Comparison study of discrete singular convolution with three other traditional numerical methods for the free torsional vibration analysis of non-prismatic shafts

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Abstract

In this paper, the free torsional vibration of non-prismatic shafts is investigated. The governing equation is nonlinear, so there is no general analytical solution for the problem. Using numerical techniques is efficient in such circumstances. There is a wide range of numerical methods which each one has some abilities and benefits. So, choosing the proper numerical method is important in the solution procedure. In this paper, the discrete singular convolution (DSC) which is a relatively novel numerical approach is applied for the first time. A comprehensive comparison study is carried out between the DSC and three other traditional numerical approaches, i.e. finite element (FE), second order finite difference (FD 2nd) and differential quadrature (DQ). The convergence, ability and efficiency of the DSC are evaluated in comparison with three other methods by solving several examples. The results indicate the efficiency of each method and beneficial conclusions are also given regarding the proper method for the torsional vibration analysis of non-prismatic shafts.

Keywords: Discrete singular convolution, Numerical analysis, Shaft, Non-prismatic, Torsional vibration.

Introduction

In systems which power transition is worked out by rotation of the shafts, the axels may be subjected to torsional vibration. This phenomenon is causing shaft failure over time and must be considered as a design factor. So the torsional vibration analysis of the shafts must be implemented. If the cross section varies along the shaft, the governing equation is non-linear and it has not a general analytical solution. The existing analytical approaches could be employed for a few shafts with some specific cross sectional areas (Rafiee et al, 2010). If a problem does not have a general analytical solution; it must be solved by use of a numerical approach. There is a wide range of numerical methods such as finite difference (FD), finite element (FE), and differential quadrature (DQ) which each one is suitable in some specific fields and inappropriate in others. For instance all the mentioned methods are numerically unstable in the high frequency domain (Langley and Bardell, 1998). Paying attention to this, choosing the proper numerical method is important in the solution procedure.
More recently, a new numerical algorithm entitled as discrete singular convolution (DSC) is introduced in 1999 (Wei,1999). DSC is highly potent in vibration analyses (Wei, 2001a). Vibration of some structural members such as rods (Xinwei et al, 2010) Euler- Bernoulli beam (Wei, 2001b), Timoshenko beam (Suming 2011, Civalek and Kiracioglu 2010), plates (Zhao et al, 2002) and membranes (Lim at el, 2005) are analyzed by use of DSC. The comparison study between DQ and DSC is carried out in vibration analysis of rectangular plates by Ng et al, 2004. In the past researches, the members were prismatic and the vibration analysis of non-prismatic members is not considered. The only researches on effect of non-prismatic property of a member in the DSC accuracy are performed by the authors of the current paper, (Shokrollahi and Zayeri, 2014) and (Civalek, 2009).
The objective of this paper is to present a proper algorithm to solve the nonlinear equation of the vibration of shafts by using of the DSC and perform a comprehensive comparison study between the new algorithm and traditional FD, FE and DQ methods. The proposed algorithm is produced in the form of a computer program, at first. To validate the algorithm, the results of the program will be compared with those of exact analytical solution for a specific example. then, the results of the new algorithm will be compared with those of traditional FD, FE and DQ methods and accuracy and convergence of the four will be investigated to determine the position of the DSC between mentioned numerical approaches. The results of the letter is helpful to choose the best method for determining the frequencies of the shaft in all frequency domains.

Discrete singular convolution

Discrete singular convolution (DSC) method is a relatively new numerical technique in applied mechanics which was originally introduced by Wei, 1999. Since then, the DSC method has been applied to various science and engineering problems. Accurate results and exact convergence have demonstrated that the DSC is a reliable and convenient numerical approach. The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis. Like some other numerical methods, the DSC method discretizes the spatial derivatives and, therefore, reduces the given partial differential equations into a system of linear algebraic equations. So, in the DSC algorithm, any function $f(x)$ and its spatial derivatives at a grid point x are approximated by a linear sum of the functional values in the narrow domain $[x\text{-}x_{-M},\ x\text{+}x_{+M}]$ in that coordinate direction. This expression can be written as follows (Wei, 1999):

$$f^{(n)}(x) \approx \sum_{i=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x - x_i)\, f(x_i) \qquad (1)$$

Where superscript n (n = 0, 1, 2...) denotes the nth-order derivative with respect to x. The $2M$ +1 is the computational bandwidth which is usually smaller than the whole computational domain. Therefore, the resulting approximation matrix has a banded structure, which makes the DSC method more efficient than normal global methods and is particularly valuable with respect to large scale computations. $\{x_i\}$ is an appropriate set of discrete points on which the DSC of Eq.

