\frac{3(1-p_1)\alpha_{11}\alpha_{12}}{t} - \frac{3p_1\alpha_{11}y_1\alpha_{12}}{t} & \frac{(1-p_1)\alpha_{11}\alpha_{22}}{2t} + \frac{(1-p_2)\alpha_{11}\alpha_{22}}{2t} + \frac{p_1\alpha_{11}y_1\alpha_{22}}{2t} + \frac{p_2\alpha_{22}y_2\alpha_{11}}{2t} \\ \frac{(1-p_1)\alpha_{11}\alpha_{21}}{2t} + \frac{(1-p_2)\alpha_{11}\alpha_{21}}{2t} + \frac{p_1\alpha_{11}y_1\alpha_{21}}{2t} + \frac{p_2\alpha_{21}y_2\alpha_{11}}{2t} & -\frac{3(1-p_2)\alpha_{21}^2}{t} - \frac{3p_2\alpha_{21}^2 y_2}{t} & \frac{(1-p_1)\alpha_{12}\alpha_{21}}{2t} + \frac{(1-p_2)\alpha_{12}\alpha_{21}}{2t} + \frac{p_1\alpha_{12}y_1\alpha_{21}}{2t} + \frac{p_2\alpha_{21}y_2\alpha_{12}}{2t} & -\frac{3(1-p_2)\alpha_{21}\alpha_{22}}{t} - \frac{3p_2\alpha_{21}y_2\alpha_{22}}{t} \\ -\frac{3(1-p_1)\alpha_{11}\alpha_{12}}{t} - \frac{3p_1\alpha_{11}y_1\alpha_{12}}{t} & \frac{(1-p_1)\alpha_{12}\alpha_{21}}{2t} + \frac{(1-p_2)\alpha_{12}\alpha_{21}}{2t} + \frac{p_1\alpha_{12}y_1\alpha_{21}}{2t} + \frac{p_2\alpha_{21}y_2\alpha_{12}}{2t} & -\frac{3(1-p_1)\alpha_{12}^2}{t} - \frac{3p_1\alpha_{12}^2 y_1}{t} & \frac{(1-p_1)\alpha_{12}\alpha_{22}}{2t} + \frac{(1-p_2)\alpha_{12}\alpha_{22}}{2t} + \frac{p_1\alpha_{12}y_1\alpha_{22}}{2t} + \frac{p_2\alpha_{22}y_2\alpha_{12}}{2t} \\ \frac{(1-p_1)\alpha_{11}\alpha_{22}}{2t} + \frac{(1-p_2)\alpha_{11}\alpha_{22}}{2t} + \frac{p_1\alpha_{11}y_1\alpha_{22}}{2t} + \frac{p_2\alpha_{22}y_2\alpha_{11}}{2t} & -\frac{3(1-p_2)\alpha_{21}\alpha_{22}}{t} - \frac{3p_2\alpha_{21}y_2\alpha_{22}}{t} & \frac{(1-p_1)\alpha_{12}\alpha_{22}}{2t} + \frac{(1-p_2)\alpha_{12}\alpha_{22}}{2t} + \frac{p_1\alpha_{12}y_1\alpha_{22}}{2t} + \frac{p_2\alpha_{22}y_2\alpha_{12}}{2t} & -\frac{3(1-p_2)\alpha_{22}^2}{t} - \frac{3p_2\alpha_{22}^2 y_2}{t} \end{bmatrix}$$

As it is clear from the above Hessian matrix supply chain, the profit function is concave. Therefore, for determining the number of sales prices the following formula is used:

As it is clear from the above Hessian matrix supply chain, the profit function is concave. Therefore, for determining the number of sales prices the following formula is used:

$$\begin{cases} \frac{\partial \pi_{sc}}{\partial r_{11}} = 0 \\ \frac{\partial \pi_{sc}}{\partial r_{21}} = 0 \\ \frac{\partial \pi_{sc}}{\partial r_{12}} = 0 \\ \frac{\partial \pi_{sc}}{\partial r_{22}} = 0 \end{cases}$$

4- Numerical example

To show the validity of the presented model, we solved the model using the data that is generated randomly and we showed in the next step that how changing important parameters of the presented model for optimal price impacts on each of the chains, the demand and supply chain total profits in the two cases of stackelberg model and cooperation model. The parameters of the model solution are provided in Table below:

Table 1. Numerical model parameters

Parameters	Example 1	Example 2	Example 3	Example 4
r	40	40	40	40
t	16	16	16	16
d	1	1	1	1
α_{11}	0.5	0.5	0.5	0.5
α_{12}	0.5	0.5	0.5	0.5
α_{21}	0.5	0.5	0.5	0.5
α_{22}	0.5	0.5	0.5	0.5
c_1	7	9	12	14
c_2	6	8	10	12
a_1	5	6	8	10
a_2	4	5	7	8
p_1	0.2	0.4	0.7	0.2
p_2	0.3	0.6	0.65	0.3
y_1	0.7	0.8	0.9	0.7
y_2	0.75	0.85	0.9	0.75
λ_1	0.15	0.15	0.15	0.15
λ_2	0.14	0.14	0.14	0.14

