



Modeling and Analysis of Thin Steel Plate Shear Walls Using the New Method

Ghasem Pachideh

Phd Candidate, civil Department, Semnan University, Semnan,Iran
Ghasem_civil@yahoo.com

Majid Gholhaki

Assistant Professor, civil Department, Semnan University, Semnan,Iran
mgholhaki@semnan.ac.ir

Alireza Yadegari

Architecture Associate's student, Islamic Azad University of Varamin Branch-Pishva,Varamin,Iran
alirezayadegari@yahoo.com

Morteza Shiri

Islamic Azad University of Varamin Branch-Pishva,Varamin,Iran
m.shiri1400@gmail.com

Abstract

Thin steel plate shear walls (SPSWs) are an effective lateral force resisting system and are increasingly being used in structures. Engineers require the ability to predict both elastic and inelastic structural responses of SPSWs accurately by a simplified model that is convenient to use. Based on the strip model, this paper proposes a new simplified model for SPSWs, called the three-strip model, in which the infill plates of SPSWs are replaced by three tension-only strips. Comparisons of experimentally obtained pushover responses of SPSWs and that predicted by the proposed model are given and reasonable agreement is observed. A parametric study examines the effect of aspect ratio and column flexibility on the accuracy of the predicted inelastic behavior from the three-strip model. In addition, the capacity of the proposed model in predicting frame forces is verified based on test and strip model results. It was found that the three-strip model can provide generally accurate results for SPSWs and meanwhile reduce modeling workload and computational expense.

Keywords: shear wall, Steel plate, Design, Model, Metal and composite structures.



Introduction

Steel plate shear walls (SPSWs) have been increasingly used as lateral load resisting systems in recent years. Traditional SPSWs are composed of unstiffened thin infill steel plates and boundary frames. The infill plates will buckle under very small shear loads, and as the plates are anchored by the boundary beams and columns, the postbuckling tension field action of SPSWs can provide considerable stiffness and strength (Thorburn et al. 1983). Experimental research has shown the SPSWs feature high elastic stiffness properties, large displacement ductility capacities, and stable hysteresis behavior under cyclic loading (e.g. Timler et al. 1983; Driver et al. 1997; Lubell et al. 2000; Behbahanifard et al. 2003).

Some simplified models for SPSWs have been developed. Thorburn et al. (1983) proposed an equivalent truss model that treats the plate at each story as a single pin-ended truss member. Although it cannot predict accurate inelastic behaviors for SPSWs, this model was recommended both by CAN/CSA S16-01 and ANSI/AISC 34105 (AISC 2005) for preliminary design purposes. Thorburn et al. (1983) also developed a strip model in which the tension field action of the infill plate is modeled by a series of tension-only strips. This model was validated to be a satisfactory approach for predicting inelastic behaviors (Timler et al. 1983; Driver et al. 1997). Both CAN/CSA S16-01 and ANSI/AISC 34105 (AISC 2005) adopted the strip model as the major analytical tool for the design of SPSWs. When using the strip model, the inclination angle and spacing of the strips should be determined separately for each panel because they are directly dependent on the properties of the boundary frames and the infill plate thickness. Therefore, it is very probable that the strips of adjacent panels will have staggered nodes at boundary beams, and every changing of the inclination angle of the strip in a design process will lead to a geometric revision of the analysis model. These make the utilization of the strip model time-consuming and troublesome. A simpler method using an average inclination angle to arrange the strips for all panels was suggested by Timler et al. (1998), by which the problem of changing the inclination angle is avoided. The accuracy loss from this method has been verified to be acceptable (Timler et al. 1998) so it was adopted by ANSI/AISC 34105 (AISC 2005). However, this method cannot guarantee the strip number is constant for all panels, and therefore, reassessments of the strip spacing may be required (Shishkin et al. 2005). Moreover, the strips need to be arranged symmetrically to reflect the tension-field action caused by bidirectional lateral loads and the nodes of strips in two directions are inevitably staggered. These may considerably increase the computational expense. To simplify the modeling, Rezai et al. (2000) proposed a multiangle model in which tension strips are placed diagonally between opposite corners and from the corners to the midspan of boundary members. However, the elastic stiffness and ultimate strength obtained from this model were found to be less than ideal. To predict the inelastic behavior of SPSWs more accurately, a modified strip model was proposed by Shishkin et al. (2005). In this model, a compression strut and deterioration strips were introduced by the strip model. This model was found to predict more accurate overall inelastic behavior for a four-story specimen (Driver et al. 1997) than the strip model. But it estimated the peak capacity and declining curve quite inaccurately for a one-story specimen (Lubell et al. 2000) because the deterioration of the specimen was not reflected correctly by the model. In addition, the compression strut may provide too much stiffness, and the designers were advised to create one more model (without compression strut) to predict the response of thin SPSWs. However, it seems a problem for engineers to judge which result is more accurate in a design progress. This paper proposes a new simplified model for SPSWs, called the three-strip model, to reduce the modeling workload and computational expense for engineers. Pushover analysis and parametric study are conducted to verify the proposed model. The model is shown to be much simpler and more efficient for application while still providing generally accurate results.

