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The biobjective single allocation star *p*-hub location problem

Nader Ghaffarinasab[a] and Ayda Movahhed Vala[b]

[a] *Department of Industrial Engineering, University of Tabriz, Tabriz, Iran*
E-mail: ngnasab@tabrizu.ac.ir

[b] *Department of railway engineering, Iran University of Science and Technology, Tehran, Iran*
E-mail: a_movahed@rail.iust.ac.ir

Abstract

This paper addresses a biobjective version of the single allocation star p-hub location problem which has many applications in transportation and telecommunications network design. The first objective is the median objective trying to minimize the total transportation costs, whereas the second objective is the center objective which aims at minimizing the maximum distance between origin-destination pairs. A mathematical formulation is proposed for the problem and the two objectives are aggregated using the weighting method. The proposed mathematical model is then solved using a standard optimization package and the computational results are discussed. The conducted numerical experiments show that the proposed mathematical model is efficient enough to obtain the optimal solutions in reasonable time.

Keywords: star hub location problem, multi-objective optimization, *p*-hub median, *p*-hub center, mathematical programming.

1. Introduction

Hubs are special facilities that serve as switching, transshipment and sorting points in many-to-many distribution systems. The hub location problem is concerned with locating hub facilities and allocating demand nodes to hubs in order to route the traffic between origin-destination (O/D) pairs [1]. Regarding the way the non-hub nodes are allocated to the hub nodes, there are two types of allocations: single allocation and multiple allocation. In single allocation networks, all the incoming/outgoing traffic to/from a non-hub node is routed through a single hub, whereas in multiple allocation networks, each non-hub node can receive and send flow through more than one hub.

In this paper, we address the biobjective single allocation star *p*-hub location which has numerous applications in telecommunications. We take both cost based and service based criteria into account by using median and center objectives, respectively.

Assume that there is a set of demand points and each of these points are going to communicate with all others. The communication is done by sending/receiving flows via some intermediate points called hub facilities. There is a fixed central hub and we want to locate *p* additional hubs from among the user nodes so that the O/D flows can be routed via these hubs efficiently. Each hub is connected by a direct link to the central hub and each remaining non-hub node is connected to exactly one hub (single allocation protocol). In the resulting network, the sub-network connecting the hub facilities (the backbone network) is a star network and therefore the final configuration is called a star hub network. A typical star hub network is shown in figure 1. In this figure, the large circle represents the central hub, whereas the triangles and the small circles represent the hubs and demand points, respectively.

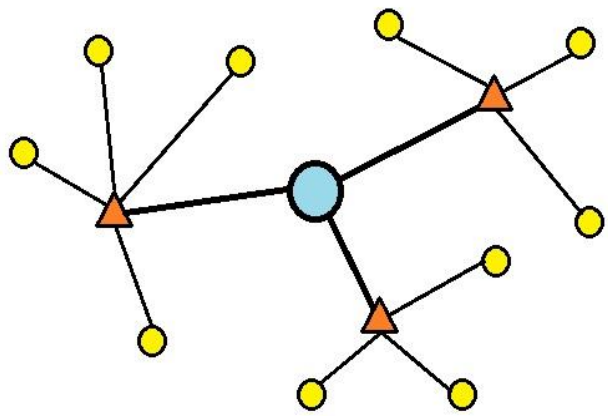

Figure 1- A typical of star hub network

The two objectives used in this paper are the median and center objectives which are widely studied in the literature of facility location. The *p*-hub median problem focuses on economic aspects of the network, whereas the *p*-hub center problem is more sensitive on the service level aspects of the network. The former tries to minimize the total transportation costs in the network, while the latter aims at minimizing the maximum distance between all the O/D pairs within the network. The application of *p*-hub center problem is to minimize the maximum travel time (or distance) between any O/D pair by placing *p* hubs in a network and allocating non-hub nodes to hub nodes. The *p*-hub center problem is important for time-sensitive or guaranteed time distribution systems, such as express mail services and emergency services. In these systems, the maximum travel time represents the best time guarantee that can be offered to all customers. To be competitive, it is important that this value is the lowest [2].

