An EOQ model with holding - ordering cost reduction and partial delay in payment under credit period- dependent demand: A reversed constraint programming

Leyla Aliabadi[a], Razieh Heidari[a], Reza Yazdanparast[a], Fariborz Jolai[a]

[a] *School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran*
(Corresponding Author E-mail: r.yazdanparast@ut.ac.ir), (leyla.aliabdi@ut.ac.ir, raziyeh.heidari@ut.ac.ir, fjolai@ut.ac.ir)

Abstract

Harmony and coordination among various departments of a company can be an important factor for obtaining competitive advantage. For many businesses, trade credit represents a main section of company finance and is used as one of coordination strategies in a company. In this study, a nonlinear model of joint partial delayed payments, pricing, and marketing strategies is presented in a supply chain including a retailer and multiple customers. Demand rate is an endogenous variable and depends on marketing cost, selling price and the length of the credit period. To obtain better inventory management, both holding and ordering costs are controllable by an added cost. The proposed problem is formulated in two cases for maximizing the retailer's profit and determines length of the credit period, marketing cost, selling price, holding cost, ordering cost, purchasing cost and order quantity simultaneously. Each case is modeled a constrained signomial geometric programming with 2 degrees of difficulty. For solving our models, we transform both models to a reversed constraint programming and can obtain the optimal solutions in closed forms for each case. The applicability of this solution method is demonstrated by a numerical example.

Keywords:
EOQ model, Partial delay in payment, Signomial geometric programming, Marketing.

Introduction

In many businesses, trade credit is used as an essential tool to financing growth. In today's business transaction, many firms use different trade credits as part of the financing strategy and offer to their customers in order to reduce and manage the capital requirement, attract new customers, increase market share, increase power purchasing, etc. One of the well-known strategies that is more applied in practice is the partial delayed payment. Under this mechanism, the retailer (customers) have to pay a special amount (e.g., μ percent)of total purchasing cost at order receipt time and the remaining amount of total purchasing cost would be paid at the end of credit period without any additional charges. For first time [1]considered delay in payments in EOQ model. After that, different types of trade credit were considered in inventory management models with additional assumptions. For instance, [2]considered delayed payments in EOQ model for deteriorating products. [3]extended the work of [2] under fully backordered shortages and inflation. [4] presented an EOQ model that a partial delayed payment is offered to the retailer if the order quantity is smaller than a specified quantity. [5] described an EOQ model in situation that shortages and trade credits were considered in partial forms. An inventory model with partial delay of payments and partial backordering was discussed by [6] under condition that partial delay in payment is linked to order quantity.

In all firms, marketing and production strategies are two interdependent and important decisions that are handled with different strategies such as separate, joint, and coordinated. In classic models, many authors assumed demand rare to be constant while in real world markets, demand for many products is sensitive different parameters such as marketing cost, selling price, trade credits, etc. the related first model that incorporated marketing strategies and production planning in a EOQ model was given by [7]. They considered a nonlinear relation between selling price, marketing costs and demand rate. This assumption also have been considered in many articles such as [8-11].

One of the most important parameters that has a positive impact on demand is trade credit period suggested by the retailer to his/her customers.[12-15] considered the impact credit period on demand in their models. But, little attention has been given to the impact credit period on demand. Since, for first time this study coordinates trade credit, marketing, and pricing strategies in an EOQ model under the condition of partial delayed payments. We represent demand rate as multivariate function of the length of the credit period, marketing cost, and selling price. For tackling real world conditions, ordering and holding cost are controllable by an added cost and can be determined in coordination with trade credit, marketing, and pricing strategies. The main objective function of this study is to maximize the retailer's profit that have been formulated as nonlinear programming (NLP) problem. These types of NLP problems have been solved by various techniques, between them, geometric programming (GP) is an effective approach to solve these problems that first time was introduced by [16] and well applied in many fields such as inventory control[8, 9, 17] , power control[18], engineering design[19, 20] , and etc. Therefore, for solving the proposed problem by using GP technique, we first convert the problem in to a constrained signomial geometric programming (SGP) problems. The past few years, several global optimization approaches for solving SGP problems were presented. Reversed constraint programming is a global optimization method that was developed by [21] for solving the SGP problems and we apply this method for solving our problem.

Problem description

In this study, a retailer presents a single product to the competitive market and provide a partial delay in payment to the his/her customers. Therefore, the customers have to pay μ percent of total purchasing cost immediately and the remaining $(1-\mu)$ percent of purchasing cost would be paid at the end of credit period (M). Shortages are not allowed. We study the effect of credit period, marketing expenditure, and selling price on the demand rate in an EOQ model that maximizes the retailer's profit. Therefore, demand rate is represented as a power function of credit period (M), marketing expenditure (E), and selling price (S) as follows:

$$D = uS^{-\alpha}E^{\lambda}M^{\sigma} \tag{1}$$

Where u is marketing size and $\alpha > 1$, $\lambda > 0$ and $\sigma > 0$ are selling price, marketing expenditure ,and the credit period elasticity, respectively.

In the proposed problem, two cases based on relation between the length of an inventory cycle time, T , and credit period , M , are considered as : case(1) $M \le T$ and case (2) $T \le M \le M_0$.