(1) is well defined and δ is a singular kernel. The DSC algorithm can be realized by using many approximation kernels (Wei et al, 2002). However, it was shown that for many problems, the use of the Regularized Shannon Kernel (RSK) is very efficient (Wei 2001b, Wei et al 2002a,b, Wei 2000, Wan and Wei 2000, Wan et al 2002). The RSK is given by (Wei, 1999):

$$\delta_{\Delta,\sigma}(x-x_k)=\frac{\sin\left(\frac{\pi}{\Delta}(x-x_k)\right)}{\frac{\pi}{\Delta}(x-x_k)}\exp\left(-\frac{(x-x_k)^2}{2\sigma^2}\right) \qquad (2)$$

In these equations, $\Delta=L/(N-1)$ is the grid spacing and N is the number of grid points. The parameter σ determines the width of the Gaussian envelope and often varies in association with the grid spacing, i.e., σ = r.Δ, here r is a parameter chosen in computations.
As the DSC kernel is symmetric, the DSC computation requires a total of M fictitious grid points (FPs) outside each edge. Furthermore, the solution carries out for the grids inside the domain, so FPs must be eliminated. More precisely, it requires function values on these FPs which could be determined from those inside the domain by applying the boundary condition equations. Some attempts have been carried out for applying boundary conditions by researchers. Wei et al 2001, 2002b and Zhao and Wei, 2002 proposed a practical method to incorporate the boundary conditions. After that, Zhao et al, 2005 applied the iteratively matched boundary method to impose the free boundary conditions for Euler beams. More recently, Wang and Xu, 2010 present a method for applying boundary conditions using the Taylor's series expansion. For gaining more details about the DSC method, interested readers may refer to the works of Wei 2001b,c, Wei et al 2002b, Xiang et al 2002, Wei 2000 and Civalek 2008.

Governing equation and boundary conditions
The governing equation of free rotational vibration of shafts with varying cross sectional area along its axial direction is (Bishop and Johnson, 1960):

$$\frac{\partial}{\partial x}\left[Gj(x)\frac{\partial\theta(x,t)}{\partial x}\right]=\rho j(x)\frac{\partial^2\theta(x,t)}{\partial t^2} \qquad (3)$$

Where x and t denote the Cartesian coordinate in the longitudinal direction of the shaft and time parameter, respectively. θ is the rotational displacement, G is the shear modules of elasticity, ρ is the mass of unit volume and $j(x)$ is the rotational inertia of the area cross section which varies along the shaft.
In order to eliminate the time parameter from Eq. (3), we can make use of the separation of variables technique:

$$\theta(x,t)=\Psi(x)e^{i\omega t} \qquad (4)$$

In the above equation, ω is the torsional frequency of the shaft. Substituting Eq. (4) in the Eq. (3) gives:

$$Gj(x)\frac{d^2\Psi(x)}{dx^2}+G\frac{dj(x)}{dx}\frac{d\Psi(x)}{dx}+\rho j(x)\omega^2\Psi(x)=0 \qquad (5)$$

To achieve a general formulation, it is better to write the governing Eq. (5) in the dimensionless form:

$$GJ(X)\frac{d^2\Psi(X)}{dX^2}+G\frac{dJ(X)}{dX}\frac{d\Psi(X)}{dX}+\rho J(X)\omega^2L^2\Psi(X)=0 \qquad (6)$$

In a more brief representation, Eq. (6) can be rewritten as follows:

$$\Psi''(X)+\frac{J'(X)}{J(X)}\Psi'(X)+\Omega^2\Psi(X)=0 \tag{7}$$

The dimensionless parameters used in Eq. (7) are defined as follows:

$$X=\frac{x}{L},\quad J(X)=\frac{j(x)}{j(0)},\quad \Omega^2=\frac{\rho\omega^2L^2}{G} \tag{8}$$