Strip Model

The strip model for a typical story of SPSWs is shown in Fig. 1(a). In this model, a series of pin-ended tension-only strips is used to model the tension field action and the prebuckling strength of the panel is neglected. The boundary beams are assumed to be infinitely rigid in bending because of the presence of the tension fields adjacent to the modeled panel. Although pinned beam-to-column connections are used in Fig. 1(a), the connections also could be rigid or semirigid to reflect the frame behavior of a real structure. The inclination angle of the tension field, α , is given by (Timler et al. 1983)

$$\alpha = \arctan \sqrt{\frac{1 + \left(\frac{A_c}{A_b}\right)}{1 + tH\left(\frac{1}{A_b} + \frac{H^3}{360I_c L}\right)}} \quad (1)$$

where t = infill plate thickness; I_c = moment of inertia of the columns; L = distance between the column centerlines; H = distance between the beam centerlines; and A_b and A_c = cross-sectional areas of the beam and column, respectively.

The story elastic stiffness of a single-story steel plate shear wall can be determined using the following equation (Thorburn et al. 1983):

$$K_s = \frac{Elt}{h} \sin^2 \alpha \cos^2 \alpha \quad (2)$$

where E = elastic modulus of the infill plate; l = length of the infill plate; and h = height of the infill plate. Eq. (2) neglects the stiffness of the boundary frame, which should be taken into consideration when rigid or semirigid beam-to-column connections are used. The ultimate shear strength of a single-story steel plate shear wall with pinned beam-to-column connections is given by (Berman et al. 2003)

$$V_s = F_y l t \sin \alpha \cos \alpha \quad (3)$$

where F_y = yield stress of the infill plate.

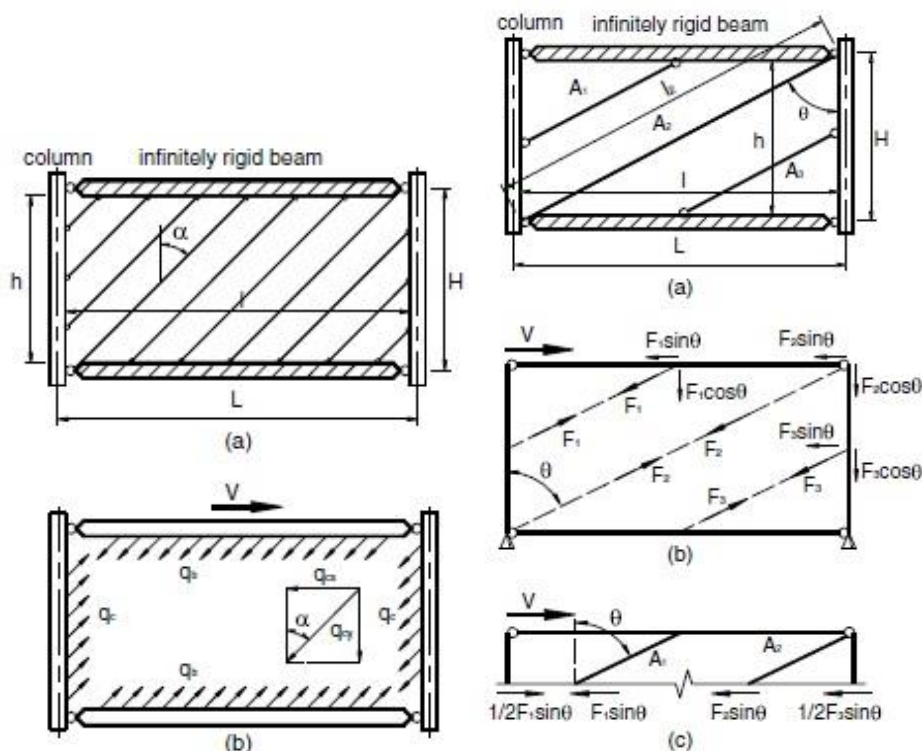


Fig. 1. Strip model and force diagram: (a) strip model (data from Thorburn et al. 1983); (b) panel force applied to the boundary fram