Notations and other assumptions have been presented to formulate the proposed problem as follows:

Assumptions:

- Lead time is zero.
- Purchasing cost per item is a decision variable.
- Replenishment is instantaneous.
- All parameters are supposed precise and constant.
- There is no deterioration.
- I_c rate of interest paid) is longer than I_e (rate of interest that can be earned) .
- Shortages are not allowed.
- Ordering and holding cost can be decrease by an added cost as follows:

$$C(h,A) = vA^{-\theta}h^{-\gamma} \tag{2}$$

Where v, θ, γ are positive parameters, A, h ,and $C(h,A)$ indicate ordering cost per order, holding cost per unit per year , and Capital investment per cycle ,receptively.

Parameters:

M_0	Upper band of trade credit
I_p	Rate of interest paid ($/year)
I_e	Rate of interest that can be earned ($/year)
α	Selling price elasticity to demand
λ	Marketing expenditure elasticity to demand
σ	Credit period elasticity to demand
θ	Ordering cost elasticity
γ	Holding cost elasticity
μ	The portion of the purchasing cost must be paid at the time of receiving.

Decision variables:

D	Demand rate per year
T	Length of an inventory cycle
S	Unit selling price
M	Credit period
E	Marketing expenditure per unit item
Q	Order quantity
C	Unit purchasing cost per unit item
h	Holding cost per unit item per year
A	Ordering cost per order is placed

The model formulation

According to Fig1, the average total profit per year for case 1 that $M \le T$ is calculated by following equation:

$$\begin{aligned} Max\ z =\ & \underbrace{SD}_{\substack{sale \\ revenue}} - \underbrace{ED}_{\substack{Marketing \\ cost}} - \underbrace{\frac{A}{T}}_{\substack{Ordering \\ cost}} - \underbrace{\frac{hDT}{2}}_{\substack{Holding \\ cost}} - \underbrace{CD}_{\substack{Purchasing \\ cost}} \\ & - \overbrace{\left(\frac{\mu C I_c DT}{2} + \frac{(1-\mu) C I_c D (T-M)^2}{2T}\right)}^{Capital\ cost} \\ & + \overbrace{\left(\frac{(1-\mu) C I_e D M^2}{2T}\right)}^{Interest\ earned} - \overbrace{vA^{-\theta}h^{-\gamma}T^{-1}}^{Capital\ investment} \\ s.t\quad & M \le T \end{aligned} \tag{3}$$

After replacing demand rate by equation (1) and expanding the quadratic components, we have:

$$\begin{aligned} Max\ z =\ & uS^{-\alpha+1}E^{\lambda}M^{\sigma} - uS^{-\alpha}E^{\lambda+1}M^{\sigma} - AT^{-1} \\ & -0.5uhS^{-\alpha}E^{\lambda}M^{\sigma}T - uCS^{-\alpha}E^{\lambda}M^{\sigma} \\ & -0.5(1-\mu)(I_c - I_e)uCS^{-\alpha}E^{\lambda}M^{\sigma+2}T^{-1} \\ & -0.5I_c uCS^{-\alpha}E^{\lambda}M^{\sigma}T - vA^{-\theta}h^{-\gamma}T^{-1} \\ & +(1-\mu)I_c uCS^{-\alpha}E^{\lambda}M^{\sigma+1} \\ s.t.\quad & M \le T \end{aligned} \tag{4}$$

We transform the above equation to a constrained signomial geometric programing (SGP) with 2 degrees of difficulty after defining an extra variable:

$$Max\ \ w = Min\ \ w^{-1}$$

$$\left\{\begin{array}{l} uS^{-\alpha+1}E^{\lambda}M^{\sigma} - uS^{-\alpha}E^{\lambda+1}M^{\sigma} - AT^{-1} \\ -0.5uhS^{-\alpha}E^{\lambda}M^{\sigma}T - uCS^{-\alpha}E^{\lambda}M^{\sigma} \\ -0.5(1-\mu)(I_c - I_e)uCS^{-\alpha}E^{\lambda}M^{\sigma+2}T^{-1} \\ -0.5I_cCS^{-\alpha}E^{\lambda}M^{\sigma}T - vA^{-\theta}h^{-\gamma}T^{-1} \\ +(1-\mu)I_cuCS^{-\alpha}E^{\lambda}M^{\sigma+1} \end{array}\right\} \geq w \tag{5}$$

$$\Rightarrow \left\{\begin{array}{l} u^{-1}wS^{\alpha-1}E^{-\lambda}M^{-\sigma} + S^{-1}E + u^{-1}AS^{\alpha-1}E^{-\lambda}M^{-\sigma}T^{-1} \\ +0.5hS^{-1}T + CS^{-1} + 0.5(1-\mu)(I_c - I_e)CS^{-1}M^{2}T^{-1} \\ +0.5I_cCS^{-1}T + vu^{-1}S^{\alpha-1}E^{-\lambda}M^{-\sigma}A^{-\theta}h^{-\gamma}T^{-1} \\ -(1-\mu)I_cCS^{-1}M \end{array}\right\} \leq 1$$

Finally, we have:

$$\begin{array}{l} Min\ f = w^{-1} \\ s.t. \\ \left\{\begin{array}{l} u^{-1}wS^{\alpha-1}E^{-\lambda}M^{-\sigma} + S^{-1}E + u^{-1}AS^{\alpha-1}E^{-\lambda}M^{-\sigma}T^{-1} \\ +0.5hS^{-1}T + CS^{-1} + 0.5(1-\mu)(I_c - I_e)CS^{-1}M^{2}T^{-1} \\ +0.5I_cCS^{-1}T + vu^{-1}S^{\alpha-1}E^{-\lambda}M^{-\sigma}A^{-\theta}h^{-\gamma}T^{-1} \\ -(1-\mu)I_cCS^{-1}M \end{array}\right\} \leq 1 \\ MT^{-1} \leq 1 \end{array} \tag{6}$$

