

## Improving surgical theater performance using an integrated and multi-objective mathematical model and data envelopment analysis: A case study

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### Abstract

*Surgical theater is one of the most sensitive and most expensive hospital sources that a high percentage of hospital admissions are related to it. Therefore, efficient planning and scheduling of the operating rooms is necessary to guarantee optimal use of resources, timely surgery and increase patients' satisfaction. Therefore, in this paper, the weekly planning and scheduling of the operating rooms is addressed to minimize the waiting time of elective patients, overutilization and underutilization cost of operating rooms and the sum of surgeries' completion time. We take into account the available hours of operating rooms and the surgeons, legal constraints, the skill level of surgeons and priority of patients in the model. A real-life example is provided to demonstrate the effectiveness and applicability of the model and is solved using  $\epsilon$ -constraint method in GAMS software. Then, data envelopment analysis is employed to obtain the preferred solution among the Pareto solutions achieved by  $\epsilon$ -constraint method, which is compared to the schedule used in the hospitals.*

**Keywords:** multi objective model, the planning and scheduling, elective patients, data envelopment analysis

### 1. Introduction

A significant portion of the health care system costs is spent in hospitals. In most hospitals, surgery theater is most

costly and most lucrative resource[1]. In addition, this sector is recognized as the bottleneck of hospitals [2] because the theater has a close relationship with other hospital sectors and improving its performance, in addition to patient satisfaction, has a great effect on increasing the efficiency of the entire hospital. In this context, techniques of operations research can be applied to improve the efficiency of surgical theater.

Planning and scheduling, as a management tool, has the effective role to improve the level of utilization and reduce costs related to the hospital. Planning is to assign a time of day, and day of week to each surgery considering the required resources being available. While scheduling is to determine the sequence of surgeries at each OR-day.

The planning and scheduling of ORs may be done by the leader of the operation room nursing team, which means the surgeries are placed on the scheduling bases on the first come, first serve policy. So, the surgical cases are organized based on their order of registration and the available sources are allocated to them in the surgeries' timetable. However, this affair is so complex and Time-consuming due to conflicting priorities (i.e. patients, surgeons, manager of surgical theater, etc.) and the scarcity of costly resources. Therefore, the planning and scheduling can be a useful tool to help the leader of the operating room to satisfy different Stakeholders.

In this paper, the planning and scheduling problem of the surgeons and elective patients is simultaneously addressed. An integrated and multi objective model is presented to reduce the cost, the sum of complication times and increase



satisfaction of patients. Moreover, the constraint such as the availability, legal constraint and job qualification of surgeons are considered in the proposed model. A real case is used to show the applicability of the model and solved by  $\epsilon$ -constraint method. Finally, the preferred solution is selected by DEA among the Pareto solution and compared to the solution proposed by the operating room manager.

The rest of This study is summarized as follows: A literature review of the previous studies in this field is provided in section 2. The mathematical model is addressed in section 3. The solution method is presented in section 4. The case study provided in section 5. The computational results are provided in section 6 and finally, in the last section the conclusion and future work are presented.

## 2. Literature review

The activities of researchers to study problems in the field of health have been started from about 60 years ago, but a considerable amount of research conducted from 2006 onwards, so that half of the literature is related to after 2006's.

The first article on the scheduling of surgery was done by Magerlein and Martin [3] that has divided the scheduling of operations into two sub-stages of advance scheduling and the allocation scheduling. Advance scheduling is the process of determining the day of surgery for each patient, while the scheduling of the allocation determines the operating room and time to surgery on the specified day. By studying more than 100 papers in this field, Cardoen, Demeulemeester and Beliën [4] have presented a comprehensive classification in seven areas (such as the type of patients, methods of solving, performance criteria, etc.) and have examined the related articles with considering these areas. In another study, Bai, Fügner, Schoenfelder and Brunner [5] have reviewed the studies related to the scheduling of the intensive care unit, from different aspect, such as a planning horizon of decisions, the type of models and solving techniques. Samudra, Demeulemeester and Cardoen [6] studying more than 200 articles from 2000 to 2014 and have presented a more comprehensive classification than the Cardoen's overview article. They have different conclusions from browsing the articles, for example, most of the performance measures used in this field has been the patient waiting time and overtime. In the last 10 years, contrary to expectations based on an increase Studies in the field of integrated systems (considering units after surgery), no significant growth has not been observed. In addition, most of recent studies have focused on constraint related to personnel (such as the availability of surgeons) and related preferences (such as priority of patients). Here, we only examine the literature that relates directly to our work.