Where L is the shaft length, $j(0)$ is the rotational inertia of the cross sectional area of shaft at the left support ($x=0$), and Ω is the non-dimensional torsional frequency of the shaft.
Two types of boundary conditions will be considered for the shaft in this paper:

a) Free edge

$$\frac{d\theta}{dx}=0 \tag{9}$$

b) Clamped support

$$\theta=0 \tag{10}$$

Solution procedure

The solution procedure of the torsional vibration of the shaft by use of the DSC is as follows:

1. Mesh of the shaft

N equidistant grids will be considered along the shaft body. The value of the function at each grid point is estimated based on the values of 2m nodes on both sides of it. So, to calculate the value of the function at the nodes near the boundaries, the value of m fictitious points outside of the solution domain is needed. So, there are a total number of $N+2m$ computational nodes inside and outside the domain. Fig. 1 shows the mesh of a non-prismatic shaft.

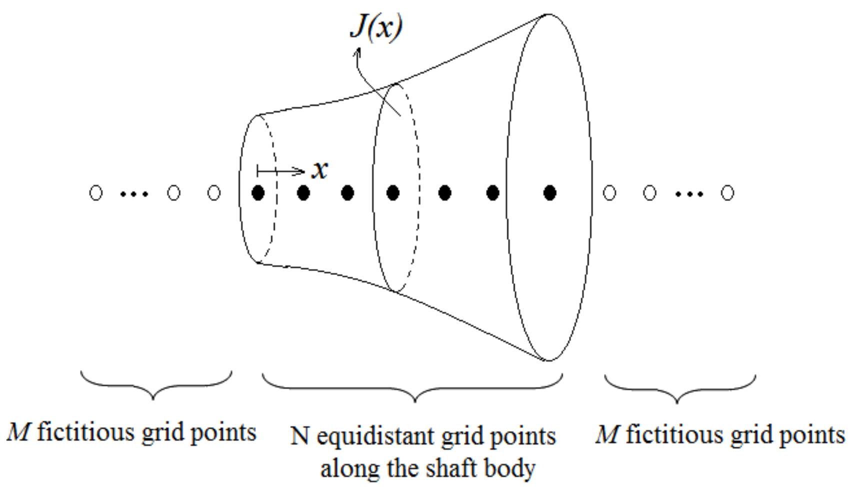


Fig 1. Geometry and grid points of a non-prismatic shaft

2. Making the weighting coefficients matrices

The terms of Eq. (7) can be written as follows by use of Eq. (1):

$$\Psi(X_i)=\sum_{k=-m}^{m}\delta_{\Delta,\sigma}(X_i-X_k)\Psi(X_{i+k})$$

$$\Psi'(X_i) = \sum_{k=-m}^{m} \delta'_{\Delta,\sigma}(X_i - X_k)\Psi(X_{i+k})$$
$$\Psi''(X_i) = \sum_{k=-m}^{m} \delta''_{\Delta,\sigma}(X_i - X_k)\Psi(X_{i+k}) \quad (11)$$

The following parameters are defined for simplification:

$$A_{ik} = \delta_{\Delta,\sigma}(X_i - X_k)$$
$$B_{ik} = \delta'_{\Delta,\sigma}(X_i - X_k)$$
$$C_{ik} = \delta''_{\Delta,\sigma}(X_i - X_k)$$
$$k = i-m,\dots, i-1, i, i+1,\dots, i+m$$
$$i = 0,1,2,\dots, N-1 \quad (12)$$

So,

$$\Psi(X_i) = \sum_{k=-m}^{m} A_{ik}\Psi(X_{i+k})$$
$$\Psi'(X_i) = \sum_{k=-m}^{m} B_{ik}\Psi(X_{i+k}),$$
$$\Psi''(X_i) = \sum_{k=-m}^{m} C_{ik}\Psi(X_{i+k}) \quad (13)$$

The A, B and C are the weighting coefficients matrices. The value of Ψ **Error! Bookmark not defined.** function and its first and second derivatives at N nodes over the shaft body is estimated using Eq. (11). So, the order of each weighting coefficient matrix is $N \times (N+2m)$.