Also, the average total profit per year for case 2 that $T \leq M \leq M_0$ is (see Fig2) :

$$\begin{array}{ll} Max\ \ z = SD - ED - \dfrac{A}{T} - \dfrac{hDT}{2} - CD & \\ \quad -\dfrac{\mu CI_cDT}{2} + \dfrac{(1-\mu)CI_eDT}{2} & \\ \quad +(1-\mu)CI_eD\,(M - T) - vA^{-\theta}h^{-\gamma}T^{-1} & \\ s.t \qquad T \leq M \leq M_0 & \end{array} \tag{7}$$

SGP problem form for this case are given as follows:

$$\begin{array}{l} Min\ f' = (w')^{-1} \\ s.t. \\ \left\{\begin{array}{l} u^{-1}wS^{\alpha-1}E^{-\lambda}M^{-\sigma} + S^{-1}E + u^{-1}AS^{\alpha-1}E^{-\lambda}M^{-\sigma}T^{-1} \\ +0.5hS^{-1}T + CS^{-1} + 0.5\left(\mu I_c + (1-\mu)I_e\right)CS^{-1}T \\ +vu^{-1}S^{\alpha-1}E^{-\lambda}M^{-\sigma}A^{-\theta}h^{-\gamma}T^{-1} - (1-\mu)I_eCS^{-1}M \end{array}\right\} \leq 1 \\ TM^{-1} \leq 1 \\ M_0^{-1}M \leq 1 \end{array} \tag{8}$$

Intrinsically, it is difficult for solving models (6) and (8) directly. Since, we transform these models to reversed constraint program in next section.

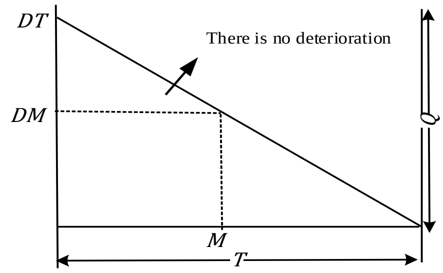

Figure 1- inventory system for case1: $M \leq T$

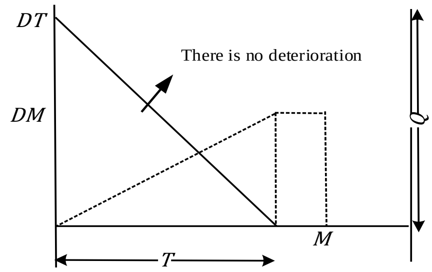

Figure 2- inventory system for case 2: $T \leq M \leq M_0$

Methodology

In this section, we apply reversed constraint program in order to solve equations (6) and (8). [21] introduced this approach to solve the SGP problems. Reversed constraint programs develop GPs to contain equal or greater than restrictions. Therefore, we formulate reversed constraint version of each case as follows:

- *Case1-primal:*

$$\begin{array}{l} Min\ f = w^{-1} \\ s.t. \\ \left\{\begin{array}{l} u^{-1}wS^{\alpha-1}E^{-\lambda}M^{-\sigma}x^{-1} + S^{-1}Ex^{-1} \\ +u^{-1}AS^{\alpha-1}E^{-\lambda}M^{-\sigma}T^{-1}x^{-1} + 0.5hS^{-1}Tx^{-1} \\ +CS^{-1}x^{-1} + 0.5(1-\mu)(I_c - I_e)CS^{-1}M^{2}T^{-1}x^{-1} \\ +0.5I_cCS^{-1}Tx^{-1} + vu^{-1}S^{\alpha-1}E^{-\lambda}M^{-\sigma}A^{-\theta}h^{-\gamma}T^{-1}x^{-1} \end{array}\right\} \leq 1 \\ x^{-1} + (1-\mu)I_cCS^{-1}Mx^{-1} \geq 1 \\ MT^{-1} \leq 1 \end{array} \tag{9}$$

- *Case1-dual:*

The dual to the reversed constraint program is:

$$\begin{array}{l} MinMax\ \varphi(\beta) = \left(\dfrac{1}{\beta_1}\right)^{\beta_1} \times \left(\dfrac{\lambda_1 u^{-1}}{\beta_2}\right)^{\beta_2} \times \left(\dfrac{\lambda_1}{\beta_3}\right)^{\beta_3} \times \left(\dfrac{\lambda_1 u^{-1}}{\beta_4}\right)^{\beta_4} \\ \quad \times \left(\dfrac{0.5\lambda_1}{\beta_5}\right)^{\beta_5} \times \left(\dfrac{\lambda_1}{\beta_6}\right)^{\beta_6} \times \left(\dfrac{0.5\lambda_1(1-\mu)(I_c - I_e)}{\beta_7}\right)^{\beta_7} \\ \quad \times \left(\dfrac{0.5\lambda_1 I_c}{\beta_8}\right)^{\beta_8} \times \left(\dfrac{\lambda_1 vu^{-1}}{\beta_9}\right)^{\beta_9} \times \left(\dfrac{\lambda_2}{\beta_{10}}\right)^{-\beta_{10}} \end{array}$$

