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Free vibration analysis of lateral carrying load systems with opening by a new algorithm

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Abstract

In this study, the free vibration analysis of lateral carrying load systems with openings (coupled shear walls) is presented by the use of discrete singular convolution (DSC). Discrete singular convolution is a relatively new numerical method which is very powerful in vibration analyses. Much research on how to apply complex boundary conditions in the analysis is carried out using this method and various solutions have been proposed. Applying the boundary conditions in the governing equations of lateral carrying load systems is a challenging issue. This paper proposes a new algorithm for applying the boundary conditions in the vibration analysis of coupled shear walls using the DSC method. To evaluate the proposed approach, several samples were analyzed. In order to verify the feasibility of the proposed method, the obtained results were compared with the values of the differential quadrature method (DQ) and finite element (FE). Strict conformity of the results of the proposed method shows the effectiveness and capability of the numerical solution in vibration analysis of coupled shear walls with opening.

Keywords: Discrete singular convolution method, vibration analysis, lateral carrying load systems, boundary conditions

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Introduction

One of the systems that are used for resisting buildings against lateral loads are shear walls. Shear walls, which have openings and vertical rows of windows and doors, due to architectural concerns, are called coupled shear walls. Due to the frequent usage of these structural elements in the building industry, a careful analysis becomes more important. The vibration analysis of coupled shear walls have been studied by a great number of researchers (Rosman 1964, Smith 1970, Cheung et al. 1977, Basu et al. 1979, Chaallal 1992, Kwan 1993, 1995, Li and Choo 1996, Chaallal and Ghmallar 1996, Chaallal et al. 1996, Kuang and Chau 1998, Ha and Tan 1999, Rashed 2000, Kim and Lee 2003, Aksogan et al. 2003, Zeidabadi et al. 2004, Wang and Wang 2005, Aksogan et al. 2007, Bozdogan et al. 2009, Resatoglu et al. 2010, Takabatake 2010, Bozdogan 2012). In summary, the methods used in previous studies can be divided into three categories: the laboratory, analytical and numerical methods. Because the laboratory methods are expensive and the analytical approaches are not general solutions, so researchers have been intended to utilize the numerical methods. There are some well-known numerical approaches such as Finite Difference Method (FDM), Differential Quadrature Method (DQM) and Finite Element Method (FEM) used to solve the problem. The solution convergence of the FDM is very slow as well as FEM and DQM suffer the numerical instability when calculating the high frequencies are considered. Although for the design objective of coupled shear walls, low structural frequencies are required, but if one plans a special research by focus on high vibration frequencies, these numerical methods cannot do a lot of help. In this paper, we utilize the discrete singular convolution (DSC) approach for the vibration analysis of the coupled shear walls. The DSC method introduced by Wei in 1999 (Wei 1999). This approach has exhibited its ability and efficiency in vibration analyses, especially in the calculation of high frequencies (Wei et al 2002a). The first structural analysis using the DSC is performed on vibration analysis of the beams by Wei, which is limited to beams with simple and clamped boundary conditions (Wei 2001b). In 2005, Zhao et al formulated the DSC for solving vibration analysis of beams with the free edge boundary condition by use of iteratively matched boundary method (Zhao et al 2005). The proposed approach was a relatively appropriate method for analysis of the Euler- Bernoulli cantilevered beam. Then in 2010, Xinwei et al presented a more efficient method for implementation of the free edge boundary condition in analyzing the beams and plates which has less computation and provides more accurate results (Xinwei and Suming 2010). They developed the proposed algorithm for analyzing the Timoshenko beam with free edge which has more complex boundary conditions (Suming and Xinwei 2011). The free edge boundary conditions of coupled shear walls have never been taken into consideration before. This study is implemented to present a convenient solution for applying the complex boundary conditions of the free edge of the coupled shear walls in the DSC algorithm. So, the DSC method, governing equations and boundary conditions of the coupled shear walls will be introduced, firstly. Then, the governing equations will be discretized and applying of the boundary conditions will be presented. Evaluation and efficiency of the proposed method will be assessed by solving some case studies.

