The 3rd International CUA Graduate Students Symposium

سومین سمپوزیوم بین‌المللی دانشجویان تحصیلات تکمیلی دانشگاه‌های عضو اتحادیهٔ قفقاز

University of Mohaghegh Ardabili

دانشگاه محقق اردبیلی

June 5-6, 2016

16–17 خردادماه

Thermodynamic Entropy Due to Lorentz Violation

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Abstract
Quantum effects of gravity in the early universe can be led to the existence of the present particles. We consider a free mass-less scalar field on a spatial lattice has been included in the early universe, so that all modes of the field are in vacuum state. Such scalar field model represents a modified dispersion relation as Lorentz violation which displays a quantum gravity model. The vacuum state in the presence of Lorentz violation can be appeared as particle creation. In this paper, the particle production due to Lorentz violation is shown as an entropic increase. Therefore, in this approach the source of the current entropy content of the universe should be directly related to particle creation stemming from Lorentz violation.

Keywords: particle creation, Lorentz violation, thermodynamics entropy

1.Introduction

In a gravity dominant regime with enormous density of the field, such big-bang and black hole theories, quantum effects are important. A quantum gravity model is a deviation from standard dispersion relation, i. e. Lorentz violation. In fact, the modified dispersion relation is achieved by the process of discrete modeling the spacetime on a lattice (Amelino-Camelia, 2013).

Study of quantum field theory in curved spacetime is shown that the initial vacuum state represents created particle due to interaction between quantum field and gravitational field (Birrell, *at al.*, 1984). We use Lorentz violation model with a time dependent Lorentz violation parameter to study how the present state containing particle is appeared. Also, we employ thermodynamics to understanding the current entropy source of universe .We show that the source of the current entropy content of the universe is directly related to created particle stemming from Lorentz violation.

2.Free scalar field on a spatial lattice

We consider a mass-less free scalar field in 1+1 dimensions without loss of generality. The Klein-Gordon equation is such as follow.

$$\ddot{\phi} = \nabla^2\phi - m^2\phi, \tag{1}$$

where exhibits the dispersion relation for plane waves as

$$\omega^2 = k^2 + m^2. \tag{2}$$

Letting Eq. (1) on a spatial lattice, we have

$$a^2\ddot{\phi} = \phi(n+1) - \phi(n-1) - 2\phi(n) - a^2m^2\phi. \tag{3}$$

where a is the space discrete-time continuum lattice. We will have a symmetric Brillouin zone, $-\pi < ka < \pi$, considering a plan-wave solution on the lattice as follows.

$$\phi = e^{ikna+i\omega t}. \tag{4}$$

Substituting the solution in Eq. (3) yields

$$-\omega^2\phi = \frac{e^{ika}+e^{-ika}-2}{a^2}\phi - m^2\phi. \tag{5}$$

Thus, the dispersion relation for the lattice model is as follows

$$\omega^2 = m^2 - \frac{2}{a^2}(\cos ka - 1). \tag{6}$$

Eq. (6) for $ka \ll 1$, is yielded

$$\omega^2 = m^2 + k^2 - O(k^4a^2), \tag{7}$$

where in the limit $a \to 0$, implies the standard dispersion relation, i. e. Eq. (2). Adding second term of the Talor extension of the cos-function in Eq. (6), represents a modified dispersion relation as follows.

$$\omega^2 = m^2 + k^2 - a^2k^4. \tag{8}$$

3.Production entropy

Let us consider a deviation from the standard dispersion relation such Eq. (8), as quantum gravity effect (Amelino-Camelia, 2013). We assume the back ground structure of space time is flat. For the early times, $t \to -\infty$, the dispersion relation is $\omega^2 = m^2 + k^2 - {a_0}^2 k^4$, where a_0 is the Lorentz violation parameter. For the late times, $t \to +\infty$, the dispersion relation has the standard form, Eq. (2). The natural solutions are $e^{\pm ikx \pm i\omega t}$, but since the dispersion relation is different for two asymptotic regions, the definitions of basis are not the same. Thus, the initial vaccum state and the final vacuum state are not equivalent, in analogy with other scenarios of quantum field theory in curved space time (Birrell, 1984).

According to above interpretation, we choose a time evolution of the Lorentz violation parameter as follows (Khosravi, 2011)

$$a(t)^2 = \frac{{a_0}^2}{1+e^t}. \qquad (9)$$

Substituting the operators $-i\partial_t$ and $-i\partial_x$ into ω and k in Eq. (7), and letting $\phi = e^{ikx} T_k(t)$, the Klien Gordon equation leads to

$$[\partial_t^2 + k^2 - a(t)^2 k^4] T_k = 0. \qquad (10)$$

The solution of Eq. (9) is

$$T_k(t) = C_1 e^{-i\left(\sqrt{k^2 - a_0^2 k^4}\right)t} {}_2F_1(a, b; c; -e^t) + C_2 e^{+i\left(\sqrt{k^2 - a_0^2 k^4}\right)t} {}_2F_1(b^*, a^*; c^*; -e^t). \qquad (11)$$

where ${}_2F_1$ is the hypergeometric function. Eq. (10) for the far past time, *in*-region and the far future time, *out*-region are as follow ${}_2F_1$

$$T_k^{in}(t) = \frac{1}{\sqrt{4\pi\omega_{in}}} e^{-i\omega_{in} t} {}_2F_1(a, b; c; -e^t),$$

