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Weak Almost Periodicity and Uniform Continuity on a Locally Compact Quantum Group

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Abstract

First we present some useful characterizations for the function spaces $AP(\mathbb{G})$, $WAP(\mathbb{G})$ and $LUC(\mathbb{G})$ on a locally compact quantum group $\mathbb{G}$ and then we study some conditions under which $AP(\mathbb{G})$ and $WAP(\mathbb{G})$ are C*-algebras. We also investigate the equality $LUC(\mathbb{G}) = WAP(\mathbb{G})$.

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1. Introduction

Let G be a locally compact group. It is known that the function spaces $AP(G)$, $WAP(G)$ and $LUC(G)$ are C*-subalgebras of $C_b(G)$. Moreover, they enjoy the inclusion relations $AP(G) \subseteq WAP(G) \subseteq LUC(G)$, where the equality hold under some additional conditions. For example, it has been known that $WAP(G) = LUC(G)$ if and only if G is compact. From the operator theory point of view, it is known that $WAP(G)$ can be identified with $WAP(L^1(G))$. Also there is a characterization of left uniform continuity, namely $LUC(G) = L^\infty(G) \cdot L^1(G)$. For complete information with more details on these funtion spaces and their inclusion relashionships one may refer to [1].

We refer to [4] for the definitions and basic facts on Hopf von Neumann algebras and locally compact quantum groups. In [2] Daws showed that for an abelian Hopf von Neumann algebra (M, Γ), $WAP(M_*) = \{x \in M : L_x : M_* \to M \text{ is weakly compact}\}$ and $AP(M_*) = \{x \in M : L_x : M_* \to M \text{ is compact}\}$ are C*-subalgebras of M. In [3] he defined $WAP(M, \Gamma)$ for an arbitrary Hopf von Neumann algebra (M, Γ) and proved that it is the largest C*-algebra contained in $WAP(M_*)$. However, in contrast to the situation for group, it was not known whether the equality $WAP(M_*) = WAP(M, \Gamma)$ holds. In this talk we show that for a coamenable locally compact quantum group $\mathbb{G}$, the spaces $WAP(L^1(\mathbb{G}))$ and $AP(L^1(\mathbb{G}))$ are C*-algebras and that $WAP(M_*) = WAP(M, \Gamma)$. We also define the notation of completely weakly compact operator for an arbitrary

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Hopf von Neumann algebra and we give a charaterization of $WAP(M, \Gamma)$ in terms of complete weak compactness of left translation operators. On the other hand Runde [5] showed that for certain locally compact quantum groups $\mathbb{G}$, $LUC(\mathbb{G})$ is a C*-algebra. In this talk we aslo improve Runde's result.

2. Main Results

For a Hopf von Neumann algebra M we define $CWAP(M_*)$ and we show the following result.

Theorem 2.1. *Let M be a Hopf von Neumann algebra. Then*

(i) *$CWAP(M_*)$ is equal to $WAP(M, \Gamma)$, and it is the largest C^*-algebra in $WAP(M_*)$.*
(ii) *If M is abelian, then $WAP(M_*) = CWAP(M_*)$, so $WAP(M_*)$ is a C^*-subalgebra of M.*

We give some new characterizations for $WAP(L^1(\mathbb{G})), AP(L^1(\mathbb{G}))$ and $LUC(\mathbb{G})$ from which we present the following results.

Theorem 2.2. *Let $\mathbb{G}$ be a coamenable locally compact quantum group. Then $WAP(L^1(\mathbb{G}))$ and $WAP(L^1(\mathbb{G}))$ are a C^*-algebras.*

Theorem 2.3. *Let $\mathbb{G}$ be a locally compact quantum group such that $L^1(\mathbb{G})$ is strongly Arens irregular. Then $WAP(L^1(\mathbb{G})) = LUC(\mathbb{G})$ implies that $\mathbb{G}$ is compact.*

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