The basic difference between single and multi-objective optimization problems is the number of the objective functions to be optimized simultaneously. A multi-objective linear problem (MOLP) can be described as follows:

$$\begin{aligned} &\min \quad [f_1(x), f_2(x), \ldots, f_K(x)] \\ &s.t. \quad x \in S \end{aligned}$$

where K is the number of objectives, $f_i(x)$ is the ith objective function ($i = 1, 2, \ldots, K$) and S is the feasible region [3]. So far several methods have been developed for solving the multi-objective optimization problems. One of the well-known methods is to combine different objectives as a single objective by multiplying each objective value with a special weight. However, since different objectives normally have different measurement units, their values must be normalized before being added to each other. We use this method in our research and since both of our objectives are of minimization type, we normalize each objective by dividing it to its best possible value.

Application of multi-objective models is popular in hub location problems because there is usual conflict between the quality and the cost of the solutions. In this paper, we introduce a new bi-objective model to deal with the limitations of star *p*-hub center and star *p*-hub median problem. The use of proposed model allows the decision maker to choose the favorite solution among the optimal solutions obtained by the model with considering different combination of weights for the objective functions. To the best of our knowledge, the biobjective star *p*-hub location problem has not been studied before.

The remainder of this paper is organized as follows. The next section briefly reviews the relevant literature to the problem. In Section 3, we will present the mathematical formulation for the problem. Computational experiments and corresponding results using CAB data set are presented in section 4. Finally, some concluding remarks and directions for future works are given in section 5.

2. Background

Over the past decades, the hub location problem has attracted growing attention form the operations re3search society and has successfully been applied in several areas such as freight and passenger transportation, telecommunication, supply chain management, etc.

There is a wide range of mathematical programming models in the field of hub location and several different objective functions have been used to make such models suitable for multiple applications. Campbell [4] proposed linear integer programming formulations for different versions of the HLP such as *p*-hub median problem, the uncapacitated hub location problem, *p*-hub center problem, and hub covering problem. For more details on HLP and recent advances in this field, the interested readers are referred to surveys [1], [5], and [6].

Design of star-type or hierarchical networks has become popular since they have many applications in transportation and telecommunications network design. Some of the related works are summarized as follows.

Chung et al. [7] addressed the design of a two-level hierarchical structure where the embedded backbone network was full-meshed, whereas the local networks attached to it were of star type. They formulated the problem as a quadratic zero-one programming model, and also linearized the model as a variant of the well-established uncapacitated facility location problem.

Helme and Magnanti [8] considered the problem of designing a star/star satellite communication network. They formulated this problem as a zero-one quadratic facility location problem (FLP) and transformed it into an equivalent zero-one integer linear program. Computational results with a branch and bound algorithm and greedy heuristics based on real data were reported.

A more general star hub location problem was studied by Chardaire et al. [9] in which each demand node is connected to a first level hub which is connected to a second level hub which is connected to a central hub They did not consider the traffic flows and fixed costs associated with connections and installing facilities were minimized. They presented two integer programming formulations and a simulated annealing algorithm for solving the problem.

Labbe and Yaman [10] tackled the star hub location problem for a telecommunication network. They did not assumed a fixed number of hubs but rather, this number was an endogenous decision made by the model. Two formulations were presented and a heuristic based on Lagrangian relaxation were developed. In another work, Yaman [11] studied the star *p*-hub median problem with modular arc capacities in which links were installed on the arcs of the network to route the traffic. The author proposed two formulations and a heuristic algorithm based on Lagrangian relaxation and local search to solve the problem.