Fig. 2. Three-strip model and force diagram: (a) three-strip model; (b) strip force; (c) free body diagram

Three-Strip Model

In the proposed model, the infill plate panels of SPSWs are replaced by three strip elements that are placed diagonally between opposite corners and midspan of boundary members, as shown in Fig. 2(a). The cross-section areas of the strips are determined by three equivalence factors, which are the elastic stiffness, the maximum axial compression force, and the maximum bending moment of the boundary column.

Elastic Stiffness

Elastic stiffness is one of the most important factors for engineers. In this section, the elastic stiffness of the three-strip model is determined using elastic mechanics, and then an equation for determining strip cross-section areas is obtained.

Fig. 2(b) shows the strip axial force caused by a horizontal shearing force V . The axial force of the strips can be obtained using elastic mechanics

$$F_i = \frac{VEA_i \sin \theta}{K_i I_2} \quad (4)$$

where F_i = axial force of the i th strip; A_i = cross-sectional area of the i th strip; l_2 = length of the strip between opposite corners; K_t = story stiffness; and $\theta = \arctan l/h$.

Fig. 2(c) shows a free body diagram of portion of the three-strip model. The horizontal forces acted on the free body incorporate the horizontal shearing force, V , the shearing force of the left column, $1=2F_1 \sin\theta$, the shearing force of the right column, $1=2F_3 \sin\theta$, and the horizontal component of the strip axial forces, $F_1 \sin\theta$ and $F_2 \sin\theta$.

Writing an equilibrium equation of the horizontal forces, the horizontal shearing force, V , can be expressed as

$$V = F_2 \sin \theta + \frac{1}{2} F_1 \sin \theta + \frac{1}{2} F_3 \sin \theta \quad (5)$$

Substituting Eq. (4) into Eq. (5) and using the geometry $l_2 = h/\cos\theta$, the story stiffness of a single-story three-strip model can be obtained

$$K_t = \frac{E \sin^2 \theta \cos \theta}{h} \cdot \left(\frac{A_1}{2} + A_2 + \frac{A_3}{2} \right) \quad (6)$$

It should be noticed that Eq. (6) neglects the stiffness of the boundary frame, which should be taken into consideration when rigid or semirigid beam-to-column connections are used. Equating Eqs. (2) and (6) gives

$$\frac{A_1}{2} + A_2 + \frac{A_3}{2} = \frac{I t \sin^2 \alpha \cos^2 \alpha}{\sin^2 \theta \cos \theta} \quad (7)$$

Maximum Axial Compression Force of the Boundary Column

The equivalent truss model (Thorburn et al. 1983), which is widely used for preliminary sizing, cannot predict the forces of the boundary members accurately. Most research shows that the axial force and moment of the boundary column caused by the tension field of the buckling panel are considerable. As the maximum value is relatively important for design purposes (the axial force in the boundary column will vary linearly from top to bottom), the maximum axial compression force is adopted as an equivalence factor.

According to Fig. 2(b), the maximum compression axial force of the boundary column in the three-strip model can be written as

$$F = F_2 \cos \theta + F_3 \cos \theta = \frac{EV \sin \theta \cos^2 \theta}{K_t h} \cdot (A_2 + A_3) \quad (8)$$

Eq. (8) neglects the vertical component of F_1 for the purpose of reflecting the presence of the opposing tension field, which is adjacent to the modeled panel.

When the horizontal shearing force, V , is applied to the strip model [Fig. 1(b)], the force acted on the boundary column can be obtained by elastic mechanics

$$q_c = \frac{EVt \sin^2 \alpha \cos \alpha}{K_s h} \quad (9)$$

The maximum axial compression force of the boundary column in the strip model can be expressed as

$$F = q_c h \cos \alpha = \frac{EVt}{K_s} \sin^2 \alpha \cos^2 \alpha \quad (10)$$

The vertical components of qb are neglected.