Discrete singular convolution

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics which was originally introduced by Wei (1999). Since then, the DSC method applied to various science and engineering problems. Accurate results and exact convergence have demonstrated that the DSC is a reliable and convenient numerical approach. The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis. Like some other numerical methods, the DSC method discretizes the spatial derivatives and, therefore, reduces the given partial differential equations into a system of linear algebraic equations. So, in the DSC algorithm, any function f(x) and its derivatives with respect to a coordinate at a grid point x are approximated by a linear sum of the functional values in the narrow domain [$x-x_M$, $x+x_M$] in that coordinate direction. This expression can be written as follows (Wei 1999):

$$f^{(n)}(x) \approx \sum_{i=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x-x_i)\, f(x_i) \qquad (1)$$

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where superscript n (n = 0, 1, 2...) denotes the nth-order derivative with respect to x. The 2M +1 is the computational bandwidth which is usually smaller than the whole computational domain. Therefore, the resulting approximation matrix has a banded structure, which makes the DSC method more efficient than normal global methods and is particularly valuable with respect to large scale computations. {xi} is an appropriate set of discrete points on which the DSC of Eq. (1) is well defined and δ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels (Wei et al. 2002 b). However, it was shown (Wei 2001a; Wei et al. 2002a; Wei 2000; Wei 2001c) that for many problems, the use of the regularized Shannon kernel (RSK) is very efficient. The RSK is given by (Wei 1999):

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin\left(\frac{\pi}{\Delta}(x-x_k)\right)}{\frac{\pi}{\Delta}(x-x_k)}\exp\left(-\frac{(x-x_k)^2}{2\sigma^2}\right) \tag{2}$$

In these equations, $\Delta = L/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., σ = r.Δ, here r is a parameter chosen in computations.
The expression of Eq. (1) provides the highest computational efficiency both on and off a grid. In fact, it provides exact results when the sampling points are extended to an infinite series. A mathematical estimation for the choice of M, r and Δ is available. For example, if the L_2 error for approximating an L2 function f(x) is set to 10η, the following relations are to be satisfied (Qian and Wei 1999).

$$r(\pi - B\Delta) > \sqrt{\ln 10 \times \eta} \quad and \quad \frac{M}{r} > \sqrt{\ln 10 \times \eta} \tag{3}$$

where B is the frequency bounded for the function of interest, f(x).
As the DSC kernel is symmetric, the DSC computation requires a total of M fictitious grid points (FPs) outside each edge. Furthermore, the solution carries out for the grids inside the domain, so FPs must be eliminated. More precisely, it requires function values on these FPs which could be determined from those inside the domain by applying the boundary condition equations. Some attempts have been carried out for applying boundary conditions by researchers. Wei et al. (2001), Wei et al. (2002), Zhao and Wei (2002), proposed a practical method to incorporate the boundary conditions. After that, Zhao et al. (2005) applied the iteratively matched boundary method to impose the free boundary conditions for Euler beams. More recently, Wang and Xu (2010) present a method for applying boundary conditions using the Taylor's series expansion. For gaining more details about the DSC method, interested readers may refer to the works of Wei et al. (2002) a, Wei et al. (2002 b); Wei (2001 b), Xiang et al. (2002), Wei (2000), Wei (2001c), Civalek (2008) and Shokrollahi and Zayeri (2014).

Governing equations and boundary conditions

The behavior of coupled shear walls is not similar to Euler - Bernoulli or Timoshenko beams. The behavior of these members is a combination of both. A beam which has this behavior is called sandwich beam. Fig.1 shows the coupled shear wall and the equivalent sandwich beam.

The governing equation of the vibrations of coupled shear wall is as follows:

$$EI\frac{\partial^4\Psi(z,t)}{\partial z^4}-K_s\frac{\partial^2\Psi(z,t)}{\partial z^2}+K_s\frac{\partial^2\Phi(z,t)}{\partial z^2}+\gamma\frac{\partial^2\Psi(z,t)}{\partial t^2}=0 \tag{4}$$

In the Eq. (4), γ, Ψ and $\frac{\partial\Phi(z,t)}{\partial z}$ are mass per unit length of the material, the total shape function and the function of circular frequency ω of the coupled shear wall, respectively.

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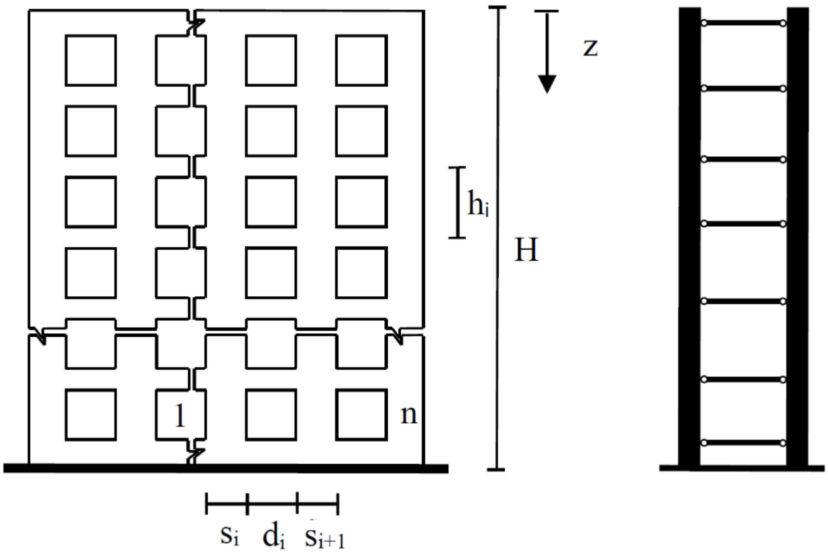