Landa, Aringhieri, Soriano, Tãnfani and Testi [7] presented a model of scheduling to maximize efficiency of operating

rooms and minimize surgical cancellation of elective patients and have solved it using a combination of neighborhood search and Monte Carlo simulations. Bouguerra, Sauvey and Sauer [8] have scheduled surgeries to maximize the utilization of the operating room and minimize the delay between surgeries. Dios, Molina-Pariente, Fernandez-Viagas, Andrade-Pineda and Framinan [9] presented a decision support system for planning and scheduling of the surgeries. In this system, for allocating operations, both approximate and exact optimization methods were used. In addition, the system is also able to create three levels of planning and scheduling in the long, medium and short term.

Only a few researchers, in their study has concerned the allocation personnel to the operating room, especially surgeons. Riise and Burke [1] presented the schedule to minimize waiting time for patients, waiting time of elective patients for children and overtime for surgeons. They have made these three goals integrated with by used the weighted sum method. Marques, Captivo and Pato [10] provided the model of planning and scheduling of elective patients with given the definite duration of surgery. The purpose of this model is the allocation of patients to operating rooms and setting a date for their surgery, so that, it increases the performance of surgery theater and reduce the number of surgeries in the waiting list, for this purpose, they optimize two objective functions of maximizing operating room occupancy. The model solved using genetic algorithms and the results indicate high efficiency of the model and the used solving method. Vijayakumar, Parikh, Scott, Barnes and Gallimore [11], have presented a bin packing model based patients' priority to maximize the efficiency of operating room, in which the selected patients will be allocated to operating rooms, day, time block and Surgeons. In their model, they have taken into account constraint such as the availability of equipment and surgeons. Aringhieri, Landa, Soriano, Tãnfani and Testi [12] studied a block planning and scheduling problem to optimize hospital utilization and waiting time of patients. In order to solve the problem, they developed a meta-heuristic based on the two-step method.

In all the articles mentioned above, the patient's surgeon is assumed pre-specified. This is despite the fact that, in many hospitals, surgeries have not a predetermined list to perform surgery. As well as other items, such as surgeons' job qualification to perform surgeries, legal constraints on working hours for surgeons, limited access to operating rooms and surgeons, hospitals and patients' satisfaction by reducing costs and waiting times for patients are taking into account in this study. Considering all these features simultaneously, has made this proposed model real and practical, which can be a useful tool for hospital administrators to manage the operating room efficiently.

## 3. The model

In this section, a three-objective linear mathematical model



is presented for the planning and scheduling of elective patients as well as surgeons according to their timetable. Based on the model, we are making three decisions: 1) Determine the start time of any surgery in the operating room 2) The allocation of surgeries to operating rooms and certain day 3) The allocation of surgeons to surgeries.

We considered some assumptions in simplifying the problem as follows:

- The number of patients in waiting list is determined before the week to schedule.
- Each patient only operates once and at a specific day and time and room.
- Non-elective patients will be operated in a separate operating room; therefore, they are not considered in the model.
- Overtime hours of operating room cannot exceed a certain limit.
- Interruption during conducting a surgery is not allowed.
- Every hour is divided 15 minute periods.
- Three priority classes are considered for patients (A, B, C), which determined based on medical status of patients. To quantify the priority, the numbers {1, 5, 10} are used, which for the patients belong class A (highest priority) are considered number 10 as their priority.