Recognizing the fact that Kt is equal to Ks, equating Eqs. (8) and (10) gives

$$A_2 + A_3 = \frac{ht \sin^2 \alpha \cos^2 \alpha}{\sin \theta \cos^2 \theta} \quad (11)$$

Maximum Bending Moment of the Boundary Column

The moment of the boundary members caused by lateral loads can be considered as the superposition of two parts (Tsai et al. 2010). One part is only due to the panel force effect and the other part is due to the frame sway action. The maximum moment could appear at either the top or bottom of the column and a reliable prediction of the end moment caused by the panel force could guarantee the maximum moment is correct. For the strip model, according to Fig. 3, the moment at the end of the column caused by the panel force can be written as

$$M_{qc} = \frac{1}{12} q_c h^2 \sin \alpha = \frac{EVth \sin^3 \alpha \cos \alpha}{12K_s} \quad (12)$$

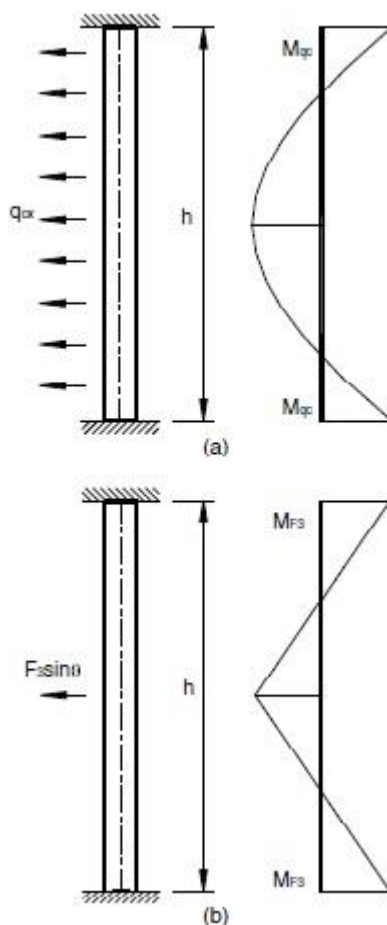


Fig. 3. Column moment caused by panel force: (a) strip model; (b) three-strip model

$$M_{F3} = \frac{1}{8} F_3 h \sin \theta = \frac{EVA_3 \sin^2 \theta \cos \theta}{8K_T} \quad (13)$$

Eqs. (12) and (13) are based on the assumption that the boundary column has fixed support on both sides. Because there are horizontal forces caused by the tension field in each story and the reactions between neighboring stories are almost the same, the boundary column could be equivalent to a continuous beam in terms of the resulting moment of the tension field. Therefore, the assumption of fixed support is adopted. Equating Eqs. (12) and (13) gives

$$A_3 = \frac{2h \cdot t \cdot \sin^3 \alpha \cdot \cos \alpha}{3 \sin^2 \theta \cdot \cos \theta} \quad (14)$$

Cross Section Areas of the Strips

Using Eqs. (7), (11), and (14), the strip areas can be obtained as

$$A_1 = A_3 = \frac{2ht \sin^2 \alpha \cos \alpha}{3 \sin^2 \theta \cos \theta} \quad (15)$$

$$A_2 = \frac{ht \sin^2 \alpha \cos^2 \alpha}{\sin \theta \cos^2 \theta} - \frac{2ht \sin^3 \alpha \cos \alpha}{3 \sin^2 \theta \cos \theta} \quad (16)$$

Maximum Shear Strength of the Three-Strip Model

Using elastic mechanics, the stress of the strips caused by a horizontal shearing force V can be written as

$$\sigma = EV \sin \theta / K_1 I_2 \quad (17)$$

Substituting Eqs. (15)–(17) into Eq. (6), assuming that the strip stress (σ) is equal to the plate yield stress in tension (F_y) and using the geometry $l/2 = h \cos \theta$, the maximum shear strength of the three-strip model can be expressed as

$$V_t = F_y l t \frac{\sin^2 \alpha \cos^2 \alpha}{\sin \theta \cos \theta} \quad (18)$$

It is found that Eq. (18) is not identical to Eq. (3), which is because the maximum shear strength is not treated as an equivalence factor to determine the strip areas. Dividing Eq. (18) by Eq. (3) gives

$$\eta = \frac{V_t}{V_s} = \frac{\sin 2\alpha}{\sin 2\theta} \quad (19)$$

The value of α is within the range of 38–50° (Shishkin et al. 2005) and the value of θ , determined by l and h , varies at a probable range of 30–60° in real structures, thus the value of η would be close to 1, so Eq. (18) could be generally used to predict the ultimate strength of SPSWs.