4-1- Numeric example in the case of exclusive market

for different values of the parameters, the values of the decision variables were obtained in two cases of the stackelberg model that retailer is leader and producer is follower and cooperation and in Table 2 have been shown that how changing in key parameters impacts on the decision variables and also benefit of the entire chain :

Table 2. The values of decision variables and objective function in exclusive market in the cooperation model

	Stackelberg model							**Cooperative model**				
	r_{11}	r_{22}	w_{11}	w_{22}	d_1	d_2	Objective function	r_{11}	r_{22}	d_1	d_2	Objective function
Example 1	۳۴/۴۴۳۶	۳۴/۲۴۶۷	۲۹/۹۶۶۷	۲۹/۰۸۴۰	۰/۸۴۱۱	۰/۸۶۵۷	۴۴/۳۱۸۵	۲۷/۴۵۷۸	۲۷/۰۴۱۲	۱/۲۷۰۹	۱/۳۲۲۹	۵۰/۱۸۷۸
Example 2	۳۵/۱۰۱۷	۳۴/۹۰۴۸	۳۱/۲۵۷۰	۳۰/۳۵۹۳	۰/۸	۰/۸۲۴۶	۳۹/۳۹۹۷	۲۸/۴۷۲۶	۲۸/۰۲۶۵	۱/۲۰۶۵	۱/۲۶۲۳	۴۴/۶۱۸۶
Example 3	۳۶/۰۲۲۷	۳۵/۶۲۸۹	۳۳/۱۴۰۲	۳۱/۶۸۶۳	۰/۷۳۶۳	۰/۷۸۵۵	۳۵/۲۷۲۱	۳۰/۰۱۳۱	۲۸/۹۸۸۳	۱/۰۹۲۲	۱/۲۲۰۳	۳۹/۹۶۰۵
Example 4	۳۶/۶۸۰۸	36.82270	۳۴/۴۳۰۵	۳۲/۹۶۱۶	۰/۶۹۵۱	۰/۷۴۴۴	۳۱/۵۴۳۱	۳۰/۹۶۳۲	۳۰/۰۳۳۶	۱/۰۳۵۷	۱/۱۵۲۰	۳۵/۷۳۱۲

Using Table 2 it can be realized that with a simultaneous increase in $c_1, c_2, p_1, p_2, y_1, y_2$ both stackelberg and cooperation models, selling price increases and with increasing prices, demand for both goods will be decreased and as a result the profit of the entire chain is reduced. In all examples it can be found that the profit rates in the chain of cooperation model is more than stackelberg model, because in stackelberg model each chain follows its own profit maximization but in the cooperation model chain , the entire supply chain act as a unified system.

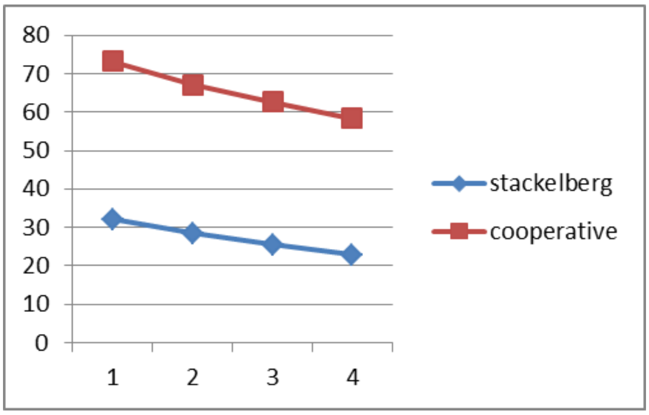


Figure 4. Comparison of total profit chain in stackelberg and cooperation models

4-2- Numerical example in the case of non-exclusive market:

For different values of the parameters, the values of the decision variables were obtained in two cases of the stackelberg model that retailer is leader and producer is follower and in cooperation model. In Table 3 and Table 4 showed that changes in key parameters impact on the decision variables as well as benefit of the entire chain:

Table 3. The values of decision variables and the objective function on the exclusive market in stackelberg model

	r_{11}	r_{12}	r_{21}	r_{22}	w_{11}	w_{12}	w_{21}	w_{22}	d_1	d_2	Objective function
Example 1	۳۹/۷۶۱۸	۳۹/۷۶۱۸	۳۹/۶۳۸۰	۳۹/۶۳۸۰	۲۳/۳۵۲۵	۲۳/۳۵۲۵	۲۲/۸۴۷۷	۲۲/۸۴۷۷	۰/۵۱۱۰	۰/۵۲۶۵	۳۲/۱۱۹۴
Example 2	۴۰/۱۶۵۲	۴۰/۱۶۵۲	۴۰/۰۳۴۶	۴۰/۰۳۴۶	۲۴/۵۳۹۱	۲۴/۵۳۹۱	۲۴/۰۶۱۲	۲۴/۰۶۱۲	۰/۴۵۸۶	۰/۵۰۱۹	۲۸/۵۵۴۶
Example 3	۴۰/۷۴۵۹	۴۰/۷۴۵۹	۴۰/۴۵۴۵	۴۰/۴۵۴۵	۲۶/۲۱۶۹	۲۶/۲۱۶۹	۲۵/۳۸۲۴	۲۵/۳۸۲۴	۰/۴۴۴۳	۰/۴۸۰۷	۲۵/۵۶۵۱
Example 4	۴۱/۱۳۴۵	۴۱/۱۳۴۵	۴۰/۸۶۴۸	۴۰/۸۶۴۸	۲۷/۴۶۱۴	۲۷/۴۶۱۴	۲۶/۵۳۹۹	۲۶/۵۳۹۹	۰/۴۲۰۷	۰/۴۵۴۴	۲۲/۸۶۱۶

Table 4. The values of decision variables and the objective function on the exclusive market in cooperation model

	r_{11}	r_{12}	r_{21}	r_{22}	d_1	d_2	Objective function
Example 1	۳۳/۶۷۸۹	۳۳/۶۷۸۹	۲۹/۱۳۱۷	۲۹/۱۳۱۷	۱/۷۸۹۸	۱/۳۲۱۴	۷۳/۱۵۸۲
Example 2	۳۴/۹۲۷۸	۳۴/۹۲۷۸	۳۰/۱۹۰۱	۳۰/۱۹۰۱	۱/۷۴۴۸	۱/۲۶۱۲	۶۷/۰۸۷۲
Example 3	۳۶/۸۲۳۸	۳۶/۸۲۳۸	۳۱/۲۵۵۲	۳۱/۲۵۲۵	۱/۶۵۹۵	۱/۲۲۰۸	۶۲/۵۷۱۲
Example 4	۳۷/۹۹۳۳	۳۷/۹۹۳۳	۳۲/۳۹۵۹	۳۲/۳۹۵۹	۱/۶۲۲۲	۱/۱۵۰۲	۵۸/۲۸۵۳

Using Tables 3 and 4 it can be realized that with a simultaneous increase in $c_1, c_2, p_1, p_2, y_1, y_2$ both stackelberg and cooperation models, selling price increases and with increasing prices, demand for both goods will be decreased and as a result the profit of the entire chain is reduced. In all examples it can be found that the profit rates in the chain of cooperation model is more than stackelberg model, because in stackelberg model each chain follows its own profit maximization but in the cooperation model chain , the entire supply chain act as a unified system.

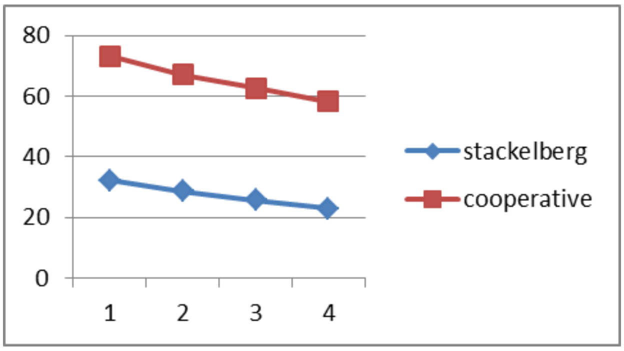


Figure 5. Comparison of total profit chain in stackelberg and cooperation models

5. Conclusion

In this paper, a two - level supply chain provided for two alternative goods, including two producers and two retailers in which the brand value considered with the possibility of disruption risk, so that every producer has its own retailer and in exclusive market retailer only can sells its own producer product and in non-exclusive market retailer can sell goods produced by the both producers. The model presented at two states of exclusive and non-exclusive market that each of these markets solved in two models the stackelberg that retailer is leader and producer is follower and the cooperation model, that finally with the help of model solution we reached to this conclusion that on exclusive and non-exclusive markets, the profit of supply chain in cooperation model is more than stekelberg model, because in cooperation model the entire rings of supply chain act for profit maximization in entire chain. Finally, with comparing exclusive and non-exclusive markets we found that profit in exclusive market is more than non-exclusive market in stackelberg model, but in cooperation model profit of non-exclusive market is more than exclusive market.In this article, preparation time and shortage were not considered, so they may be considered for future surveys.

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