3. Implementation of the non-prismatic coefficient matrices

In this stage, the non-prismatic coefficient matrix D must be realized. The order of this matrix is of $N \times (N+2m)$ and its ith row elements are identical which can be calculated by $J'(X_i)/J(X_i)$. The index i varies from zero to $N-1$ which denotes the node number over the shaft body. To implement the non-prismatic property of the shaft, the weighting coefficient matrix B must be rewritten in the following form:

$$\overline{B}_{ik} = D_{ik} \otimes B_{ik} \quad (14)$$

In the above equation the function $\otimes$ denotes the Hadamard product.

4. Implementation of the boundary conditions

In order to carry the boundary conditions out, the values of the fictitious values for the function Ψ over the nodes outside the solution domain should be calculated with regard to the inner nodes. In the present work the proposed algorithm by Subrahmanyam and Leissa, 1985 is applied for both free edges and clamped supports. So, the Eq. (9) and Eq. (10) can be written in the bellow discrete form:

The boundary condition for free edge:

$$\Psi(X_{i+k}) = \Psi(-X_{i+k})$$
$$k = -m, -m+1,\dots,-1,1,\dots, m-1, m \quad (15)$$

The boundary condition for clamped support:

$$\Psi(X_{i+k}) = -\Psi(-X_{i+k})$$
$$i = 0 \quad OR \quad N-1$$
$$k = -m, -m+1, ..., -1, 1, ..., m-1, m \tag{16}$$

After applying the boundary conditions by use of Eq. (15) and Eq. (16) the order of the weighting coefficients of the A, $\bar{B}$ and C matrices reduces to $N \times N$.

5. Implementing the system of equations

Eq. (17) is the discrete form of the governing Eq. (7) after implementing boundary condition and non-prismatic property of the shaft:

$$[\boldsymbol{C}]_{N\times N}\{\Psi\}_{N\times 1} + [\bar{\boldsymbol{B}}]_{N\times N}\{\Psi\}_{N\times 1} + \Omega^2[\boldsymbol{A}]_{N\times N}\{\Psi\}_{N\times 1} = 0 \tag{17}$$

Eq. (17) can be rewritten in the following compact form:

$$[\boldsymbol{C} + \bar{\boldsymbol{B}} + \Omega^2 \boldsymbol{A}]\{\Psi\} = 0 \tag{18}$$

The Eq. (18) is an eigenvalue problem and the $\{\Psi\}$ can be evaluated using a standard solver.

Numerical results

To validate the mentioned algorithm in the previous section, free vibration of a non-prismatic shaft with rotational inertia $j(x) = (b + ax/L)^n$ is studied. The chosen shaft is one of the few cases that has analytical solution (Rafiee et al 2010). Several examples are studied with various grid numbers and various values of the parameters *a, b* and *n*. In all cases parameters ρ, *G* and *L* set to unit value. Two types of boundary conditions i.e. clamped- clamped and free- clamped shafts have been considered.

Tables 1 and 2 tabulate values of first five frequencies obtained from DSC written computer program and those from the analytical solution for non-prismatic shaft for *b*=1, *a*=0, 1, 2 and *n*= 2, 4. In all cases *N*=15 and *m*=13 is considered. As it is seen, the numerical results are in good agreement with exact solutions and the maximum relative error is of order 10^{-2}.