Pushover Analysis for Model Validation

Pushover analysis is a primary tool for engineers to assess the inelastic behavior of a structure. To verify the three-strip model, pushover analyses (including the consideration of material nonlinearities and geometric nonlinearities) using the program SAP2000 are performed. The plastic hinge arrangement method taken for the modified strip model (Shishkin et al. 2005) is adopted in this paper. Flexural plastic hinges of frame members are located at a distance of one-half the member depth from the connection nodes and the base support nodes. Axial plastic hinges are placed in the middle of strips to model the yielding of the infill plates. The axial force versus elongation curve for the axial plastic hinge is defined as a bilinear elastic-perfectly plastic stress versus strain curve. For the beam and column flexural plastic hinges, the moment versus rotation curves are determined based on FEMA 356 (FEMA 2000). The interaction between axial load and moment of the column flexural plastic hinge is described by the following equation [FEMA 356 (FEMA 2000)]:

$$M_{pc} = 1.18ZF_y \left(1 - \frac{P}{P_y}\right) \leq ZF_y \quad (20)$$

where M_{pc} = expected flexural strength; Z = plastic section modulus; P = axial load; and P_y = expected axial yield force of the member.

Four-Story Specimen

A pushover analysis using the three-strip model is conducted on a four-story specimen (SPSW4), which was tested by Driver et al. (1997), as shown in Fig. 4(a). Fig. 4(b) shows the loading arrangement and geometry of the analysis model. The material properties of the specimen, based on ancillary tests, are input into the analysis model. Plastic hinges are arranged as the rule elucidated previously. The analysis is carried out by applying the full gravity loads (720 kN) to the column tops, followed by the lateral loads, as in the test. From the analysis, a pushover curve of the base shear versus top lateral displacements is obtained.

The pushover curve obtained by the three-strip model is compared to the envelope curve of the specimen in Fig. 5. The initial stiffness of the three-strip model is less than that measured during the test, as may be seen by comparing the initial slopes of the curves. The differences in initial stiffness is about 25%, reflecting the fact that the three-strip model neglects the prebuckling shear resistance of the infill plate. However, a slight decrease in stiffness of the test specimen was observed with each cycle of loading, even at very low load levels. The effective stiffness of the specimen (after some cycling at low load levels) is somewhat less than the initial stiffness, so the discrepancies are actually smaller than those reported previously (the difference in effective stiffness is about 12%). The ultimate strength predicted by the three-strip model is 2,985 kN, which is only 3.1% below the test value (3,080 kN). For comparison purposes, a pushover curve obtained by Driver et al. (1997) using the strip model is also shown in Fig. 5. The curve of the three-strip model is nearly identical to the strip model curve. Compared to the test result, a clear discrepancy is that the descending branch is not characterized by the two simplified models.

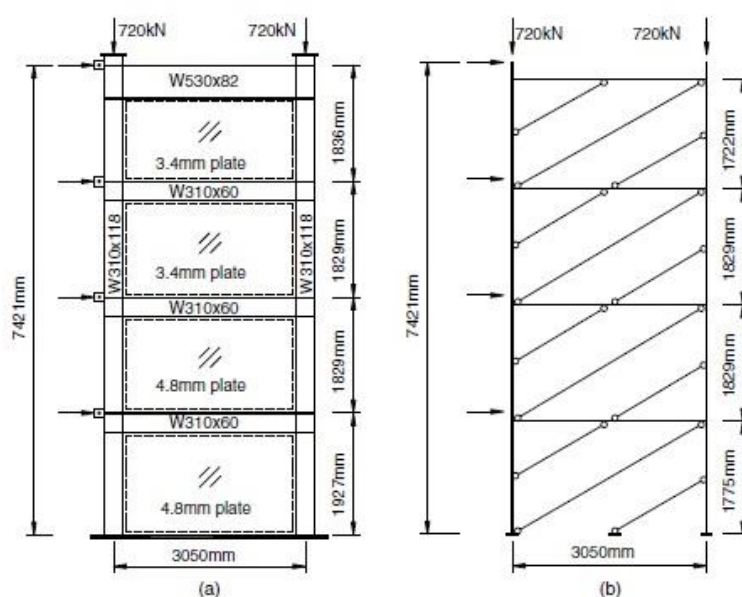


Fig. 4. SPSW4 (data from Driver et al. 1997): (a) specimen; (b) three-strip model

