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Wavelet Collocation Method for Solving The Model of Drug Release

Zahra Kalateh Bojdi* and Ataollah Askari Hemmat

Abstract

In this paper we investigate wavelet collocation-finite difference method for solving two-dimensional model of drug release in the cardiovascular tissue from the stent.

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1. Introduction

Arterial diseases are among the leading causes of death in the industrialized world. They may cause a reduction of the blood flow to important organs and to muscles, because of the narrowing or occlusion of the affected arteries. Drug release depends on many factors, such as the geometry and location of the vessel, the geometry of the stent, the coating properties as its chemical composition and porosity, and drug characteristics as for example its diffusivity. Due to the involvement of so many factors, prediction of drug release represents an important issue and mathematical models are a useful tool to design an appropriate drug delivery system.

The paper is organized as follows. Section 2 is devoted to the description of the model. In Section 3 we explain wavelet collocation method for solving the two-dimensional model of drug release from the stent.

2. Description of the model

The drug release system in the arterial wall Ω_w can be modeled as follows [4]:

$$\begin{aligned}
&\frac{\partial a}{\partial t} + \frac{K_{lag}}{k_w} u_w \nabla a - D_w \triangle a = 0, && in\ \Omega_w,\\
&D_w \frac{\partial a}{\partial n_w} + \alpha(t) a = \beta(t) c_0, && on\ \Gamma,\\
&D_w \frac{\partial a}{\partial n_w} + P_w \frac{a}{\epsilon_w k_w} = 0, && on\ \Gamma_{adv},\\
&a = 0, && on\ \Gamma_{bl} \cup \Gamma_s.
\end{aligned} \tag{1}$$

* speaker

where a are the volume averaged solid concentration of the free drug inside the arterial wall. D_w denotes the diffusion coefficient of the considered drug in the tissue, K_{lag} denotes the decrease of convective transport due to collisions of the solid particles with the structure of the porous wall ($0 \leq K_{lag} \leq 1$), k_w is an additional partition coefficient that defines the ratio between the drug bound to the tissue matrix and that dissolved in the fluid, P_w is the permeability of the tissue and ϵ_w is the its porosity. Finally, $u_w = -\frac{k_b}{\mu_b}\nabla p$, where k_b and μ_b are the hydraulic permeability of the arterial wall and the viscosity of the blood plasma respectively and p is the pressure.

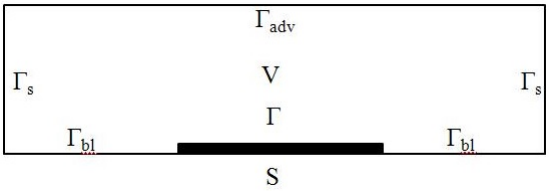

FIGURE 1. Stent S in contact with the vessel wall V.

3. Multiresolution analysis and wavelet collocation method

Let φ be a Daubechies wavelet's scaling function. Therefore φ is compact support and

$$\varphi(x) = \sum_{k=0}^{N-1} a_k \varphi(2x-k), \tag{2}$$

where $\{a_k\}$ are the filter coefficients and N is an even positive integer. Suppose $\varphi(x)$ is normalized such that: $\int_{-\infty}^{\infty} \varphi(x)\,dx = 1$. We introduce[1]

$$\theta(x) := (\varphi * \varphi(-\cdot))(x), \tag{3}$$

the function θ is called autocorrelation function of φ.

Theorem 3.1. *The function θ, have the following properties[1, 3]:*

1. $\theta(x) = \sum_{k=-N+1}^{N-1} c_k \theta(2x-k)$, *that* $c_k = c_{-k} = \frac{1}{2}\sum_{i=0}^{N-1-k} a_i a_{k+i}, \quad k \geq 0$,
2. $supp\ (\theta) \subseteq [-N+1, N-1]$,
3. $\theta(k) = \delta_{0,k},\ k \in Z$,
4. $c_{2k} = \delta_{0,k},\ c_k = \theta(\frac{k}{2}),\ k \in Z$, *therefore* $\theta(x) = \sum_{k=-N+1}^{N-1} \theta(\frac{k}{2})\theta(2x-k)$,

where N is an even positive integer in Daubechies wavelet, the sequence $\{c_k\}_{k\in Z}$ is called the scaling filter and $\delta_{0,k}$ is the Kronecker delta function.

Definition 3.2. *A sequence of subspaces $\{V\,j\}_{j\in Z}$ in $L^2(R)$ is called a Multiresolution analysis (MRA) for $L^2(R)$ with scaling function φ, if[2]:*

1. $V_j \subseteq V_{j+1} \subseteq L^2(R)$,
2. $\bigcap_{j\in Z} V_j = \{0\}$, *and* $\overline{\bigcup_{j\in Z} V_j} = L^2(R)$

3. $f(\cdot) \in V_j \Leftrightarrow f(2^{-j}\cdot) \in V_0$,

4. $f(\cdot) \in V_0 \Leftrightarrow f(\cdot - n) \in V_0$, *for all* $n \in Z$,

5. There exist a function $\varphi \in V_0$, *called scaling function, such that* $\{\varphi(\cdot - k)\}_{k\in Z}$ *is an orthonormal basis for* V_0.