$$\times\left(\frac{\lambda_2(1-\mu)I_c}{\beta_{11}}\right)^{-\beta_{11}}\times\left(\frac{\lambda_3}{\beta_{12}}\right)^{\beta_{12}}$$

$$\begin{aligned}
&s.t.\\
&\beta_1=1\\
&-\beta_1+\beta_2=0\\
&(\alpha-1)\beta_2-\beta_3+(\alpha-1)\beta_4-\beta_5-\beta_6-\beta_7+(\alpha-1)\beta_9+\beta_{11}=0\\
&-\lambda\beta_2+\beta_3-\lambda\beta_4-\lambda\beta_9=0\\
&-\sigma\beta_2-\sigma\beta_4+2\beta_7-\sigma\beta_9-\beta_{11}+\beta_{12}=0\\
&-\beta_4+\beta_5-\beta_7+\beta_8-\beta_9-\beta_{12}=0\\
&-\beta_2-\beta_3-\beta_4-\beta_5-\beta_6-\beta_7-\beta_8-\beta_9+\beta_{10}+\beta_{11}=0\\
&\beta_6+\beta_7+\beta_8-\beta_{11}=0\\
&\beta_4-\theta\beta_9=0\\
&\beta_5-\gamma\beta_9=0\\
&+\beta_2+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7+\beta_8+\beta_9=\lambda_1\\
&+\beta_{10}+\beta_{11}=\lambda_2\\
&+\beta_{12}=\lambda_3\\
&\beta_i\ge 0,\quad i=1,2,\dots,12
\end{aligned}\tag{10}$$

After solving the above constraints in terms of β_7 and β_8, we have:

$$\beta_1=1,\tag{11.a}$$

$$\beta_2=1,\tag{11.b}$$

$$\beta_3=\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.c}$$

$$\beta_4=\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.d}$$

$$\beta_5=\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.f}$$

$$\beta_6=\frac{(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.g}$$

$$\beta_9=\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.h}$$

$$\beta_{10}=\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma},\tag{11.i}$$

$$\beta_{11}=\frac{(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8,\tag{11.j}$$

$$\beta_{12}=\frac{(\gamma-\theta-1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7+\beta_8,\tag{11.k}$$

$$\lambda_1=\frac{(\theta-\gamma+1)(\alpha-\lambda-1)-\gamma(\alpha-\sigma)}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8,\tag{11.l}$$

$$\lambda_2=\frac{(\theta-\gamma+1)(\alpha-\lambda-1)-\gamma(\alpha-\sigma)}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8,\tag{11.m}$$

$$\lambda_3=\frac{(\gamma-\theta-1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7+\beta_8.\tag{11.n}$$

Since, model (10) converts to the following problem after substituting variables in the objective function by equations (11.a)-(11. n):

$$MinMax\,\varphi(\beta)=(\lambda_1u^{-1})\times\left(\frac{\lambda_1}{\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{u^{-1}\lambda_1}{\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{0.5\lambda_1}{\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{\lambda_1}{\frac{(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{0.5\lambda_1(1-\mu)(I_c-I_e)}{\beta_7}\right)^{\beta_7}$$

$$\times\left(\frac{0.5\lambda_1I_c}{\beta_8}\right)^{\beta_8}\times\left(\frac{vu^{-1}\lambda_1}{\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{\lambda_2}{\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$

$$\times\left(\frac{(1-\mu)I_c\lambda_2}{\frac{(\gamma-\theta-1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8}\right)^{-\left(\frac{(\gamma-\theta-1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8\right)}$$

$$\begin{aligned}
&s.t.\\
&(\alpha-\lambda-1)(\theta+1)-\gamma<0\\
&1-\alpha+\lambda\le 0\\
&(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma\le 0\\
&\frac{(\gamma-\theta-1)(1-\alpha+\lambda)+\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8\ge 0
\end{aligned}$$

$$\frac{(\gamma-\theta-1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7+\beta_8\geq 0$$

$$\lambda_1=\lambda_2=\frac{(\theta-\gamma+1)(\alpha-\lambda-1)-\gamma(\alpha-\sigma)}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_7+\beta_8\geq 0$$

$$\beta_i\geq 0,\quad i=7,8 \tag{12}$$

Therefore, after taking logarithm of the objective function in model (12) and using its derivatives with respect to β_7 and β_8, the optimal amount of dual variables β_7 and β_8 are obtained by any search method. Hence, the optimal amount of dual variables β_{11}, β_{12} and also objective function $(\varphi(\beta))$ would be achieved. In reversed constraint program, the dual variables and the primal variables are linked by following equation:

$$\omega_j=\frac{\beta_i}{\lambda_i},\qquad \begin{array}{l}\forall i=2,3,...,12\\ j=1,2,..,11.\end{array} \tag{13}$$