The indices, parameters and decision variables are shown in Table (1).

Table 1- index, parameters and decision variables

Index	Descriptions
$s$ :	Index of Surgical operations
$i$ :	Index of surgeons
$r$ :	Index of Operating rooms
$d$ :	Index of Days in the horizon time the Week
$t$ :	Index of time periods in a day

Parameters	Descriptions
$p_s$ :	Duration of surgery $s$
$S_i$ :	The set of surgeries that the surgeon $i$ can perform them
$i_s$ :	The set surgeons that are eligible for performing surgery $s$
$T_{sd}$ :	The set of periods which surgery $s$ can be started on day $d$ so that it can finish before closing the surgical theater

$T_{id}$ :	Set of periods which the surgeon $i$ is available on day $d$
$T_{id}^{max}$ :	The maximum period numbers which surgeon can work at day $d$
$T_{id}^{min}$ :	The minimum period numbers which surgeon should work at day $d$ if he/she are available at day $d$
$A_i^{max}$ :	The maximum period numbers which surgeon can work per week
$A_i^{min}$ :	The minimum period numbers which surgeon should work at day $d$
$H_{rd}$ :	Regular time of opening operating room $r$ at day $d$
$O_{rd}$ :	Maximum overtime allowed
$cu^{rd}$ :	The underutilization cost of room $r$ at day $d$ per periods
$co^{rd}$ :	The overtime cost of the room $r$ , on day $d$ per period
$NB$ :	Number of Periods per hour
$priority_s$ :	Priority of surgical operation $s$

Decision variables	Description
$X_{srdt}^i$ :	if surgery $s$ is started by $i$ surgeon in the room $i$ on day $d$ at the beginning of $t$ period, takes 1, otherwise 0
$over_r^d$ :	Overtime of the operating room $r$ on day $d$
$under_r^d$ :	The undertime of the operating room $r$ on day $d$

$$MinZ1: \sum_i \sum_{s \in S_i} \sum_r \sum_d \sum_{t \in (T_{id} \cap T_{sd})} ((t - 1) + (d - 1) \times 24 \times NB) X_{srdt}^i \times priority_s \tag{1}$$

$$MinZ2: \sum_r \sum_d under_r^d \times cu^{rd} + \sum_r \sum_d over_r^d \times co^{rd} \tag{2}$$

$$MinZ3: \sum_s C_s$$



(3)

$$\sum_{i \in I_s} \sum_r \sum_d \sum_{t \in (T_{id} \cap T_{sd})} X_{srdt}^i = 1 \quad \forall s \in S \quad (4)$$

$$\sum_i \sum_{s \in S_i} \sum_{\substack{t'=t-p_s+1 \\ t' \in (T_{id} \cap T_{sd})}}^t X_{srdt'}^i \leq 1 \quad \forall r \in R, \forall t \in T, \forall d \in D \quad (5)$$

$$\sum_{s \in S_i} \sum_r \sum_{\substack{t'=t-p_s+1 \\ t' \in (T_{id} \cap T_{sd})}}^t X_{srdt'}^i \leq 1 \quad \forall i \in I, \forall t \in T, \forall d \in D \quad (6)$$

$$\sum_i \sum_{s \in S_i} \sum_{t \in (T_{id} \cap T_{sd})} P_s X_{srdt}^i \leq H_{rd} + O_{rd} \quad \forall r \in R, \forall d \in D \quad (7)$$

$$T_{id}^{min} \leq \sum_{s \in S_i} \sum_r \sum_{t \in (T_{id} \cap T_{sd})} P_s X_{srdt}^i \leq T_{id}^{max} \quad \forall i \in I, \forall d \in D \quad (8)$$

$$A_i^{min} \leq \sum_{s \in S_i} \sum_r \sum_{t \in (T_{id} \cap T_{sd})} \sum_d P_s X_{srdt}^i \leq A_i^{max} \quad \forall i \in I \quad (9)$$