Fig.1 coupled shear wall and the equivalent sandwich beam

Using the separation of variables technique, the Ψ and Φ could be written as the functions of circular frequency of the coupled shear wall:

$$\Psi(z,t) = \overline{\Psi}(z)\sin(\omega t) \tag{5}$$

$$\Phi(z,t) = \overline{\Phi}(z)\sin(\omega t) \tag{6}$$

Substituting the Eqs. (5)-(6) in the governing Eq. (4) will get to:

$$EI\frac{d^4\overline{\Psi}}{dz^4} - K_s\frac{d^2\overline{\Psi}}{dz^2} + K_s\frac{d^2\overline{\Phi}}{dz^2} + \gamma\omega^2\overline{\Psi} = 0 \tag{7}$$

The shear force equation of the equivalent sandwich beam is written as follows:

$$D\frac{d^3\overline{\Phi}}{dz^3} + K_s\frac{d\overline{\Psi}}{dz} - K_s\frac{d\overline{\Phi}}{dz} = 0 \tag{8}$$

In the above equations, EI is the total bending rigidity of the shear walls and D represents the global bending rigidity of the coupled shear wall that can be calculated as:

$$D = \sum_{j=1}^{n} EA_j r_j^{\,2} \tag{9}$$

where A_j is the cross sectional area of the jth shear wall, n is the number of walls and r_j is the distance of the jth shear wall from the center of the cross sections.
K_s is the equivalent shear rigidity of coupled shear walls and can be calculated as:

$$K_s = \frac{1}{\frac{1}{R_c} + \frac{1}{R_b}} \tag{10}$$

For coupled shear walls which consists of n walls and n−1 connecting beams, R_c and R_b can be calculated from Eqs. (11)-(12):

$$R_c = \sum_{j=0}^{n}\frac{12EI_{cj}}{h_i^2} \tag{11}$$

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$$R_b = \sum_{j=1}^{n-1} \frac{6EI_{bj}\left[(d_j+s_j)^2+(d_j+s_{j+1})^2\right]}{d_j^{\,3} h_i\left(1+\dfrac{12kEI_{bj}}{GA_{bj}d_j^{\,2}}\right)} \tag{12}$$

EI_{bj} and GA_{bj} represent the flexural rigidity of the connecting beam and the shear rigidity of connecting them, respectively and K is the constant that depends on the shape of the cross-section of the beams which is 1.2 for rectangular cross sections.

For generality, the Eqs. (7)-(8) could be written in non- dimensional form. The non-dimensional forms of the governing equations are as follows:

$$\frac{d^4\overline{\Psi}}{d\xi^4} - K^2\frac{d^2\overline{\Psi}}{d\xi^2} + K^2\frac{d^2\overline{\Phi}}{d\xi^2} + m\overline{\Psi} = 0 \tag{13}$$

$$\frac{d^3\overline{\Phi}}{d\xi^3} + s^2\frac{d\overline{\Psi}}{d\xi} - s^2\frac{d\overline{\Phi}}{d\xi} = 0 \tag{14}$$

The parameters ξ , K , s , m are defined as:

$$\xi = \frac{z}{H} \tag{15-a}$$

$$K = H\sqrt{\frac{k_s}{EI}} \tag{15-b}$$

$$s = H\sqrt{\frac{k_s}{D}} \tag{15-c}$$

$$m = \frac{\gamma}{EI}H^4\omega^2 \tag{15-d}$$

The boundary conditions of clamped support are presented in Eqs. (16).

$$\overline{\Psi} = 0 \tag{16-a}$$

$$\frac{d\overline{\Psi}}{d\xi} = 0 \tag{16-b}$$

$$\frac{d\overline{\Phi}}{d\xi} = 0 \tag{16-c}$$

And free edge boundary conditions are presented in Eqs. (17).

$$\frac{d^2\overline{\Psi}}{d\xi^2} = 0 \tag{17-a}$$

$$\frac{d^2\overline{\Phi}}{d\xi^2} = 0 \tag{17-b}$$

$$\frac{d^3\overline{\Psi}}{d\xi^3} - K^2\left(\frac{d\overline{\Psi}}{d\xi} - \frac{d\overline{\Phi}}{d\xi}\right) = 0 \tag{17-c}$$