Where ω_j are the weights of the components of the restrictions in model (9). According to this relation we have:

$$\omega_1=\frac{\beta_2^*}{\lambda_1}=u^{-1}w^*S^{*\alpha-1}E^{*-\lambda}M^{*-\sigma}x^{*-1}, \tag{14.a}$$

$$\omega_2=\frac{\beta_3^*}{\lambda_1}=S^{*-1}E^*x^{*-1}, \tag{14.b}$$

$$\omega_3=\frac{\beta_4^*}{\lambda_1}=u^{-1}A^*S^{*\alpha-1}E^{*-\lambda}M^{*-\sigma}T^{*-1}x^{*-1}, \tag{14.c}$$

$$\omega_4=\frac{\beta_5^*}{\lambda_1}=0.5h^*S^{*-1}T^*x^{*-1}, \tag{14.d}$$

$$\omega_5=\frac{\beta_6^*}{\lambda_1}=C^*S^{*-1}x^{*-1}, \tag{14.e}$$

$$\omega_6=\frac{\beta_7^*}{\lambda_1}=0.5(1-\mu)(I_c-I_e)C^*S^{*-1}M^{*2}T^{*-1}x^{*-1}, \tag{14.f}$$

$$\omega_7=\frac{\beta_8^*}{\lambda_1}=0.5I_cC^*S^{*-1}T^*x^{*-1}, \tag{14.g}$$

$$\omega_8=\frac{\beta_9^*}{\lambda_1}=vu^{-1}S^{*\alpha-1}E^{*-\lambda}M^{*-\sigma}A^{*-\theta}h^{*-\gamma}T^{*-1}x^{*-1}, \tag{14.h}$$

$$\omega_9=\frac{\beta_{10}^*}{\lambda_2}=x^{*-1}, \tag{14.i}$$

$$\omega_{10}=\frac{\beta_{11}^*}{\lambda_2}=(1-\mu)I_cC^*S^{*-1}M^*x^{*-1}, \tag{14.j}$$

$$\omega_{11}=\frac{\beta_{12}^*}{\lambda_3}=M^*T^{*-1}. \tag{14.k}$$

Using equations (14.a)-(14. k), the optimal value of decision variables can be achieved in closed forms as follows:

$$M^*=\frac{\omega_{11}\omega_7}{0.5I_c\omega_5}, \tag{15.a}$$

$$T^*=\frac{\omega_7}{0.5I_c\omega_5}, \tag{15.b}$$

$$S^*=\begin{pmatrix} v\,(0.5)^{\gamma}u^{-\theta-1}\omega_3^{-\theta}\omega_4^{-\gamma}\omega_8^{-1}\omega_2^{-\theta(\lambda+1)}\\ \times M^{*-\sigma\theta-\sigma}T^{*-\theta+\gamma-1}\omega_9^{(\lambda+1)(\theta+1)+\gamma}\end{pmatrix}^{\frac{1}{(\lambda+1-\alpha)(\theta+1)+\gamma}}, \tag{15.c}$$

$$C^*=\omega_5\omega_9^{-1}S^*, \tag{15.d}$$

$$E^*=\omega_2\omega_9^{-1}S^*, \tag{15.e}$$

$$h^*=I_c\omega_4\omega_5\omega_7^{-1}\omega_9^{-1}S^*, \tag{15.f}$$

$$A^*=\left(vI_c^{-\gamma}\omega_3\omega_4^{-\gamma}\omega_5\omega_7^{\gamma}\omega_8^{-1}\omega_9^{\gamma}S^{*-\gamma}\right)^{\frac{1}{1+\theta}}, \tag{15.g}$$

$$w^*=(0.5)^{-\sigma}uI_c^{-\sigma}\omega_1\omega_2^{\lambda}\omega_5^{-\sigma}\omega_7^{\sigma}\omega_9^{-1-\lambda}\omega_{11}^{\sigma}S^{*1-\alpha+\lambda}, \tag{15.h}$$

$$Q^*=DT=u\left(\frac{\omega_{11}^{\sigma}\omega_7^{1+\sigma}}{(0.5)^{1+\sigma}I_c^{1+\sigma}\omega_5^{1+\sigma}}\right)\omega_2^{\lambda}\omega_9^{-\lambda}S^{*-\alpha+\lambda}. \tag{15.i}$$

We can obtain the optimal values of decision variables for case 2 by following similar steps applied for case 1.

- *Case2-primal:*

$$Min\ f'=(w')^{-1}$$

$$s.t.$$

$$\left\{\begin{array}{l} u^{-1}wS^{\alpha-1}E^{-\lambda}M^{-\sigma}(x')^{-1}+S^{-1}E(x')^{-1}+\\ u^{-1}AS^{\alpha-1}E^{-\lambda}M^{-\sigma}T^{-1}(x')^{-1}+0.5hS^{-1}T+CS^{-1}(x')^{-1}\\ +0.5(\mu I_c+(1-\mu)I_e)CS^{-1}T(x')^{-1}\\ +vu^{-1}S^{\alpha-1}E^{-\lambda}M^{-\sigma}A^{-\theta}h^{-\gamma}T^{-1}(x')^{-1}\end{array}\right\}\leq 1$$