Alumur et al. [12] addressed a hierarchical multimodal hub location problem with time-definite deliveries. They proposed an MIP formulation and solved it efficiently to optimality using the commercial solver CPLEX. Recently, Yaman and Elloumi [13] proposed several mathematical models for the star *p*-hub center problem and star *p*-hub median problem with bounded path lengths. They used some linearization methods to linearize their models and showed the efficiency of their models using extensive numerical experiments.

3. Mathematical Formulation

Let $G = (N_0, E)$ be a graph in which $N_0 = N \cup \{0\}$ is the set of nodes and E is the set of edges ($E \subseteq N_0 \times N_0$). The node 0 is assumed to be a fixed central hub. Assume that all nodes in the set N are candidate for opening hubs. Also, for all $i, j \in N$, let w_{ij} and c_{ij} denote respectively the amount of flow originated at node i and destined to node j, and the transportation cost of a unit flow from node i to node j. Also, let c_{j0} denote the unit transportation cost between node j and the central hub 0. We also assume that O_i and D_i represent respectively the total output flow and total input flow to and from node i, i.e.:

$$O_i = \sum_{j \in N} w_{ij}$$
$$D_i = \sum_{j \in N} w_{ji}$$

We aim at locating p hubs in the set of nodes N in such a way that the two objective functions reach a satisfactory level.
The two objectives used in this paper are the median and center objectives which are widely studied in the literature of facility location. The p-hub median problem focuses on economic aspects of the network, whereas the p-hub center problem is more sensitive on the service level aspects of the network. The former tries to minimize the total transportation costs in the network, while the latter aims at minimizing the maximum distance between all the O/D pairs within the network. The following two sets of binary decision variables are used in our model:

$$x_{ik} = \begin{cases} 1, & \text{if node } i \text{ is assigned to hub } k \\ 0, & \text{otherwise} \end{cases}$$

$$y_{ijk} = \begin{cases} 1, & \text{if both nodes } i \text{ and } j \text{ are assigned to hub } k \\ 0, & \text{otherwise} \end{cases}$$

Based on the notations and variables defined above, the mathematical model for the biobjective star p-hub location problem can be written as follows:

$$\min \ \lambda \frac{Z_1}{Z_1^*} + (1-\lambda)\frac{Z_2}{Z_2^*} \qquad (1)$$

s.t.

$$Z_1 = \sum_{i \in N} \sum_{k \in N \setminus \{i\}} \left[c_{ik}(O_i + D_i) + c_{k0}(O_i + D_i - w_{ik} - w_{ki}) \right] x_{ik} + \sum_{k \in N} c_{k0}(O_k + D_k) x_{kk} - \sum_{k \in N} \sum_{i \in N \setminus \{k\}} \sum_{j \in N \setminus \{k\}: i<j} 2c_{k0}(w_{ij} + w_{ji}) y_{ijk} \qquad (2)$$

$$Z_2 = \beta \qquad (3)$$

$$\sum_{k \in N} x_{ik} = 1 \qquad \forall i \qquad (4)$$

$$\sum_{k \in N} x_{kk} = p \qquad (5)$$

$$x_{ik} \le x_{kk} \qquad \forall i, k \qquad (6)$$

$$y_{ijk} \le x_{ik} \qquad \forall i, j, k : i < j, i \ne k, j \ne k \qquad (7)$$

$$y_{ijk} \le x_{jk} \qquad \forall i, j, k : i < j, i \ne k, j \ne k \qquad (8)$$

$$\sum_{k \in N} (c_{ik} + c_{k0}) x_{ik} + \sum_{k \in N} (c_{jk} + c_{k0}) x_{jk} - 2 \sum_{k \in N \setminus \{i,j\}} c_{k0} y_{ijk} - 2c_{i0} x_{ji} - 2c_{j0} x_{ij} \le \beta \qquad \forall i, j : i < j \qquad (9)$$

$$x_{ik}, y_{ijk} \in \{0,1\} \qquad \forall i, j, k : i < j, i \ne k, j \ne k \qquad (10)$$

The objective function (1) minimize the weighted sum of the normalized values of the two main objectives. (2) and (3) calculate the exact values of the median and center objectives, respectively. Constraints (4) ensure that each demand node is assigned to exactly one hub node. Constraint (5) sets the number of installed hubs to p hubs. Constraints (6) ensure that nodes can only be assigned to hub nodes. Constraints (7) and (8) define the relationship between the binary decision variables. Constraint (9) calculates the maximum distance between all the O/D pairs in the network. Finally, (10) is the standard domain constraints for the decision variables.