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DSC formulation

For the DSC discretizing of the governing equations using Eq. (1), the computational domain must be meshed. Since the DSC formulation is required values of $2m$ points adjacent to the solution point, the m mesh points must be continued outside the boundaries. So, if there are n mesh points inside the media, we have $n+2m$ total number of mesh points. Here we assume that the computational domain $0 \le \xi \le 1$ is divided by n-1 equal intervals with coordinates of grid points as, $\xi_0 < \ldots < \xi_{n-1}$, with a total of 2m fictitious grid points, $\xi_{-m} < \ldots < \xi_{-1}$ and $\xi_n < \ldots < \xi_{n+m-1}$, out of the domain.

Based on the defined mesh nodes of the computational domain, the elements of $\overline{\Psi}$ and $\overline{\Phi}$ vectors are:

$$\{\overline{\Psi}\} = \begin{bmatrix} \overline{\Psi}(\xi_{-m}) \\ \vdots \\ \overline{\Psi}(\xi_0) \\ \vdots \\ \overline{\Psi}(\xi_{n-1}) \\ \vdots \\ \overline{\Psi}(\xi_{n-1+m}) \end{bmatrix} \quad , \quad \{\overline{\Phi}\} = \begin{bmatrix} \overline{\Phi}(\xi_{-m}) \\ \vdots \\ \overline{\Phi}(\xi_0) \\ \vdots \\ \overline{\Phi}(\xi_{n-1}) \\ \vdots \\ \overline{\Phi}(\xi_{n-1+m}) \end{bmatrix} \tag{18}$$

DSC coefficient matrixes

The Eq. (1) can be rewritten for the node with coordinate ξ_i as:

$$\left.\frac{d^{(n)} f(\xi)}{d\xi^{(n)}}\right|_{\xi=\xi_i} \approx \sum_{k=-m}^{m} \delta_{\Delta,\sigma}^{(n)}(\xi_i - \xi_{i+k}) f(\xi_{i+k}) = \sum_{k=-m}^{m} \lambda_{i,k}^{(n)} f(\xi_{i+k}) \tag{19}$$

where $\lambda_{i,k}^{(n)}$ are weighting coefficients of the nth derivative of the f function at the node with coordinate ξi. From the Eq. (19), there are 2m+1 weighting coefficients for estimating the nth derivative of the f function at the node with coordinate ξ_i. These coefficients can be shown by the vector $\left[\lambda_{i,-m}^{(n)}, \ldots, \lambda_{i,0}^{(n)}, \ldots, \lambda_{i,m}^{(n)}\right]$. The distances between the mesh points are equal, so weighting coefficients of the all nodes are similar. On the other word the subscript i can be eliminated from the $\lambda_{i,k}^{(n)}$:

$$\lambda_{i,k}^{(n)} = \lambda_k^{(n)} \qquad k = -m, \ldots, 0, \ldots, m \qquad \forall i \in \{0,1,2,\ldots,n-1\} \tag{20}$$

So the Eq. (19) will be rewritten as bellow:

$$f^{(n)}(\xi_i) = \sum_{k=-m}^{m} \lambda_k^{(n)} f(\xi_{i+k}) \tag{21}$$

The DSC coefficient matrix $\left[\Gamma^{(n)}\right]$ will be defined for discretizing of the nth derivative of the function f(ξ) all over the domain nodes:

$$\left[\Gamma^{(n)}\right]_{i,j} = \begin{cases} \lambda_{j-i-m}^{(n)} & 0 \le j-i \le 2m \\ 0 & otherwise \end{cases} \quad , \quad i = 1,2,\ldots,n \quad , \quad j = 1,2,\ldots n+2m \tag{22}$$

So, the derivative statement of the Eqs. (13)-(14) can be written discretely at the mesh points as follows:

$$\frac{d^4\overline{\Psi}(\xi_i)}{d\xi^4} = \left[\Gamma^{(4)}\right]\{\overline{\Psi}\} \tag{23}$$

$$\frac{d^2\overline{\Psi}(\xi_i)}{d\xi^2} = \left[\Gamma^{(2)}\right]\{\overline{\Psi}\} \tag{24}$$