Table 1. Non-dimensional frequencies of the C-C shaft with $j(x) = (b + ax/L)^n$

n	Ω	*a*=0 DSC	*a*=0 Rafiee 2010	*a*=0 Relative error	*a*=1 DSC	*a*=1 Rafiee 2010	*a*=1 Relative error	*a*=2 DSC	*a*=2 Rafiee 2010	*a*=2 Relative error
	1	3.14159	3.14159	0	3.14159	3.14159	0	3.14159	3.14159	0
	2	6.28318	6.28319	1.59E-6	6.28318	6.28319	1.59E-6	6.28318	6.28319	1.59E-6
2	3	9.42478	9.42478	0	9.42478	9.42478	0	9.42478	9.42478	0
	4	12.56637	12.56637	0	12.56637	12.56637	0	12.56637	12.56637	0
	5	15.70801	15.70796	3.18E-6	15.70801	15.70796	3.18E-6	15.70801	15.70796	3.18E-6
	1	3.14159	3.14159	0	3.28631	3.13349	4.88E-2	3.34598	3.12565	7.05E-2
	2	6.28318	6.28319	1.59E-6	6.27564	6.27892	5.22E-4	6.27564	6.27225	5.41E-4
4	3	9.42478	9.42478	0	9.45935	9.42191	3.97E-3	9.45935	9.41726	4.47E-3
	4	12.56637	12.56637	0	12.52352	12.56421	3.24E-3	12.52352	12.56067	2.96E-3
	5	15.70801	15.70796	3.18E-6	15.70710	15.70623	5.57E-5	15.70710	15.70337	2.38E-4

Table 2. Non-dimensional frequencies of the C-F shaft with $j(x)=(b+ax/L)^n$

n	Ω	a=0 DSC	a=0 Rafiee 2010	a=0 Relative error	a=1 DSC	a=1 Rafiee 2010	a=1 Relative error	a=2 DSC	a=2 Rafiee 2010	a=2 Relative error
2	1	1.57080	1.57080	0	1.16578	1.16556	1.89E-4	0.96704	0.96740	3.72E-4
	2	4.71239	4.71239	0	4.57383	4.60422	6.60E-3	4.53787	4.56745	6.48E-3
	3	7.85389	7.85398	0	7.75562	7.78988	4.40E-3	7.73488	7.76837	4.31E-3
	4	10.99558	10.99557	9.09E-7	10.94917	10.94994	7.03E-5	10.93242	10.93468	2.07E-4
	5	14.13718	14.13717	7.07E-7	14.19981	14.10173	6.96E-3	14.18125	14.08989	6.48E-3
4	1	1.57080	1.57080	0	0.82497	-	-	0.51315	-	-
	2	4.71239	4.71239	0	4.60045	4.48748	2.52E-2	4.66260	4.40407	5.87E-2
	3	7.85389	7.85398	0	7.75270	7.72175	4.01E-3	7.80518	7.67293	1.72E-2
	4	10.99558	10.99557	9.09E-7	10.91119	10.90163	8.77E-4	10.94383	10.86697	7.07E-3
	5	14.13718	14.13717	7.07E-7	14.06471	14.06426	3.18E-5	14.09559	14.03736	4.15E-3

As this problem is solvable by other numerical approaches, so, in this letter the free torsional vibration of the non-prismatic shaft is analyzed by use of traditional FDM-2nd, FEM and DQM. So, three computer programs are provided based on each method. The first five frequencies obtained from the DSC method are compared with those of three other numerical approaches in tables 3 and 4. The value of N is fixed to be 15 in all four methods. It is observed from these tables that numerical errors of the four are relatively identical. But as the number of vibration mode increases, the results of the DSC are more accurate in contrast to three other numerical approaches.

Table 3. Comparison between results of four numerical approaches for C-C shaft with $j(x)=(1+x/L)^4$

Ω	DSC	Relative error	FEM	Relative error	DQM	Relative error	FDM-2nd	Relative error	Rafiee 2010
1	3.28631	4.88E-2	3.29461	5.14E-2	3.28600	4.87E-2	3.28724	4.91E-2	3.13349
2	6.27564	5.22E-4	6.41808	2.22E-2	6.36068	1.30E-2	6.36178	1.32E-2	6.27892
3	9.45935	3.97E-3	9.66316	2.56E-2	9.47720	5.87E-3	9.46955	5.06E-3	9.42191
4	12.52352	3.24E-3	13.04072	3.79E-2	12.60589	3.32E-3	12.56523	8.09E-5	12.56421
5	15.70710	5.57E-5	16.58356	5.59E-2	15.73969	2.13E-3	15.61462	5.83E-3	15.70623

Table 4. Comparison between results of four numerical approaches for C-F shaft with $j(x)=(1+x/L)^4$