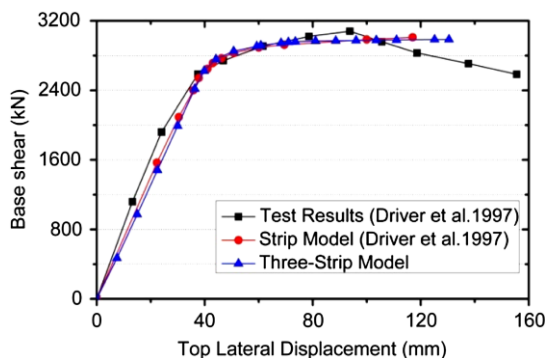


Fig. 5. Response curves of SPSW4 specimen

Parameters and Grouping of Models

The parameters considered in this study are the aspect ratio, $L=H$, and the column flexibility parameter, ω_H , defined as CSA S16-01 (CSA 2001)

$$\omega_H = 0.7H \sqrt{\frac{t}{2LI_c}} \leq 2.5 \quad (21)$$

The value of H and the thickness of all the infill plates are at constant value of 3,800 and 3 mm, respectively. Different values of L and column cross sections are used to reflect the variation of aspect ratio and ω_H . For comparison, the analysis models are arranged in groups (Table 1). Groups 1-A and 1-B are all arranged with various values of ω_H for single-story models. The aspect ratio of models in Groups 1-A and 1-B are 1.5 and 0.75, respectively. Groups 3-A and 3-B are arranged with various values of aspect ratio for three-story models, whereas the beam-to-column connections of the models in Group 3-B are all pinned. Groups 5-A and 5-B are arranged with various values of aspect ratio for five-story models, whereas the beam-to-column connections of models in Group 5-B are all pinned. All models meet the limit of the beam flexibility parameter, ω_L , which was derived by Dastfan et al. (2008) as follows:

$$\omega_L = 0.7 \sqrt{\left(\frac{H^4}{I_c} + \frac{L^4}{I_b}\right) \frac{t}{4L}} \leq 2.5 \quad (22)$$

where I_b = moment of inertia of the top beam. For all cases, the material properties of the specimen tested by Driver et al. (1997), with little simplification, are input into the analysis model, as shown in Table 2.

Analysis Results

A pushover analysis is carried out for each model. Gravity loads, which are 200, 150, and 100 kN for single-story, three-story, and five-story models, respectively, are first applied to each column at every story level, followed by the lateral load applied to the beam at the top story only. Plastic hinges are arranged as the rule used in the pushover analyses of the test specimens, and each model has fixed column bases. Because the strip model has been verified

as an accurate analytical tool, a pushover analysis using the strip model is also carried out for each model and the results are used to examine the three-strip model.

Fig. 8(a) shows the pushover curves for Group 1-A. In all cases, the initial stiffness predicted by the three-strip model is slightly higher than those from the strip model, and the ultimate strengths are in good agreement with one another (Table 1). For column flexibility parameter of 1.73 and 2.48, the initial stiffnesses obtained from the two simplified models are approximately equal. For column flexibility parameter of 3.10 and 3.35 (the column flexibility did not meet the limit), the initial stiffness differences are relatively bigger. The response curves for Group 1-B are shown in Fig. 8(b).

