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Stochastic comparisons of redundancy allocation at component level versus systems level

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Abstract

In this paper, we consider a coherent system of n components and n active spares. Then, we investigate stochastic comparisons of the lifetimes of series-parallel systems and discuss that component redundancy offer greater reliability than the system redundancy with respect to the hazard rate order and the reversed hazard rate order for two scenarios, matching spares and non-matching spares.

Keywords: Stochastic orders; Coherent systems; System redundancy; Active redundancy.

1 Introduction

In some applications, one way of improving the reliability of a system is the allocation of an active redundancy. On the other words, the allocation of an active redundancy is the efficient method to add redundancy components to a system. To allocate the spare components (component) to the system reliability, one natural question that arises in this direction is that, is it better to allocate the spare components (component) in parallel or series with the weakest components (component) of the system?. In general, there are two commonly used types of redundancies called active redundancy and standby redundancy commonly used in reliability engineering and system security. For active redundancy, available spares are put in parallel to components of the system and these spares start functioning simultaneously as original components. For standby redundancy, spares are attached to components of the system in a way that a spare starts functioning right after the component to which it is attached failed. In two cases, for the measurement of the performance of different allocations to the system reliability, we use the various types of the stochastic orders. For some recent results on stochastic comparisons in system

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reliabilities with active redundancy and standby redundancy, one can refer to Boland et al. (1992), Singh and Misra (1994), da Costa Bueno and do Carmo (2007), Zhao, Chan, and Ng (2012) and Laniado and Lillo (2014) and the references therein.

Boland and El-Neweihi (1995) concluded that, in the sense of the usual stochastic order, redundancy at component level is better than redundancy at system level for series systems, while the reverse is true for parallel systems. Consider a system with two components and two active spares with the random lifetimes X_1, X_2 and Y_1, Y_2, respectively. Consider the lifetimes of two coherent systems as $U_1 = \min\{\max\{X_1, Y_1\}, X_2\}$ and $U_2 = \min\{X_1, \max\{X_2, Y_1\}\}$, in which is observed that there is one spare allocation. Maybe for the first time (in our knowledge), Boland et al. (1992) showed that if $X_2 \geq_{st} X_1$ then $U_1 \geq_{st} U_2$, which reveals that, it is better to allocate the spare component in parallel with the weakest component, where $\geq_{st}$ denotes the usual stochastic order (for its formal definition please see Shaked and Shanthikumar (2007), Page 3). Then, Valdés and Zequeira (2003) extended the result in Boland et al. (1992) for the likelihood rate order (for its formal definition please see Shaked and Shanthikumar (2007), Page 16) to compare the lifetimes $U_1 = \min\{\max\{X_1, Y_1\}, X_2\}$ and $U_2 = \min\{X_1, \max\{X_2, Y_2\}\}$, in which is observed that there are two spare allocations, Y_1 and Y_2. After that for two spare allocations, Valdés and Zequeira (2006) gave conditions under which the allocation of the strongest spare with the weakest component is optimal in the sense of the hazard rate order.

Consider a general coherent systems ϕ with n components having independent lifetimes $X_1, \ldots, X_n$ and n active spares having independent lifetimes $Y_1, \ldots, Y_n$. Suppose that $X_1, \ldots, X_n$ and $Y_1, \ldots, Y_n$ are statistically independent. Recall that a system is said to be coherent system if it has no irrelevant components and the structure function of the system is monotone in each argument (that is, an improvement of a component cannot lead to a deterioration in system performance).