$$(x')^{-1}+(1-\mu)I_eCS^{-1}M(x')^{-1}\geq 1$$

$$TM^{-1}\leq 1$$

$$M_0^{-1}M\leq 1 \tag{16}$$

- *Case2-dual:*

$$MinMax\ \varphi'(\beta')=\left(\frac{1}{\beta_1'}\right)^{\beta_1'}\times\left(\frac{\lambda_1'(u')^{-1}}{\beta_2'}\right)^{\beta_2'}\times\left(\frac{\lambda_1'}{\beta_3'}\right)^{\beta_3'}\times\left(\frac{\lambda_1'(u')^{-1}}{\beta_4'}\right)^{\beta_4'}$$
$$\times\left(\frac{0.5\lambda_1'}{\beta_5'}\right)^{\beta_5'}\times\left(\frac{\lambda_1'}{\beta_6'}\right)^{\beta_6'}\times\left(\frac{0.5(\mu I_c+(1-\mu)I_e)\lambda_1'}{\beta_7'}\right)^{\beta_7'}$$
$$\times\left(\frac{vu^{-1}\lambda_1'}{\beta_8'}\right)^{\beta_8'}\times\left(\frac{\lambda_2'}{\beta_9'}\right)^{-\beta_9'}\times\left(\frac{\lambda_2'(1-\mu)I_e}{\beta_{10}'}\right)^{-\beta_{10}}$$

$$\times\left(\frac{\lambda_3'}{\beta_{11}'}\right)^{\beta_{11}'}\times\left(\frac{\lambda_4'}{\beta_{12}'}\right)^{\beta_{12}'}$$

$s.t.$

$$\beta_1'=1$$
$$-\beta_1'+\beta_2'=0$$
$$(\alpha-1)\beta_2'-\beta_3'+(\alpha-1)\beta_4'-\beta_5'-\beta_6'-\beta_7'+(\alpha-1)\beta_8'+\beta_{10}'=0$$
$$-\lambda\beta_2'+\beta_3'-\lambda\beta_4'-\lambda\beta_8'=0$$
$$-\sigma\beta_2'-\sigma\beta_4'-\sigma\beta_8'-\beta_{10}'-\beta_{11}'+\beta_{12}'=0$$
$$-\beta_4'+\beta_5'+\beta_7'-\beta_8'+\beta_{11}'=0$$
$$-\beta_2'-\beta_3'-\beta_4'-\beta_5'-\beta_6'-\beta_7'-\beta_8'+\beta_9'+\beta_{10}'=0$$
$$\beta_6'+\beta_7'-\beta_{10}'=0$$
$$\beta_4'-\theta\beta_8'=0$$
$$\beta_5'-\gamma\beta_8'=0$$
$$+\beta_2'+\beta_3'+\beta_4'+\beta_5'+\beta_6'+\beta_8'+\beta_9'=\lambda_1'$$
$$+\beta_9'+\beta_{10}'=\lambda_2'$$
$$+\beta_{11}'=\lambda_3'$$
$$+\beta_{12}'=\lambda_4'$$
$$\beta_i'\geq 0,\quad i=1,2,\ldots,12 \tag{17}$$

Solving the above constraints in terms of β_6', and β_7', we have:

$$\beta_1'=1, \tag{18.a}$$
$$\beta_2'=1, \tag{18.b}$$
$$\beta_3'=\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}, \tag{18.c}$$
$$\beta_4'=\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma}, \tag{18.d}$$
$$\beta_5'=\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}, \tag{18.e}$$
$$\beta_8'=\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}, \tag{18.f}$$
$$\beta_9'=\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}, \tag{18.g}$$
$$\beta_{10}'=\beta_6'+\beta_7' \tag{18.h}$$
$$\beta_{11}'=\frac{(\theta-\gamma+1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7, \tag{18.i}$$
$$\beta_{12}'=\frac{(\gamma-\theta-1)(\alpha-\lambda-1)-\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_6, \tag{18.j}$$
$$\lambda_1'=\lambda_2'=\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_6'+\beta_7' \tag{18.k}$$
$$\lambda_3'=\frac{(\theta-\gamma+1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7, \tag{18.l}$$
$$\lambda_4'=\frac{(\gamma-\theta-1)(\alpha-\lambda-1)-\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_6. \tag{18.m}$$

After replacing variables in the objective function by equations (18.a)-(18. m), we have:

$$MinMax\,\varphi'(\beta')=\left(\lambda_1'(u')^{-1}\right)$$
$$\times\left(\frac{\lambda_1'}{\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{-\gamma\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$
$$\times\left(\frac{u^{-1}\lambda_1'}{\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\theta-\alpha\theta+\lambda\theta}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$
$$\times\left(\frac{0.5\lambda_1'}{\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\gamma-\alpha\gamma+\lambda\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$
$$\times\left(\frac{\lambda_1'}{\beta_6'}\right)^{\beta_6'}\times\left(\frac{0.5\left(\mu I_c+(1-\mu)I_e\right)\lambda_1'}{\beta_7'}\right)^{\beta_7'}$$
$$\left(\frac{vu^{-1}\lambda_1'}{\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{1-\alpha+\lambda}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$
$$\times\left(\frac{\lambda_2'}{\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}}\right)^{\left(\frac{\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}\right)}$$
$$\times\left(\frac{\lambda_2'(1-\mu)I_e}{\beta_6'+\beta_7'}\right)^{-(\beta_6'+\beta_7')}$$

$s.t.$

$$(\alpha-\lambda-1)(\theta+1)-\gamma<0$$
$$1-\alpha+\lambda\leq 0$$
$$\beta_6'+\beta_7'\geq 0$$
$$\frac{(\theta-\gamma+1)(1-\alpha+\lambda)}{(\alpha-\lambda-1)(\theta+1)-\gamma}-\beta_7\geq 0$$
$$\frac{(\gamma-\theta-1)(\alpha-\lambda-1)-\sigma\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_6\geq 0$$
$$\lambda_1'=\lambda_2'=\frac{-\alpha\gamma}{(\alpha-\lambda-1)(\theta+1)-\gamma}+\beta_6'+\beta_7'\geq 0$$
$$\beta_i'\geq 0,\quad i=6,7 \tag{19}$$