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$$\frac{d\overline{\Psi}(\xi_i)}{d\xi}=\left[\Gamma^{(1)}\right]\left\{\overline{\Psi}\right\} \tag{25}$$

$$\frac{d^3\overline{\Phi}(\xi_i)}{d\xi^3}=\left[\Gamma^{(3)}\right]\left\{\overline{\Phi}\right\} \tag{26}$$

$$\frac{d^2\overline{\Phi}(\xi_i)}{d\xi^2}=\left[\Gamma^{(2)}\right]\left\{\overline{\Phi}\right\} \tag{27}$$

$$\frac{d\overline{\Phi}(\xi_i)}{d\xi}=\left[\Gamma^{(1)}\right]\left\{\overline{\Phi}\right\} \tag{28}$$

For clarifying, the extended form of the Eq. (23) is presented by Eq. (29):

$$\frac{d^4\overline{\Psi}(\xi_i)}{d\xi^4}=\left[\Gamma^{(4)}\right]\left\{\overline{\Psi}\right\}=\begin{bmatrix}\lambda_{-m}^{(4)} & \cdots & \lambda_m^{(4)} & & & \\ & \lambda_{-m}^{(4)} & \cdots & \lambda_m^{(4)} & & \\ & & \ddots & & \ddots & \\ & & & \lambda_{-m}^{(4)} & \cdots & \lambda_m^{(4)}\end{bmatrix}_{(n)(2m+n)} \times \begin{bmatrix}\overline{\Psi}(\xi_{-m}) \\ \vdots \\ \overline{\Psi}(\xi_0) \\ \vdots \\ \overline{\Psi}(\xi_{n-1}) \\ \vdots \\ \overline{\Psi}(\xi_{n-1+m})\end{bmatrix}_{(2m+n)(1)} \tag{29}$$

Applying of the boundary conditions

Since the governing equations only apply to n mesh points inside the solution domain, we have n equations and $n+2m$ unknowns. The $\overline{\Psi}$ and $\overline{\Phi}$ values at $2m$ fictitious grid points, must be determined as functions of inside point values. This will be implemented by applying the boundary condition equations. For connecting the values of outside points to inside ones, the Taylor expansion is used which is presented in Eq. (30):

$$\mathrm{T}(-\xi)=\mathrm{T}(\xi)+\mathrm{T}'(0)(-2\xi)+\mathrm{T}'''(0)(-\frac{\xi^3}{3}) \tag{30}$$

where T may be taken as each of $\overline{\Psi}$ and $\overline{\Phi}$.

Each one of the boundary conditions can be applied as follows:

a) Free edge boundary condition

Substituting the Eq. (21) into the Eq. (17-a) will produce the Eq. (31):

$$\frac{d^2\overline{\Psi}(0)}{d\xi^2}=\overline{\Psi}''(0)=\sum_{k=-m}^{k=m}\lambda_k^{(2)}\overline{\Psi}(\xi_k)=0 \tag{31}$$

where $\lambda_k^{(2)}$ is the weighting coefficients of the free edge boundary node (ξ=0) that obtain by the second order derivatives of the RSK with respect to ξ:

$$\lambda_k^{(2)}=\delta_{\Delta,\sigma}^2(0-\xi_k) \tag{32}$$

Since $\lambda_{-k}^{(2)}=\lambda_k^{(2)}$, substituting Eq. (30) in Eq. (31) lead to:

$$\overline{\Psi}''(0)=\sum_{k=1}^{k=m}\lambda_k^{(2)}\left[2\overline{\Psi}(\xi_k)+\overline{\Psi}'(0)(-2\xi_k)+\overline{\Psi}'''(0)(\frac{-\xi_k^{\;3}}{3})\right]+\lambda_0^{(2)}\overline{\Psi}(0)=0 \tag{33}$$

The Eq. (17-c) can be written as:

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$$\overline{\Psi}'''(0) = K^2\overline{\Psi}'(0) - K^2\overline{\Phi}'(0) \tag{34}$$

After substituting Eq. (34) in Eq. (33) we have:

$$\overline{\Psi}'(0) = \frac{\lambda_0^{(2)}\overline{\Psi}(0) + \sum_{k=1}^{k=m} 2\lambda_k^{(2)}\overline{\Psi}(\xi_k) + K^2\overline{\Phi}'(0)(BN)}{BH} \tag{35}$$

where

$$BN = \sum_{k=1}^{k=m} \lambda_k^{(2)} \frac{\xi_k^{\,3}}{3} \tag{36}$$

$$BH = \sum_{k=1}^{k=m} \lambda_k^{(2)} (2\xi_k + K^2 \frac{\xi_k^{\,3}}{3}) \tag{37}$$

Substituting the Eq. (21) in the Eq. (17-b) will produce the Eq. (38):

$$\frac{d^2\overline{\Phi}(0)}{d\xi^2} = \overline{\Phi}''(0) = \sum_{k=-m}^{k=m} \lambda_k^{(2)}\overline{\Phi}(\xi_k) = 0 \tag{38}$$