In this paper, we consider two scenario, matching spares and non-matching spares. In a matching spares problem, $X_i =_{st} Y_i$ for all $i = 1, \ldots, n$ where $=_{st}$ stands that the random variables have the same distribution, and in a non-matching spares problem $X_i \neq_{st} Y_i$ for all $i = 1, \ldots, n$. Denote $\tau(\mathbf{X}) = \tau(X_1, \ldots, X_n)$ the lifetime of the coherent system ϕ. When all components and spares lifetimes are independent and identically distributed, Boland and El-Neweihi (1995) proved, for general coherent systems, under some mild conditions, that $\tau(\mathbf{X} \vee \mathbf{Y}) \geq_{hr} \tau(\mathbf{X}) \vee \tau(\mathbf{Y}).wherethesymbols$‘$\wedge$’ and ‘$\vee$’ mean min and max, respectively. Noting that, they said that (1) can not be applied to the general k-out-of-n systems but they showed the result is true for the special case of 2-out-of-n system. After that, Singh and Singh (1997) extend (1) to a stronger result in the sense of the likelihood ratio order (for its formal definition please see Shaked and Shanthikumar (2007), Page 42) for the general k-out-of-n system. Gupta and Nanda (2001) obtained a similar result for the reversed hazard rate ordering. Also, there remains an open problem that whether this result still holds for the matching spares problem. Also, Misra, Dhariyal, and Gupta (2009) consider the non-matching problem and obtained some new results in this direction. Recently Zaho et al. (2015) studied stochastic comparisons of series systems at component level and system level with n exponential components under the set-up of matching spares problem.

In this paper, we focus our attention to the matching spares and non-matching spares and obtained some interesting results to compare the coherent systems with n exponential components and with n proportional reversed hazard rate ($PRHR$) components that redundancies as active spares at the system level is better than the system level in terms of the hazard rate order and the reversed hazard rate order.

2 Main results

In this section, we discuss matching spares and non-matching spares, respectively. We obtain some results under which component redundancy to be superior to the system redundancy with respect to the hazard rate order and the reversed hazard rate order, for any n-component series system. Before presenting our main results, let us first introduce the $PRHR$ model. Independent random variables $X_1, \ldots, X_n$ are said to follow the $PRHR$ model if for $i = 1, \ldots, n$, the distribution function of X_i can be expressed as,

$$F_i(x) = F^{\lambda_i}(x),\ \lambda_i > 0,$$

where F is the distribution function of the base line distribution and λ_i is the parameter of the model.

Theorem 1. *Let $X_1, \ldots, X_n$ be independent random variables following the $PRHR$ model with distribution functions $F^{\lambda_1}, \ldots, F^{\lambda_n}$, respectively. Let $Y_1, \ldots, Y_n$ be another independent random variables following the $PRHR$ model with parameters $F^{\lambda_1}, \ldots, F^{\lambda_n}$, respectively. Then*

$$\wedge\{X_1 \vee Y_1, \ldots, X_n \vee Y_n\} \geq_{hr} \{\wedge(X_1, \ldots, X_n)\} \vee \{\wedge(Y_1, \ldots, Y_n)\}.$$

Theorem 2. *Let $X_1, \ldots, X_n$ be independent random variables following the $PRHR$ model with common distribution function F^{λ}. Let $Y_1, \ldots, Y_n$ be another independent random variables following the $PRHR$ model with common distribution function F^{β}. Then*

$$\wedge\{X_1 \vee Y_1, \ldots, X_n \vee Y_n\} \geq_{rh} \{\wedge(X_1, \ldots, X_n)\} \vee \{\wedge(Y_1, \ldots, Y_n)\},$$

where $\geq_{rh}$ is the reversed hazard rate order (for its formal definition please see Shaked and Shanthikumar (2007), Page 36).

Theorem 3. *Let $X_1, \ldots, X_n$ be independent random variables following the exponential distribution with common parameter λ. Let $Y_1, \ldots, Y_n$ be another independent random variables following the exponential distribution with common parameter β. Then*

$$\wedge\{X_1 \vee Y_1, \ldots, X_n \vee Y_n\} \geq_{hr} \{\wedge(X_1, \ldots, X_n)\} \vee \{\wedge(Y_1, \ldots, Y_n)\}.$$

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