4. Numerical Experiments

In order to test the efficiency of the proposed mathematical model, we use the CAB data set which is introduced by O'Kelly [14]. The CAB data set is based on the airline passenger interactions between 25 US cities in 1970 evaluated by the Civil Aeronautics Board. This data set has been used by most of the hub location researchers in the literature. We use four different values as the number of hubs to be opened in the network (p=2,3,4,5).
The proposed mathematical model is coded in GAMS and solved using CPLEX solver. All the experiments have been run on a computer with Intel(R) Core(TM) i7-4500U CPU of 2.0 GHz and 8 GB of RAM, using the Microsoft Windows 8 operating system.
We first solved each problem instance with respect to both the objective functions (median and center) separately to obtain the best possible vales for these objectives (Z_1^* and Z_2^*). The results are shown in Table 1. The first column

shows the number of hubs which are going to be opened. The second column show the type of objective which is considered in solving the problem, whereas the third column presents the optimal value for the corresponding optimal solution. The last column shows the nodes that are selected as hub in the optimal solution.

Table 1- Computational results with separate objectives

p	Objective	Optimum	Hubs
2	Z_1	1532.03	5,19
	Z_2	3046.83	11,21
3	Z_1	1546.37	8,19,20
	Z_2	3047.24	5,11,21
4	Z_1	1562.94	8,13,19,20
	Z_2	3073.44	5,11,13,21
5	Z_1	1567.26	7,8,10,19,20
	Z_2	3074.69	4,5,11,13,21

As can be seen from the above table, solving the problem with respect to the two objectives result in completely different solutions regarding the opened hubs. Only in one of the case (p=4) a hub is opened in both the median and center problems (node 13). This can be regarded as an indication of conflicting objectives which reveals the need for using a multi-objective approach for solving the star hub network problems.

Table 2 presents the results for solving the problem considering both the objectives at the same time using the formulation (1)-(10) for the case of p=2. The relative weight factor values (λ) are shown in the first column. The next two column show the corresponding objective function values for Z_1 and Z_2 in the resulted optimal solution. Fourth column presents the opened hubs and the last column show the computational time (in seconds) needed to reach the final solution.

Table 2- Computational results for p=2

λ	Z_1	Z_2	Hubs	CPU(s)
0.0	2144.35	3046.83	11,21	140.04
0.1	1844.21	3047.24	5,11	21.20
0.2	1844.21	3047.24	5,11	18.45
0.3	1844.21	3047.24	5,11	17.58
0.4	1844.21	3047.24	5,11	17.36
0.5	1844.21	3047.24	5,11	14.25
0.6	1672.49	3448.07	13,21	15.35
0.7	1580.10	3790.38	5,8	10.58
0.8	1540.51	4003.38	5,13	9.47
0.9	1540.51	4003.38	5,13	8.12
1.0	1532.03	4173.67	5,19	5.10