By applying the Taylor expansion with only first order derivative for circular frequency (Eq. 39) the Eq. (40) will be produced:

$$\overline{\Phi}(-\xi) = \overline{\Phi}(\xi) + \overline{\Phi}'(0)(-2\xi) \tag{39}$$

$$\overline{\Phi}'(0) = \frac{\lambda_0^{(2)}\overline{\Phi}(0) + \sum_{k=1}^{k=m} 2\lambda_k^{(2)}\overline{\Phi}(\xi_k)}{BM} \tag{40}$$

where

$$BM = \sum_{k=1}^{k=m} \lambda_k^{(2)} (2\xi_k) \tag{41}$$

So by use of Eqs. (34)-(35) and (40) the third order derivative of the deflection function in free edge can be written as the form of the Eq. (42):

$$\overline{\Psi}'''(0) = \frac{K^2\left(\lambda_0^{(2)}\overline{\Psi}(0) + \sum_{k=1}^{k=m} 2\lambda_k^{(2)}\overline{\Psi}(\xi_k)\right) + K^2\overline{\Phi}'(0)(BM)}{BH} \tag{42}$$

So by use of Eqs. (35) and (40)-(42) in Eq. (30) the values of all points outside the solution domain can be determined as functions of values of inside points.

b) Clamped boundary condition

The Eq. (16-b) can be easily applied by the symmetric expansion of the shape function values of the outside points, $\overline{\Psi}$, in term of the inside points. For applying the Eq. (16-c), Symmetric expansion of the circular frequency values of outside points, $\overline{\Phi}$, in term of the inside points can be utilized. Applying the Eq. (16-a) will occur some difficulty. Because by applying this equation, number of the unknown circular frequencies will be one less than the deflections. The deflection of the coupled shear wall at this boundary equals to zero, although the circular frequency is an unknown value at this boundary. For overcoming this challenging issue, the Eqs. 16(a)-(c) must be applied in the Eq. (14). So the Eq. (14) will get the bellow equation:

$$\frac{d^3\overline{\Phi}(1)}{d\xi^3} = 0 \tag{43}$$

By applying the Eq. (21) in Eq. (43), the following equation will be produced:

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$$\frac{d^3\overline{\Phi}(1)}{d\xi^3} = \overline{\Phi}'''(1) = \sum_{k=-m}^{k=m} \lambda_k^{(3)} \overline{\Phi}(\xi_{n+k-1}) = 0 \tag{44}$$

where $\lambda_k^{(3)}$ is the weighting coefficients of the clamped boundary node ($\xi = \xi_{n-1} = 1$) that obtain by the third order derivatives of the RSK with respect to ξ:

$$\lambda_k^{(3)} = \delta_{\Delta,\sigma}^3 (\xi - \xi_{n+k-1})\Big|_{\xi=\xi_{n-1}} \tag{45}$$

Since $\lambda_{-k}^{(3)} = -\lambda_k^{(3)}$ and symmetric expansion of outside and inside points is applicable about $\overline{\Phi}$ the Eq. (44) will due to Eq. (46):

$$\overline{\Phi}(1) = 0 \tag{46}$$

Discretizing of the governing equations

After applying the boundary conditions in the DSC coefficient matrixes $\left[\Gamma^{(n)}\right]$, the derivatives terms of the function values $\overline{\Psi}$ and $\overline{\Phi}$ at the n inner grid points of the Eqs. (13)-(14) can be written as follows:

$$\frac{d^4\overline{\Psi}(\xi_i)}{d\xi^4} = \left[\overline{\Gamma}_{\overline{\Psi}}^{(4)}\right]\left\{\overline{\Psi}\right\} \tag{47}$$

$$\frac{d^2\overline{\Psi}(\xi_i)}{d\xi^2} = \left[\overline{\Gamma}_{\overline{\Psi}}^{(2)}\right]\left\{\overline{\Psi}\right\} \tag{48}$$

$$\frac{d\overline{\Psi}(\xi_i)}{d\xi} = \left[\overline{\Gamma}_{\overline{\Psi}}^{(1)}\right]\left\{\overline{\Psi}\right\} \tag{49}$$

$$\frac{d^3\overline{\Phi}(\xi_i)}{d\xi^3} = \left[\overline{\Gamma}_{\overline{\Phi}}^{(3)}\right]\left\{\overline{\Phi}\right\} \tag{50}$$