Ω	DSC	Relative error	FEM	Relative error	DQM	Relative error	FDM-2nd	Relative error	Rafiee 2010
1	0.82497	-	0.82745	-	0.82497	-	0.82840	-	-
2	4.60045	2.52E-2	4.62704	3.11E-2	4.60045	2.52E-2	4.61009	2.73E-2	4.48748
3	7.75270	4.01E-3	7.90027	2.31E-2	7.78910	8.72E-3	7.80149	1.03E-2	7.72175
4	10.91119	8.77E-4	11.24580	3.16E-2	10.94966	4.41E-3	10.95012	4.45E-3	10.90163
5	14.06471	3.18E-5	14.72373	4.69E-2	14.10158	2.65E-3	14.05432	7.07E-4	14.06426

In order to compare the application of the four numerical methods in computing high frequencies, the first 50 frequencies of the non-prismatic shaft are investigated. Fig. 2 and Fig. 3 plot the percent relative error against mode number for all of four approaches.

In all methods number of nodes set to be N= 51 and the parameter m=N-1 in the DSC algorithm. According to these figures, the maximum error of the DSC is 2.4 percent for clamped- clamped shaft, while this value for the FEM and FDM-2nd is 20.13 and 25.04 percent, respectively. The DQ method gives accurate results up to 35th vibration mode. But from then the numerical instability gradually occurs in the method. For the clamped- free shaft, the maximum numerical error in calculating the first 50 modes for the DSC, FEM and FDM-2nd is 2.48, 20.16 and 24.31 percent, respectively. The DQM presents acceptable results up to 35th mode number and then the numerical instability occurs gradually in the solution procedure. As it is observed from these figures, the DSC method is preferred for calculating high modes.

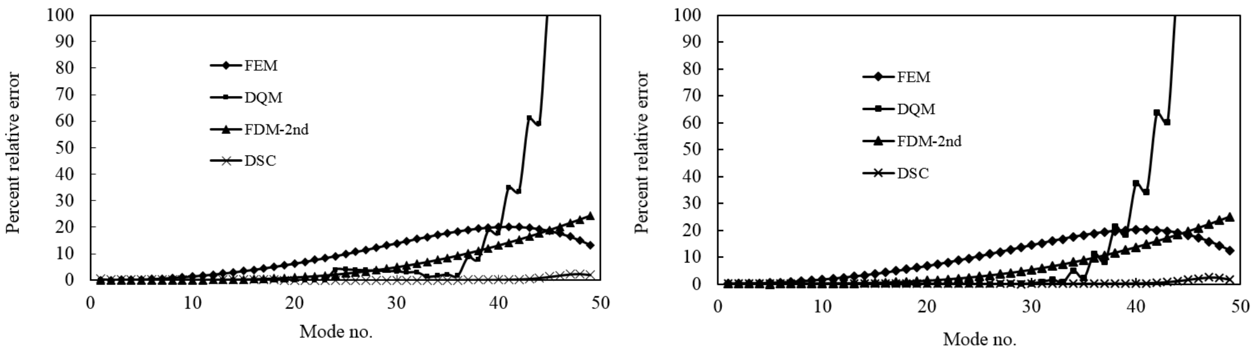


Fig 2 & 3. Percent relative error against mode number for C-F (left) and C-C (right) shaft with $j(x) = (1 + x / L)^4$

One of the advantages of a promising numerical approach is to present acceptable results with few grid points and so has less computational operations. The convergence trend of the four numerical approaches against increase in the number of grid points is shown for calculating the first and the second vibration modes of the non-prismatic shaft in Fig. 4 and Fig. 5. As it is seen, the DQ and after that the DSC methods converge with less grid points. However, the FEM and FDM-2nd are accurate methods; they need a finer grid to converge.