Corollary 3.3. *Let* $\{V_j\}_{j\in Z}$ *be MRA for* $L^2(R)$ *with scaling function* φ. *There exist coefficients* $\{a_k\}_{k\in Z}$ *such that*

$$\varphi(x) = \sum_{k\in Z} a_k \varphi(2x - k).$$

and for any $j, k \in Z$ *define* $\varphi_{jk}(x) = 2^{j/2}\varphi(2^j x - k)$. *Then* $\{\varphi_{jk}(x)\}_{k\in Z}$ *is an orthonormal basis for* V_j *[2].*

If $\{V_j\}_{j\in Z}$ is a multiresolution anaysis for $L^2(R)$ with scaling function ϕ and wavelet ψ, then $\{V_j^{'} = V_j \otimes V_j\}_{j\in Z}$ is a multiresolution analysis of $L^2(R^2)$.

We can easily show that

$$\begin{aligned} V_1^{'} &= V_1^{(x)} \otimes V_1^{(y)} = (V_0^{(x)} \oplus W_0^{(x)}) \otimes (V_0^{(y)} \oplus W_0^{(y)}) \\ &= (V_0^{(x)} \otimes V_0^{(y)}) \oplus (V_0^{(x)} \otimes W_0^{(y)}) \oplus (W_0^{(x)} \otimes V_0^{(y)}) \oplus (W_0^{(x)} \otimes W_0^{(y)}) \\ &= V_0^{'} \oplus W_0^{'1} \oplus W_0^{'2} \oplus W_0^{'3}. \end{aligned} \tag{4}$$

This 2-D multiresolution analysis requires one scaling function

$$\Phi(x, y) = \phi(y)\phi(x) \in V_0^{'},$$

and three wavelets

$$\Psi^1(x, y) = \phi(x)\psi(y), \quad \Psi^2(x, y) = \phi(y)\psi(x), \quad \Psi^3(x, y) = \psi(x)\psi(y),$$

where Ψ^i is the wavelet associated to $W^{'i}$ for $i = 1, 2, 3$, respectively.

Define $V_j = span\{\theta(2^j \cdot -k), k\}$, that $j \in Z$. So $\{V_j\}_{j\in Z}$ generates an MRA with scaling function θ [2, 3].

The derivatives of the function θ defined by $\theta(x) = \int \varphi(t)\varphi(t - x)dt$ are

$$\theta'(l) = -\int \varphi(t)\varphi'(t - l)dt, \ \theta''(l) = -\int \varphi'(t)\varphi'(t - l)dt.$$

Thus we compute derivatives of the function θ at the point $x_l = l2^{-j}$.

Let J be arbitrary. We estimate the solution for equation (1) with corresponding initial and boundray conditions using the following expansion:

$$a(x, y) \approx \sum_{k\in Z} \sum_{l\in Z} a_{kl}\theta(2^J x - k)\theta(2^J y - l), \tag{5}$$

where $a_{kl} = a(x_k^J, y_l^J)$, $x_k^J = k2^{-J}$ and $y_l^J = l2^{-J}$. The first derivative of a with respect to time, are estimated by $\frac{\partial a}{\partial t} \simeq \frac{a^{n+1} - a^n}{\Delta t}$.

Thus the discretization of Eq. (1) at given collocation points $x_p^J = \frac{p}{2^J}$ and $y_q^J = \frac{q}{2^J}$, $p, q = 1, \cdots, 2^J - 1$, is

$$\begin{aligned} a^{n+1}(x_p, y_q) &= \frac{K_{lag}}{k_w}\frac{k_b}{\mu_b} 2^J \Delta t \left(p_1 \sum_{k\in Z} a_{kq}^n \theta'(p-k) + p_2 \sum_{l\in Z} a_{pl}^n \theta'(q-l) \right) \\ &+ D_w 2^{2J} \Delta t \left(\sum_{k\in Z} a_{kq}^n \theta''(p-k) + \sum_{l\in Z} a_{pl}^n \theta''(q-l) \right) \\ &+ \Delta t S^n(x_p, y_q) + a_{pq}^n. \end{aligned} \tag{6}$$

Now, we can write $a^{n+1} = Aa^n + B^n$, where the vector B^n is generated by the boundary conditions.

4. Conclusion

Wavelet collocation-finite difference approximation to the solution of the two-dimensional model of drug release from the stent is constructed. The equation (1) with corresponding initial and boundary conditions, can be solved successfully using the proposed method in this paper. The numerical results will be presented at the speech.

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Zahra Kalateh Bojdi,
Department of Mathematics,
University of Advanced Technology,
City Kerman, Iran
e-mail: z.kalatehbojdi@student.kgut.ac.ir

Ataollah Askari Hemmat,
Department of Applied Mathematics,
University of Shahid Bahonar,
City Kerman, Iran
e-mail: askari@uk.ac.ir