$$\frac{d^2\overline{\Phi}(\xi_i)}{d\xi^2} = \left[\overline{\Gamma}_{\overline{\Phi}}^{(2)}\right]\left\{\overline{\Phi}\right\} \tag{51}$$

$$\frac{d\overline{\Phi}(\xi_i)}{d\xi} = \left[\overline{\Gamma}_{\overline{\Phi}}^{(1)}\right]\left\{\overline{\Phi}\right\} \tag{52}$$

where $\left[\overline{\Gamma}_{\overline{\Psi}}^{(n)}\right]$ and $\left[\overline{\Gamma}_{\overline{\Phi}}^{(n)}\right]$ are DSC coefficient matrixes which boundary conditions are applied to them. For example the extended form Eq. (47) can be shown by:

$$\frac{d^4\overline{\Psi}(\xi_i)}{d\xi^4} = \left[\overline{\Gamma}_{\overline{\Psi}}^{(4)}\right]\left\{\overline{\Psi}\right\} = \begin{bmatrix} \overline{\Gamma}_{\overline{\Psi},11}^{(4)} & \overline{\Gamma}_{\overline{\Psi},12}^{(4)} & \cdots & \overline{\Gamma}_{\overline{\Psi},(1)(n-1)}^{(4)} \\ \overline{\Gamma}_{\overline{\Psi},21}^{(4)} & \overline{\Gamma}_{\overline{\Psi},22}^{(4)} & \cdots & \overline{\Gamma}_{\overline{\Psi},(2)(n-1)}^{(4)} \\ \vdots & \vdots & \ddots & \vdots \\ \overline{\Gamma}_{\overline{\Psi},(n-1)(1)}^{(4)} & \overline{\Gamma}_{\overline{\Psi},(n-1)(2)}^{(4)} & \cdots & \overline{\Gamma}_{\overline{\Psi},(n-1)(n-1)}^{(4)} \end{bmatrix}_{(n-1)(n-1)} \times \begin{Bmatrix} \overline{\Psi}(\xi_0) \\ \vdots \\ \overline{\Psi}(\xi_{n-2}) \end{Bmatrix}_{(n-1)(1)} \tag{53}$$

Using Eqs. (47)-(52), the governing Eqs. (13)-(14) can be discretized in final form as below:

$$\left(\left[\overline{\Gamma}_{\overline{\Psi}}^{(4)}\right] - K^2\left[\overline{\Gamma}_{\overline{\Psi}}^{(2)}\right]\right)\left\{\overline{\Psi}\right\} + K^2\left[\overline{\Gamma}_{\overline{\Phi}}^{(2)}\right]\left\{\overline{\Phi}\right\} + m[I]\left\{\overline{\Psi}\right\} = 0 \tag{54}$$

$$\left(\left[\overline{\Gamma}_{\overline{\Phi}}^{(3)}\right] - s^2\left[\overline{\Gamma}_{\overline{\Phi}}^{(1)}\right]\right)\left\{\overline{\Phi}\right\} + s^2\left[\overline{\Gamma}_{\overline{\Psi}}^{(1)}\right]\left\{\overline{\Psi}\right\} = 0 \tag{55}$$

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In Eq. (54), [I] is the unique matrix.
For simplicity, the Eqs. (54)-(55) can be rewritten as below:

$$[A]\{\overline{\Psi}\}+[B]\{\overline{\Phi}\}+m[I]\{\overline{\Psi}\}=0 \quad (56)$$

$$[C]\{\overline{\Phi}\}+[D]\{\overline{\Psi}\}=0 \quad (57)$$

where

$$[A]=\left[\overline{\Gamma}_{\Psi}^{(4)}\right]-K^2\left[\overline{\Gamma}_{\Psi}^{(2)}\right] \quad (58)$$

$$[B]=K^2\left[\overline{\Gamma}_{\Phi}^{(2)}\right] \quad (59)$$

$$[C]=\left[\overline{\Gamma}_{\Phi}^{(3)}\right]-s^2\left[\overline{\Gamma}_{\Phi}^{(1)}\right] \quad (60)$$

$$[D]=s^2\left[\overline{\Gamma}_{\Psi}^{(1)}\right] \quad (61)$$

Then Eq. (57) can be rewritten as follows:

$$\{\overline{\Phi}\}=-[C]^{-1}[D]\{\overline{\Psi}\} \quad (62)$$

Substituting Eq. (62) in Eq. (56) will produce the Eq. (63) which its eigenvalues are the non- dimensional natural frequencies of the coupled shear wall.

$$\left([A]-[B][C]^{-1}[D]\right)\{\overline{\Psi}\}+m[I]\{\overline{\Psi}\}=0 \quad (63)$$

Numerical results

In order to investigate the efficiency of the DSC algorithm in vibration analysis of the coupled shear walls and verification of the proposed method in applying the free edge boundary conditions, several case studies have been studied. Shear walls number one and two are 56 and 36 meters high one bay models, respectively. Structural and geometric properties of these models are presented in Table 1.
Non- dimensional natural frequencies of the coupled shear walls obtained from DSC and finite element (FE) are tabulated in Table 2. As it is seen, DSC results are in good agreement with those of the FE method.