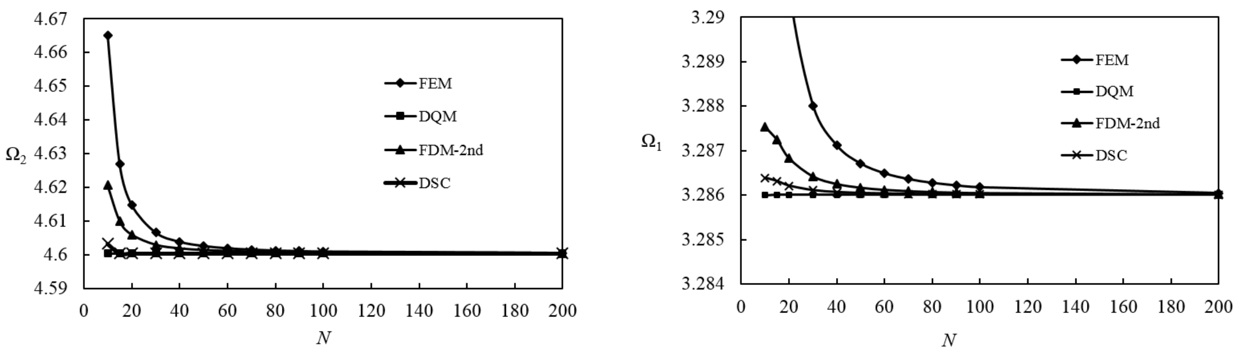


Fig 4 & 5. Convergence trend against increase in the number of grid points for C-F (left) and C-C (right) shaft with $j(x) = (1 + x / L)^4$

Data in tables 5 and 6 exhibit the impact of non-prismatic property (increase in power of the polynomial $j(x) = (b + ax / L)^n$) on application of the four numerical approaches. The first 5 frequencies of free vibration of a non-prismatic shaft with rotational inertia of $j(x) = (1 + x / L)^{100}$ is presented by use of the four methods for various numbers of grid points, in these two tables.

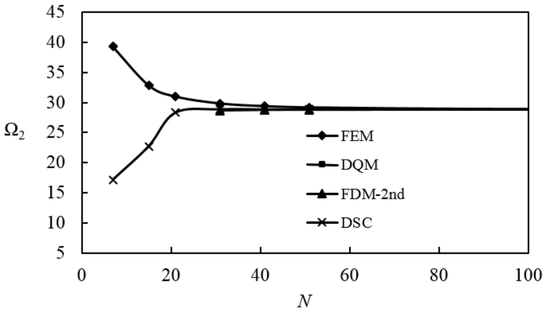
As it is observed, acceptable accurate results achieved with a few grid points by using DSC and FEM. However, the DQM and FDM-2nd yield unacceptable results (imaginary numbers) with few grid points and they need finer grids to produce acceptable accurate results. To enhance the comparison between results Fig. 6 and Fig. 7 are presented. It is obvious that the FEM and DSC give more accurate results.

Table 5. Impact study of non-prismatic property for C-C shaft with $j(x)=(1+x/L)^{100}$

	Ω	N=7	N=15	N=21	N=31	N=41	N=51	N=101	N=501
DSC	1	16.174809	14.685018	4.6228666	0.2949395	0.1776063	0.4915659	0.0114434	3.10E-05
	2	17.172898	22.710867	28.309866	28.856159	28.841682	28.825444	28.824514	28.819578
	3	17.685763	24.967824	30.173606	31.717607	31.696844	31.676607	31.677989	31.672561
	4	18.067563	26.985152	31.846043	34.177557	34.149294	34.124713	34.129109	34.12326
	5	18.557489	28.897977	33.687491	36.438908	36.402065	36.37263	36.380877	36.374639
DQM	1	-	-	-	-	-	7.60E-06	1.40E-05	3.156E-04
	2	-	-	-	-	-	28.819345	28.819344	28.81933
	3	-	-	-	-	-	31.672242	31.672291	31.672725
	4	-	-	-	-	-	34.123748	34.12315	34.117489
	5	-	-	-	-	-	36.368866	36.373006	36.419593
FEM	1	1.15E-06	6.46E-07	-	-	1.30E-06	-	-	1.71E-05
	2	39.298119	32.877596	31.009059	29.844246	29.406422	29.198252	28.915142	28.823174
	3	43.599128	36.903001	34.546967	33.02862	32.451141	32.17552	31.799706	31.6774
	4	48.381037	40.470893	37.679642	35.814119	35.096257	34.752425	34.282532	34.129366
	5	51.838612	43.81606	40.635026	38.416591	37.552255	37.136816	36.567825	36.382088
FDM-2nd	1	-	-	-	-	-	-	-	4.14E-06
	2	-	-	-	28.64746	28.773069	28.806464	28.822249	28.819568
	3	-	-	-	31.692216	31.691427	31.68693	31.677258	31.672553
	4	-	-	-	34.349849	34.209758	34.165759	34.12998	34.123253
	5	-	-	-	36.828966	36.532051	36.446352	36.383433	36.374635