Now we can take logarithm of the objective function in model (19) and use its derivatives with respect to β_6'

and β_7' to obtain the optimal values of β_6', β_7', β_{10}' β_{11}', β_{12}', also objective function $\left(\varphi'(\beta')\right)$ by any search method. The weights of ω_i' are calculates as according to equation (13):

$$\omega_1' = \frac{\beta_2'^*}{\lambda_1'} = (u')^{-1}(w')^* S^{*\alpha-1} E^{*-\lambda} M^{*-\sigma} (x')^{*-1}, \quad (20.a)$$

$$\omega_2' = \frac{\beta_3'^*}{\lambda_1'} = S^{*-1} E^* (x')^{*-1}, \quad (20.b)$$

$$\omega_3' = \frac{\beta_4'^*}{\lambda_1'} = (u')^{-1} A^* S^{*\alpha-1} E^{*-\lambda} M^{*-\sigma} T^{*-1} (x')^{*-1}, \quad (20.c)$$

$$\omega_4' = \frac{\beta_5'^*}{\lambda_1'} = 0.5h^* S^{*-1} T^* (x')^{*-1}, \quad (20.d)$$

$$\omega_5' = \frac{\beta_6'^*}{\lambda_1'} = C^* S^{*-1} (x')^{*-1}, \quad (20.e)$$

$$\omega_6' = \frac{\beta_7'^*}{\lambda_1'} = 0.5\left(\mu I_c + (1-\mu) I_e\right) C^* S^{*-1} T^* (x')^{*-1}, \quad (20.f)$$

$$\omega_7' = \frac{\beta_8'^*}{\lambda_1'} = v u^{-1} S^{*\alpha-1} E^{*-\lambda} M^{*-\sigma} A^{*-\theta} h^{*-\gamma} T^{*-1} (x')^{*-1} \quad (20.g)$$

$$\omega_8' = \frac{\beta_9'^*}{\lambda_1'} = (x')^{*-1}, \quad (20.h)$$

$$\omega_9' = \frac{\beta_{10}'^*}{\lambda_2'} = (1-\mu) I_e C^* S^{*-1} M^* (x')^{*-1}, \quad (20.i)$$

$$\omega_{10}' = \frac{\beta_{11}'^*}{\lambda_3'} = T^* M^{*-1}, \quad (20.j)$$

$$\omega_{11}' = \frac{\beta_{12}'^*}{\lambda_4'} = M_0^{-1} M^*. \quad (20.k)$$

Finally, the optimal solutions of case 2 according to above elations are:

$$M^* = M_0 \omega_{11}, \quad (21.a)$$

$$T^* = M_0 \omega_{10} \omega_{11}, \quad (21.b)$$

$$S^* = \begin{pmatrix} v\,(0.5)^{\gamma} u^{-\theta-1} \omega_3^{-\theta} \omega_4^{-\gamma} \omega_7^{-1} \omega_2^{-\theta(\lambda+1)} \\ \times M^{*-\sigma\theta-\sigma} T^{*-\theta+\gamma-1} \omega_8^{(\lambda+1)(\theta+1)+\gamma} \end{pmatrix}^{\frac{1}{(\lambda+1-\alpha)(\theta+1)+\gamma}}, \quad (21.c)$$

$$C^* = \omega_5 \omega_8^{-1} S^*, \quad (21.d)$$

$$E^* = \omega_2 \omega_8^{-1} S^*, \quad (21.e)$$

$$h^* = 2M_0^{-1} \omega_4 \omega_8^{-1} \omega_{10}^{-1} \omega_{11}^{-1} S^*, \quad (21.f)$$

$$A^* = \left(2^{-\gamma} M_0^{\gamma} v\, \omega_3 \omega_4^{-\gamma} \omega_7^{-1} \omega_8^{\gamma} \omega_{10}^{\gamma} \omega_{11}^{\gamma} S^{*-\gamma}\right)^{\frac{1}{1+\theta}}, \quad (21.g)$$

$$w^* = M_0^{\sigma} u\, \omega_1 \omega_2^{\lambda} \omega_8^{-1-\lambda} \omega_{11}^{\sigma} S^{*1-\alpha+\lambda}, \quad (21.h)$$

$$Q^* = u M_0^{\sigma+1} \omega_{10} \omega_{11}^{\sigma+1} \omega_2^{\lambda} \omega_8^{-\lambda} S^{*\lambda-\alpha}. \quad (21.i)$$

Numerical results

A retailer presents a single product to the market. He offers a partial delayed payment to his customers. Decision maker wants to find the optimal credit period, selling price, marketing costs, ordering , purchasing and holding costs, demand ,order quantity and optimal total average profit in situation that demand rate as: $D = 10^6 S^{-3.1} E^{0.05} M^{0.25}$ and the capital investment for reducing ordering and holding cost is as $C(h,A) = 1.1A^{-1.5} h^{-8.3}$.The value of parameters for this product are given as: $I_p = 0.1$, $I_e = 0.05$, $\mu = 0.5$, and $M_0 = 0.9$.Considering the solution procedure described in previous section ,we first solve problem (12) for case (1) and problem (19) for case (2) and the optimum results are reported in Table 1. Then, we must compare objective functions to determine which is higher and those amounts associated with the higher profit must be selected as optimal solutions. According to Table 1, case 2 has higher profit. Therefore, the optimal solution of decision variables must be selected based on case 2.