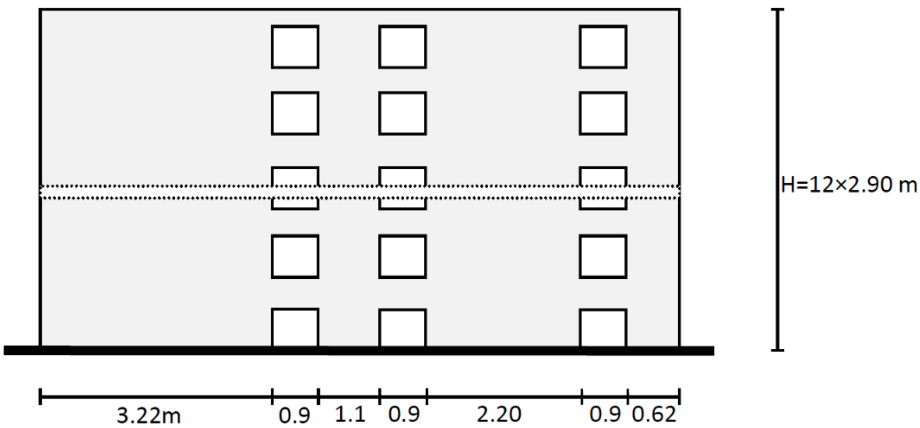

Fig.2 Coupled shear wall with 3 bays and 12 stories

Table 1. mechanical and geometric properties of the coupled shear walls

	Model (1)	Model (2)
Young Modulus	3.6×1010 N/m^2	2.1×1010 N/m^2
Poisson Ratio	0.15	0.15
Mass Density	2.4×103 kg/m^3	2.4×103 kg/m^3
Total storeys	20	12
Total Height	56 m	36 m

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Thickness of Left Side Wall	0.3 m	0.5 m
Width of Left Side Wall	5.0 m	7.0 m
Thickness of Right Side Wall	0.3 m	0.5 m
Width of Right Side Wall	7.0 m	6.0 m
Thickness of Lintel	0.3 m	0.5 m
Height of Lintel	0.4 m	0.45 m
Length of Lintel	2.0 m	1.8 m

Table 2. Non- dimensional frequencies of coupled shear walls

Mode	Model (1)		Model (2)	
	DSC	FE (Takabatake 2010)	DSC	FE (Takabatake 2010)
1	12.90	13.09	19.76	19.97
2	58.03	55.55	84.27	82.15
3	133.42	129.00	195.24	191.00
4	241.12	224.90	327.56	313.64

Another model which is studied in this paper is a coupled shear wall with 3bays and 12 stories which is shown in fig.2. The mechanical and geometric properties of the model are presented in Table 3.
Table 4 represents the non- dimensional natural frequencies of the coupled shear wall obtained from DSC, FE and differential quadrature (DQ) methods. As it is observed the DSC results are again in good agreement with those of two other numerical approaches.

Table 3. mechanical and geometric properties of the coupled shear wall3 bays and 12 stories

Young Modulus	20×1010 N/m^2
Poisson Ratio	0.15
Mass Density	2.4×103 kg/m^3
Total storeys	12
Total Height	38.4 m
Thickness of Wall	0.16 m
Height of Lintel	0.31 m

Table 4. Non- dimensional frequencies of coupled shear wall with 3 bays and 12 stories

Mode	DSC	DQM (Bozdogan 2011)	FE(Aksogan et al 2007)
1	3.036	3.070	3.030
2	12.091	12.288	12.004
3	25.974	26.536	25.969
4	42.863	42.739	42.884

Conclusions

In this paper, the free vibration analysis of the coupled shear walls using the DSC method has been studied. Both clamped and free edge boundary conditions of this structural element have some complexities to apply in DSC algorithm. To overcome the challenging issue, Taylor expansion has been employed. So, fictitious points outside the solution domain can be defined as functions of the inside points. To verify the solution, the non- dimensional natural frequencies of some case studies obtained from DSC have been compared with those of FE and DQ methods. Good agreement between DSC results and two other numerical approaches emphasizes the accuracy and efficiency of the proposed method.

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