Table 6. Impact study of non-prismatic property for C-F shaft with $j(x)=(1+x/L)^{100}$

	Ω	N=7	N=15	N=21	N=31	N=41	N=51	N=101	N=501
DSC	1	16.3936	31.902	32.12085	28.20905	28.25103	28.26897	28.29096	28.29725
	2	17.17444	32.53995	33.31442	31.03633	31.08373	31.10374	31.12809	31.13502
	3	17.33816	33.36223	34.59186	33.46684	33.51825	33.54009	33.56653	33.57399
	4	17.53472	34.39826	36.08386	35.68763	35.75535	35.77918	35.8075	35.81555
	5	18.26021	35.59856	37.83658	37.90862	37.87212	37.89605	37.9266	37.9346
DQM	1	-	-	-	-	28.30736	28.29749	28.29749	28.29749
	2	-	-	-	-	30.89083	31.1352	31.13527	31.13521
	3	-	-	-	-	-	33.57532	33.57444	33.57555
	4	-	-	-	-	-	35.80857	35.81478	35.80295
	5	-	-	-	-	-	37.97041	37.93948	38.03332
FEM	1	36.01065	31.7369	30.1942	29.196	28.81437	28.63175	28.38222	28.30089
	2	39.73381	35.63174	33.66343	32.34292	31.83174	31.58617	31.24974	31.13988
	3	43.61681	39.08229	36.73457	35.09599	34.45387	34.14428	33.71915	33.58011
	4	48.3812	42.31689	39.63135	37.6683	36.889	36.51191	35.99295	35.82298
	5	54.38809	45.40429	42.43482	40.13907	39.21468	38.76573	38.14654	37.94345
FDM-2nd	1	8.689595	19.63668	26.5836	28.00225	28.1787	28.23756	28.28852	28.29723
	2	-	-	-	31.00507	31.07061	31.09669	31.1271	31.13501
	3	-	-	-	33.62495	33.56711	33.55842	33.56706	33.57399
	4	-	-	-	36.06665	35.86995	35.82418	35.80973	35.81554
	5	-	-	-	38.41106	38.05387	37.96872	37.93001	37.93461



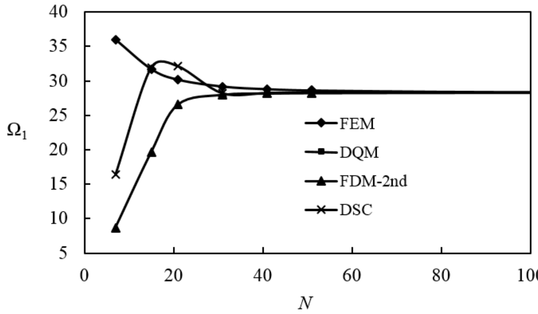


Fig 6 & 7. Convergence trend against increase in the number of grid points for C-F (left) and C-C (right) shaft $j(x) = (1 + x/L)^{100}$

Conclusions

In this paper, a numerical algorithm based on the DSC is proposed for free torsional vibration analysis of non-prismatic shafts. To validate the proposed algorithm, vibration of a specific non-prismatic shaft which has an analytical solution is investigated. Comparing the results indicated that the novel algorithm gives acceptable accurate results. As the problem of the vibration of non-prismatic shafts is solvable by other numerical approaches, some examples have been solved using FEM, DQM and FDM-2nd to determine the ability and position of the DSC in comparison with other numerical methods. Investigations indicated that the DQM gives relatively more accurate results with less grid points in low frequency domain. The DSC and FEM produce more accurate results with increasing of the non-prismatic property of the shaft and the DSC yields best results in the high frequency domain, which other approaches encounter numerical instability.

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