$$under_r^d \geq (H_{rd} - \sum_{\substack{t \in (T_{id} \cap T_{sd}) \\ |t| \leq H_{rd}}} \sum_t \sum_{s \in S_i} \sum_{t'=t}^{\min(t+p(s)-1, H_{rd})} X_{srdt'}^i) \quad \forall r \in R, \forall d \in D \quad (10)$$

$$over_r^d \geq \sum_i \sum_{s \in S_i} \sum_{\substack{t \in (T_{id} \cap T_{sd}) \\ (t+p_s-1) > H_{rd}}} (t + P_s - 1) X_{srdt}^i - H_{rd} \quad \forall r \in R, \forall d \in D \quad (11)$$

$$C_s \geq \sum_{i \in I_s} \sum_r \sum_d \sum_t X_{srdt}^i (t + p_s) \quad \forall s \in S \quad (12)$$

$$X_{srdt}^i \in \{0,1\}, under_r^d \geq 0, over_r^d \geq 0, C_s \geq 0 \quad (13)$$

Three objective functions are considered in this study. The first objective function (Equation1) minimizing the sum of the waiting time of patients according to patients' priority. The second objective function (Equation2) minimizes the costs, including the costs of underutilization (the first part) and overutilization (the second part). The third objective function (Equation 3) minimizes the sum of completion times of the surgeries.

Constraint (4) ensures every surgery is started in only one room, one day and one period by an eligible surgeon. Constraint (5) states at most one surgery by a qualified surgeon is performing on each day and room in a time period. Constraint (6) ensures that there should be no overlap in times and operating rooms on a particular day for a particular surgeon, to do the operations by the surgeon at the time they were done. Constraint (7) states that at a particular day and operating room, the time of using the operating room should be less than their available hour. Constraint (8) ensures that the total time of a surgeon's work in one day should be less than a certain limit and more than a certain limit which determined based on hospital regulation. Constraint (9) ensures that the total time of a surgeon's work in one week should be less than an upper bound and more than a lower bound. Constraints (10) and (11), respectively, show the underutilization and overutilization time of every operating room per day and Constraint (12) shows the completion time of surgeries. The domain of variables is shown in constraint (13).

### 3.1. Model validation

In order to validate the proposed model, we designed a small example ( $|S| * |I| * |R| * |T| * |D| = 8 * 3 * 2 * 40 * 1$ ), where  $|S|$ ,  $|I|$ ,  $|R|$ ,  $|T|$ ,  $|D|$  indicates number of surgeries, the number of surgeons, number of rooms, number of time periods in a day and number of days in horizon time respectively. Then, we solved the model as three single-objective problems considering each objective separately. Each single-objective problem has been solved by CPLEX solver in Gams software, that their solutions are shown in Figures 1 to 3. (A to C represent surgeons, and 1 to 8, indicate surgeries) According to the figures, the optimal solutions obtained by each single objective model are different with regard to each of the objective functions, individually, and this shows a conflict between the objective functions and the effectiveness of the proposed model.



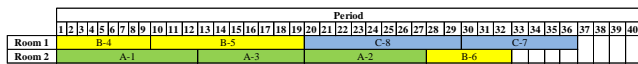


Figure 1- The optimal solution obtained by solving the model considering only the first-objective function

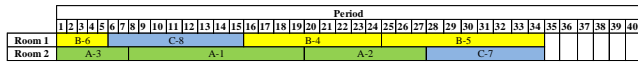


Figure 2- The optimal solution obtained by solving the model considering only the second-objective function

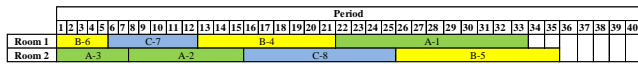


Figure 3- The optimal solution obtained by solving the model considering only the third-objective function

## 4. Solution method

### 4.1. $\epsilon$ -constraint method

The basic idea of this method is based on selecting one of the multiple objectives as the main objective function in the optimization problem and the rest of objective functions are converted to constraints. Assume the following model with K objective functions.