It should be noted that the solutions correxponding to λ=0 and λ=1 are merely single objective solutions and hence they are the same as the solution reported in Table 1. It can be seen from the above table that as the value of λ increases, the first objective (median) gets better, whereas the second objective (center) gets worse. In other words, the decision maker can make a good trade-off between the two objectives by selecting a suitable value for λ. This is because of the fact the by selecting larger values for λ, we put much weight on the first objective and hence the model finds solutions with better Z_1 values. It can also be observed that the optimal hubs for the intermediate values of the parameter λ are quite different from the hubs resulted under its extreme values (0 or 1). This shows that the proposed biobjective model can obtain a range of solutions (pareto optimal solutions) with varying qualities (in terms of objective function values) and different network configurations (in terms of location of hubs and allocation of nodes). Therefore, decision makers with different preferences with regard to these objectives can benefit by the flexibility provided by the proposed model. From solution time perspective, it can be seen that all the instances are solved in a short CPU time which indicates that the proposed mathematical model is efficient enough for being directly used for solving problem instances with small and medium sizes. Another interesting observation is that the solution time for smaller values of λ is higher than the solution time when λ get closer to 1. This means that the time required for solving the star p-hub center problem is considerably larger than the time needed for solving the star p-hub median problem of the same size.

The objective function values shown in Table 2 are plotted to form an efficient frontier curve (pareto frontier). This curve is illustrated in Figure 2. In this figure, the horizontal axis represents the first objective (Z_1), whereas the vertical axis represents the second objective (Z_2).

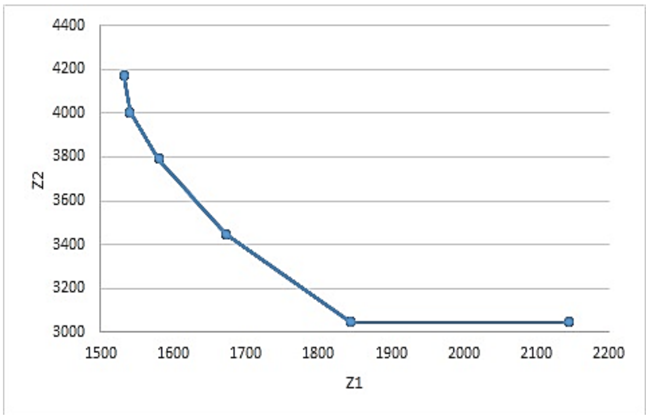

Figure 2- Efficient frontier curve for p=2

Table 3 shows the results for solving the problem for p=3. The presented results show a similar behavior as the case of

p=2. Solution times, however, are slightly higher for the case of p=3 comparing to the case of p=2.

Table 3- Computational results for p=3

λ	Z_1	Z_2	Hubs	CPU(s)
0.0	2175.09	3047.24	5,11,21	175.05
0.1	1915.39	3047.24	5,11,21	32.25
0.2	1915.39	3047.24	5,11,21	33.26
0.3	1870.34	3073.44	5,11,13	28.14
0.4	1870.34	3073.44	5,11,13	25.78
0.5	1870.34	3073.44	5,11,13	24.18
0.6	1701.76	3448.07	2,13,21	18.46
0.7	1572.28	3879.57	5,19,23	19.27
0.8	1572.28	3879.57	5,19,23	15.43
0.9	1572.28	3879.57	5,19,23	15.20
1.0	1546.37	4463.88	8,19,20	9.45

The efficient frontier curve for p=3 is shown in Figure 3.

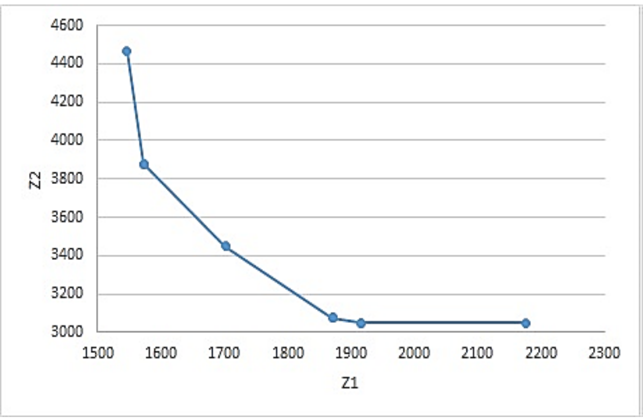

Figure 3- Efficient frontier curve for p=3

The results for solving the problem with p=4 are shown in Table 4. Also the corresponding efficient curve is illustrated in Figure 4.