Table 1- The optimal solutions

Decision variables	Case 1	Case 2
D^*	1305.081	2125.518
S^*	7.7500	6.97
E^*	0.0848	0.11443
M^*	0.337	0.9
T^*	0.337	0.9
Q^*	439.8122	1912.9716
A^*	47.156	43.189
C^*	3.835	3.312
h^*	0.874	0.956
Total profit	5757.9254	5907.8045

Conclusion

In this research, we developed an EOQ model under partial delayed payments for a retailer where demand rate was considered as power function of marketing costs, the length of the credit period suggested by retailer to his/ her customers, and selling price. In this work, ordering and holding costs can be controlled by more investment. The proposed problem was formulated in two cases as nonlinear programming problem pf profit maximization. For model solving, we first transformed our models to a constrained SGP problem with 2 degrees of difficulty; and late, we converted the SGP problem in to the reversed constrained programming to achieve the optimal decisions in closed forms. The applicability of this solution method is demonstrated by a numerical example. This problem can be extended to investigate the environmental issues, uncertain environment, and multi products.

References

[1] Goyal, S.K., "Economic order quantity under conditions of permissible delay in payments". *Journal of the operational research society*, 1985. **36**(4): p. 335-338.

[2] Aggarwal, S. and C. Jaggi, "Ordering policies of deteriorating items under permissible delay in payments". *Journal of the operational Research Society*, 1995. **46**(5): p. 658-662.

[3] Sarker, B.R., A. Jamal, and S. Wang, "Supply chain models for perishable products under inflation and permissible delay in payment". *Computers & Operations Research*, 2000. **27**(1): p. 59-75.

[4] Huang, Y.-F., "Economic order quantity under conditionally permissible delay in payments". *European Journal of Operational Research*, 2007. **176**(2): p. 911-924.

[5] Taleizadeh, A.A., et al., "An EOQ model with partial delayed payment and partial backordering". *Omega*, 2013. **41**(2): p. 354-368.

[6] Sharma, B., "An EOQ model for retailers partial permissible delay in payment linked to order quantity with shortages". *Mathematics and Computers in Simulation*, 2016. **125**: p. 99-112.

[7] Lee, W.J. and D. Kim, "Optimal and heuristic decision strategies for integrated production and marketing planning". *Decision Sciences*, 1993. **24**(6): p. 1203-1214.

[8] Samadi, F., A. Mirzazadeh, and M.M. Pedram, "Fuzzy pricing, marketing and service planning in a fuzzy inventory model: A geometric programming approach". *Applied Mathematical Modelling*, 2013. **37**(10): p. 6683-6694.

[9] Sadjadi, S.J., et al., "Joint pricing and production management: a geometric programming approach with consideration of cubic production cost function". *Journal of Industrial Engineering International*, 2015. **11**(2): p. 209-223.

[10] Sadjadi, S.J., M.-B. Aryanezhad, and A. Jabbarzadeh, "Optimal marketing and production planning with reliability consideration". *African Journal of Business Management*, 2010. **4**(17): p. 3632.

[11] Nezami, F.G., S.J. Sadjadi, and M.B. AryaNezhad. *"A geometric programming approach for a nonlinear joint production-marketing problem"*. in *Computer Science and Information Technology-Spring Conference, 2009. IACSITSC'09. International Association of*. 2009. IEEE.

[12] Dye, C.-Y. and C.-T. Yang, "Sustainable trade credit and replenishment decisions with credit-linked demand under carbon emission constraints". *European Journal of Operational Research*, 2015. **244**(1): p. 187-200.

[13] Ho, C.-H., "The optimal integrated inventory policy with price-and-credit-linked demand under two-level trade credit". *Computers & Industrial Engineering*, 2011. **60**(1): p. 117-126.

[14] Jaggi, C.K., S. Goyal, and S.K. Goel, "Retailer's optimal replenishment decisions with credit-linked demand under permissible delay in payments". *European Journal of Operational Research*, 2008. **190**(1): p. 130-135.

[15] Jaggi, C.K. and A. Khanna, "The retailer's procurement policy with credit-linked demand under inflationary conditions". *International Journal of Procurement Management*, 2009. **2**(2): p. 163-179.

[16] Zener, C., *Engineering design by geometric programming*. 1971: John Wiley & Sons.

[17] Mandal, N.K., T.K. Roy, and M. Maiti, "Multi-objective fuzzy inventory model with three constraints: a geometric programming approach". *Fuzzy Sets and Systems*, 2005. **150**(1): p. 87-106.

[18] Chiang, M., et al., "Power control by geometric programming". *Resource Allocation in Next Generation Wireless Networks*, 2005. **5**: p. 289-313.

[19] Xu, G., "Steady-state optimization of biochemical systems through geometric programming". *European Journal of Operational Research*, 2013. **225**(1): p. 12-20.

[20] Marin-Sanguino, A., et al., "Optimization of biotechnological systems through geometric programming". *Theoretical Biology and Medical Modelling*, 2007. **4**(1): p. 1.

[21] Duffin, R.J. and E.L. Peterson, "Reversed geometric programs treated by harmonic means". 1971.