$$\text{Min } (f_1(x), f_2(x), \dots, f_k(x)) \quad (14)$$

s.t.

$$x \in S,$$

Where x is the vector of decision variables and S is the feasible region. Applying  $\epsilon$ -constraint method, the model (14) is converted to the following model. To reach the efficient solution for the model (15) we must change the RHSs values of the constrained objective functions, parametrically [13].

$$\text{Max } f_1(x) \quad (15)$$

s.t.

$$f_2(x) \geq \epsilon_2$$

$$f_3(x) \geq \epsilon_3$$

...

$$f_k(x) \geq \epsilon_k$$

This method produces a Pareto optimal solution set, including non-dominated solutions. Then, to determine the most efficient solution among the Pareto solutions' set, obtained from  $\epsilon$ -constraint method, data envelopment analysis is used that is explained in the next section.

### 4-2 Data envelopment analysis

DEA is a mathematical technique based on linear

programming, which was introduced by Charnes, Cooper and Rhodes [14] for the first time. By this method, the performance of a group of decision-making units (DMU) with multiple inputs and outputs can be determined [14], [15]. This performance will be used as an indicator for evaluating the performance of DMU and to compare between them.

To determine the weight of each input and output, DEA allows each DMU to specify a set of weights, which show that unit, in the most favorable situation than other units. The problem for the unit DMU<sub>o</sub> is defined as model (A).

$$\text{Max } \theta_o = \frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \quad (A)$$

s. t.

$$\frac{\sum_{r=1}^s u_r y_{ro}}{\sum_{i=1}^m v_i x_{io}} \leq 1$$

$$j = 1, \dots, n$$

$$u_r, v_i \geq 0$$

$$i = 1, \dots, m, \quad r = 1, \dots, s$$

where r and i are index of outputs and inputs respectively. The variables of model (A) are  $u_r$  and  $v_i$  weights which indicate weight of input i and output r, and  $\theta_o$  is the efficiency of DMU<sub>o</sub>.

This model (A) is not linear, but can be easily converted to a linear programming problem as follows:

$$\text{Min } \theta = \sum_{i=1}^m v_i x_{io} \quad (B)$$

s. t.

$$\sum_{r=1}^s u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0$$

$$j = 1, \dots, n$$

$$\sum_{r=1}^s u_r y_{r0} = 1$$

$$u_r, v_i \geq 0$$

$$i = 1, \dots, m, \quad r = 1, \dots, s$$

There are the four conventional and more used DEA model, namely BCC input oriented, BCC output oriented, CCR input oriented, CCR output oriented. According to the minimizing nature of objectives functions, we employed BCC input oriented DEA model with a VRS frontier type because the percentage change in output values are not a function of direct change in input values [16], [17]. Model (C) shows BCC input oriented DEA model.

$$\text{Min } \theta \quad (C)$$

s. t.

$$\theta x_{i0} \geq \sum_{j=1}^n \lambda_j x_{ij}$$

$$i = 1, \dots, m$$

$$y_{r0} \leq \sum_{j=1}^n \lambda_j y_{rj}$$





$$\sum_{j=1}^n \lambda_j = 1$$

$$\lambda_j \geq 0$$

$$r = 1, \dots, s$$

$$j = 1, \dots, n$$

Given that the proposed model (model C) consider the efficiency of efficient units equal to one, hence, they are not able to complete ranking of efficient units. In order to have full ranking units. We convert model (C) to a Super efficiency model by eliminating  $DMU_o$  from the model [18].