Table 4- Computational results for p=4

λ	Z_1	Z_2	Hubs	CPU(s)
0.0	2195.75	3073.44	5,11,13,21	185.95
0.1	1939.21	3073.44	5,11,13,21	47.69
0.2	1939.21	3073.44	5,11,13,21	42.16
0.3	1939.21	3073.44	5,11,13,21	29.08
0.4	1939.21	3073.44	5,11,13,21	31.40
0.5	1608.54	3628.16	5,19,22,23	29.79
0.6	1608.54	3628.16	5,19,22,23	23.89
0.7	1608.54	3628.16	5,19,22,23	26.17
0.8	1608.54	3628.16	5,19,22,23	25.00
0.9	1564.56	3996.21	8,19,20,23	17.41
1.0	1562.94	4463.88	8,13,19,20	19.10

Figure 4- Efficient frontier curve for p=4

Finally, the results for solving the problem with p=5 are shown in Table 5. Corresponding efficient curve is also illustrated in Figure 5.

Table 5- Computational results for p=5

λ	Z_1	Z_2	Hubs	CPU(s)
0.0	2247.34	3074.69	4,5,11,13,21	190.08
0.1	1986.48	3104.51	5,6,11,13,21	49.30
0.2	1986.48	3104.51	5,6,11,13,21	41.95
0.3	1964.23	3119.21	5,6,11,13,20	40.84
0.4	1964.23	3119.21	5,6,11,13,20	34.39
0.5	1603.07	3628.16	5,12,19,22,23	35.13
0.6	1603.07	3628.16	5,12,19,22,23	28.48
0.7	1603.07	3628.16	5,12,19,22,23	22.79
0.8	1603.07	3628.16	5,12,19,22,23	19.00
0.9	1581.78	3920.43	8,10,19,20,23	18.13
1.0	1567.26	4354.00	7,8,10,19,20	17.11

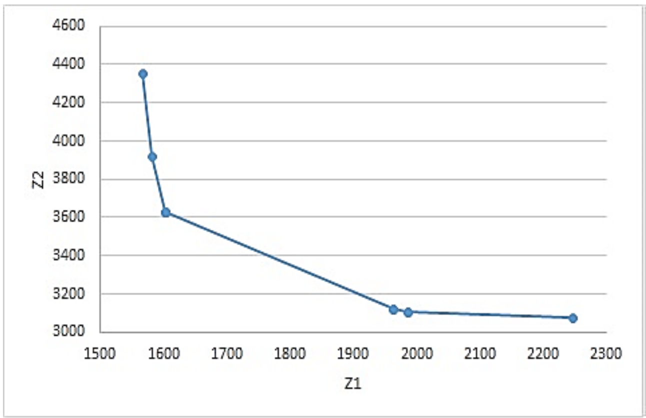

Figure 5- Efficient frontier curve for p=5

The above results show that the decision makers can use the model developed in this paper easily and solve their problem in a reasonable computational time. However, if the problem size (number of nodes in the network) gets large, solving the problem suing commercial solvers might take much longer times. In such cases, one will need to try faster solution methods such as metaheuristic algorithms.

5. Conclusions

In this paper, we considered the biobjective star p-hub location problem. The problem is formulated as a mixed integer linear model and solved using an standard optimization package. Computational experiments were conducted on the well-known CAB data sets and the proposed model was solved for all the tested instances in short and reasonable CPU times. The results showed that as the value of weight factor change, a good trade-off can be reached between the two objectives. Based on these findings, efficient frontier curves were drawn for the problem under different settings.

This research can further be extended by developing efficient solution algorithm (exact, heuristic, or metaheuristic) for solving the problem with large-sized instances in reasonable time. Another potential research direction is to incorporate uncertainty in data (demands, costs, etc.) into the modeling framework in order to better reflect the real world problems.

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