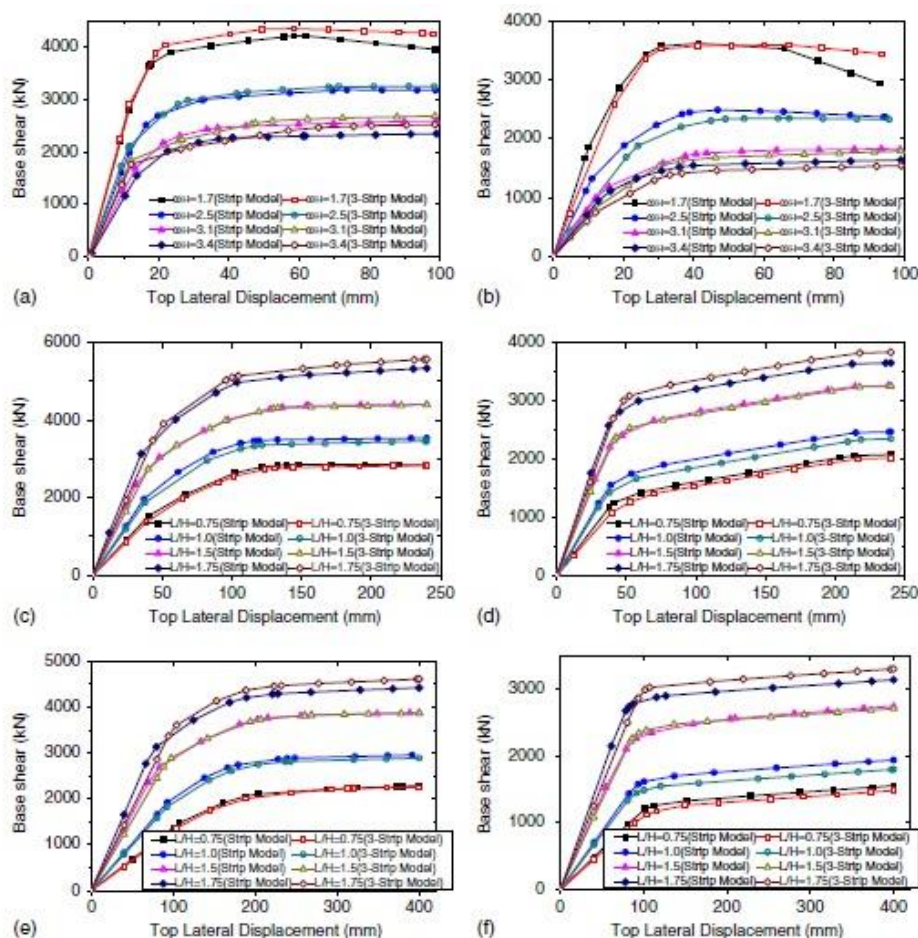


Fig. 8. Response curves for one-story, three-story, and five-story models: (a) Group 1-A; (b) Group 1-B; (c) Group 3-A; (d) Group 3-b; (e) Group 5-A; (f) Group 5-B

In all cases, the three-strip model provides slightly lower estimates of the ultimate strength and initial stiffness than the strip model. For the column flexibility parameter of 0.75, the declining rate of the strip model curve is bigger than that of the three-strip model curve. The response curves for Group 3-A are shown in Fig. 8(c). For each aspect ratio, the predicted ultimate strengths of the two simplified models are nearly identical, and the initial stiffnesses from the three-strip model are slightly lower than those from the strip model. The response curves for Group 3-B are shown in Fig. 8(d). In all cases, both the initial stiffness and ultimate strength predicted by the two simplified models are nearly identical. The curves for Group 5-

A and Group 5-B manifest similar results to those of Group 3-A and Group 3-B, respectively, as illustrated in Figs. 8(e and f).

Table 1 shows a comparison of the predicted ultimate strength and initial stiffness for all parametric study models. In all cases, the differences in ultimate strength predicted by the two simplified models are within 7.5%. For single-story models (Groups 1-A and B), the differences in initial stiffness vary from 2.5 to 28.6%, which are relatively bigger than the three-story and five-story models. One reason is that the vertical component of the strip force F_1 ($F_1 \cos \theta$) that was neglected in Eq. (11) has an influence on the lateral stiffness of the boundary column. However, as $F_1 \cos \theta$ takes quite a small proportion of total column axial force in multistory models, the impact is limited. In terms of three-story and five-story models (Groups 3-A and 3-B, Groups 5-A and 5-B), the differences in initial stiffness are all within 10%, with half of the cases being within 5%, and differences in ultimate strength are all within 8%, with seven out of eight cases being within 5%. It is found that the ultimate strengths provided by the three-strip model are sometimes larger and sometimes smaller than those from the strip model. This is mainly because the shear strength was not included in the equivalence factors and the ultimate strength ratio of the two models [Eq. (19)] will fluctuate around 1.0 as the SPSWs change. Nevertheless, the differences in ultimate strength are small and acceptable. The parametric study reveals that varying the aspect ratio and boundary column flexibility within a reasonable range does not affect the accuracy of the predicted results significantly, and the three-strip model and strip model can predict nearly identical inelastic pushover behavior for multistory SPSWs.

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