$$T_k^{out}(t) = \frac{\sqrt{4\pi\omega_{out}}}{4\pi\omega_{in}}\left(e^{-i\omega_{in}t}\frac{\Gamma(c^*)\Gamma(a^*-b^*)}{\Gamma(a^*)\Gamma(c^*-b^*)}\,{}_2F_1(a,b;c;-e^t) - \frac{\Gamma(a)\Gamma(b-a)}{\Gamma(b)\Gamma(c-a)}e^{+i\omega_{in}t}\,{}_2F_1(b^*,a^*;c^*;-e^t)\right), \tag{12}$$

where Γ is the Gamma function and

$$a = -ik - i\sqrt{k^2 - a_0^2k^4} = -i(\omega_{in} + \omega_{out}),$$
$$b = +ik - i\sqrt{k^2 - a_0^2k^4} = -i(\omega_{in} - \omega_{out}), \tag{13}$$
$$c = 1 - 2i\sqrt{k^2 - a_0^2k^4} = 1 - i2\omega_{in}.$$

Also, $\omega_{in} = \sqrt{k^2 - a_0^2k^4}$. The solution of *in*-region and *out*-region are related as follow

$$u_k^{in}(t,x) = \alpha_k u_k^{out}(t,x) + \beta_k u_{-k}^{out*}(t,x), \tag{14}$$

where $u_k^{in}(t,x) = e^{ikx}\,T_k^{in}(t)$, $u_k^{out}(t,x) = e^{ikx}\,T_k^{out}(t)$ and α_k and β_k are the Bogoliubov coefficients. Therefore, the created particle is in the following form

$$N_k = \beta_k^2 = \frac{\sinh^2(\pi(\omega_{out}-\omega_{in}))}{\sinh(2\pi\omega_{in})\sinh(2\pi\omega_{out})}. \tag{15}$$

Then the initial vacuum state converts to an exited state which the context of the particle is given by Eq. (15).

Considering the vacuum state in the infinity future as a two mode squeezed state, the reduced density matrix will be as

$$\rho_k = \frac{1}{|\alpha_k^2|}\sum_{n_k}|\frac{\beta_k}{\alpha_k}|^{2n_k}|n_k\rangle\langle n_k|. \tag{16}$$

The density matrix of the created particle can be written in the thermal form such as follows

$$\rho_k = 2\sinh\frac{\omega_{out}}{2T}\sum_{n_k} e^{-\frac{\omega_{out}}{2T}(n_k+\frac{1}{2})}\,|n_k\rangle\langle n_k|. \qquad (17)$$

Therefore, the created entanglement mode pairs are in a thermal equilibrium with an equilibrium temperature as follows

$$T = \frac{\omega_{out}}{\log\left(|\frac{\beta_k}{\alpha_k}|^2\right)}. \qquad (18)$$

It has been shown the production entropy of the created particle is given by

$$s = \frac{\omega_{out}}{T} N_k, \qquad (19)$$

where T is the equilibrium temperature assigned to created particle, Eq. (18). Substituting Eq. (18) into Eq. (19) yields

$$s = N_k \log\left(\frac{1+N_k}{N_k}\right), \qquad (20)$$

where N_k is given by Eq. (15). Regarding to the definition of $\omega_{in} = \sqrt{k^2 - a_0^2 k^4}$, there is an upper bound on the k, for positive value of a_0^2, in the event that the Eq. (20) is always valid for negative value of a_0^2. The case $a_0^2 = 0$ corresponding to $s = 0$, as expected.

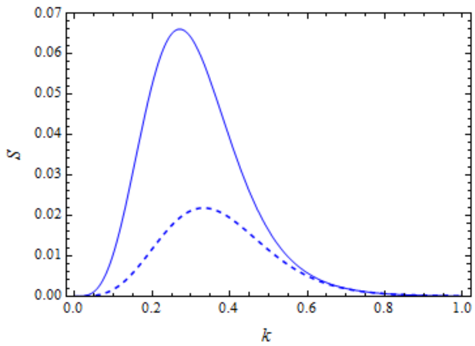

Figure 16: The production entropy with respect to k for $a_0^2 = -15$, the solid line, and for $a_0^2 = -5$, the dotdashed line.

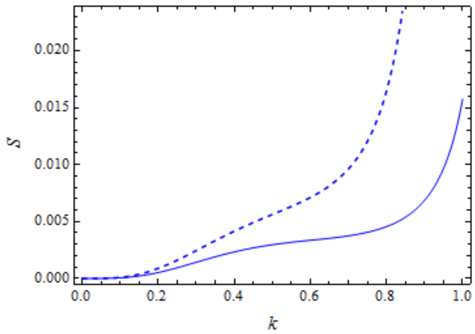

Figure 17: The production entropy with respect to k for $a_0^2 = 0.75$, the solid line, with the upper bound $k = 1$ and for $a_0^2 = 1$, the dotdashed line, with the upper bound $k = 1.16$.

Figure 1 shows the production entropy with respect to k for two different negative values of a_0^2. The production entropy has maxima, where for each Lorentz parameter is different. Increasing the Lorentz parameter, the maximum value of the production entropy forward to larger value of k. Figure 2 represents the production entropy with respect to k for two different positive values of a_0^2. It is worth mentioning that for each case, valid values of k are less than the upper bound on the k.

4.**Results and Discussion**

We investigated the effects of Lorentz violation by a time dependent modified dispersion relation. In the past infinity, The vacuum state in the presence of Lorentz violation can be represented the created particles in the absence of Lorentz violation in the future infinity.The particle production due to Lorentz violation was shown as an entropic increase. Therefore, in this approach the source of the current entropy content of the universe was directly related to particle creation stemming from Lorentz violation.

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