### 5- Case study

This study focuses on the operating rooms devoted the orthopedic surgeries the Baqiyatallah Alazm hospital in Tehran. The hospital was founded in 1984 with an area of over 52,000 square meters and it is one of the largest and most advanced hospitals in the Middle East with an annual average of 42000 Surgeries. In hospital nine type surgeries, including Orthopedic, Urology, General, ophthalmology, ENT, brain and nerves, kidney transplant, maternity and cardiac and vascular surgeries are performed. This study, as mentioned, is focused based on a real week in the orthopedic surgery unit includes two operating rooms. In this unit, operating rooms are 8:30 hours on Saturdays until Wednesday, and 7 hours on Thursday and Friday regularly. In addition, the maximum overtime hours in the days are 1.5 from Saturday to Wednesday and 1 hour in Thursday and Friday. According to data collected in the considered week, there are 54 elective patients for the surgery that during this week, 8 Surgeons are available according to a timetable on a special hour and also the time interval that the operating room can access to them. Based on hospital regulation, the surgeons must not perform surgery more than 8.5 and 35 hours in a day and week respectively. Moreover, if a surgeon is ready in surgical theater, the surgical theater manager must reserve at least two vacant hours for conducting surgeries which he/she can perform it on that day.

### 6-Computational results

The case study solved using  $\epsilon$ -constraint method and CPLEX solver in GAMS software. Table 2 shows 16 schedules obtained by  $\epsilon$ -constraint method as well as the schedules offered by the surgical theater manager. These schedules are considered as DMUs and are ranked by BCC input oriented model. Due to the minimizing nature of the objective functions, i.e. Z1, Z2 and Z3, they have been considered as input in DEA model and labeled as input1, input2 and input 3 respectively which are shown in Table (2). The ranks achieved by the DEA model are indicated in Table 3.

As Table 3, the DMU 14 is selected by the DEA model as the preferred solution while the schedule offered by

operating room manager (the DMU 17) takes rank 14. The best schedule obtained by DEA model and the schedule offered by operating room manager are shown in Figures (4) and (5) respectively.

Table 2- The non-dominated solutions obtained by  $\epsilon$ -constraint method and the actual schedule used in hospital

DMU	Z1 (15min)	Z2 ( $\times 10^4$ Rials)	Z3 (15 min)
1	50691	6300	990
2	40136	6300	1030
3	40120	6900	1030
4	40128	6600	1030
5	37852	6300	1066
6	37472	6600	1067
7	37468	6900	1067
8	37045	6300	1101
9	36941	6600	1109
10	36879	6900	1107
11	36875	7200	1107
12	36786	8100	1110
13	37000	6300	1140
14	36865	6600	1150
15	36847	6900	1143
16	36762	7800	1138
17	36839	6900	1170

Table 3- The rank of the schedules achieved by the DEA model

DMU	The rank	Efficiency
1	15	0.99998
2	7	1.01002
3	8	1.01000
4	6	1.01120
5	17	0.98999
6	5	1.01210
7	9	1.00121
8	2	1.03110
9	4	1.01310
10	12	1.000006
11	16	0.99984
12	13	1.00000
13	3	1.02875
14	1	1.10037
15	10	1.00021
16	11	1.00003
17	14	0.99999





- [14] A. Charnes, W.W. Cooper, E. Rhodes, Measuring the efficiency of decision making units, *European journal of operational research*, 2 (1978) 429-444.
- [15] R.D. Banker, H. Chang, W.W. Cooper, Equivalence and implementation of alternative methods for determining returns to scale in data envelopment analysis, *European Journal of Operational Research*, 89 (1996) 473-481.
- [16] A. Azadeh, Z. Gaeini, B. Moradi, Optimization of HSE in maintenance activities by integration of continuous improvement cycle and fuzzy multivariate approach: A gas refinery, *Journal of Loss Prevention in the Process Industries*, 32 (2014) 415-427.
- [17] R.D. Banker, A. Charnes, W.W. Cooper, Some models for estimating technical and scale inefficiencies in data envelopment analysis, *Management science*, 30 (1984) 1078-1092.
- [18] P. Andersen, N.C. Petersen, A procedure for ranking efficient units in data envelopment analysis, *Management science*, 39 (1993) 1261-1264.

