The 3rd International CUA Graduate Students Symposium

سومین سمپوزیوم بین‌المللی دانشجویان تحصیلات تکمیلی دانشگاه‌های عضو اتحادیهٔ قفقاز

University of Mohaghegh Ardabili

دانشگاه محقق اردبیلی

June 5-6, 2016

16–17 خردادماه

New Approach for Solution of Volterra-Fredholm Integral Equations

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Abstract

In this paper we analyze two methods to approximate of the solution of the mixed volterra-fredholm integral equations, by making use of the expression of a function of a basis spline and schauder. The first is collocation method and second one is fixed point method. We analyze some problems of convergence and stdy the error in each approximation. Numerical results show the theoretical statements.

Keywords: Volterra-Fredholm equation, collocation method, fixed point method. spline, schauder basis

Introduction

Let us consider the following Volterra–Fredholm linear integral equation of the second kind:

$$\lambda u(x) = f(x) + \int_a^x k_1(x,y)u(y)dy + \int_a^b k_2(x,y)u(y)dy, \quad x \in [a,b], \tag{1}$$

Where $\lambda \in \mathbb{R} \setminus \{0\}$; $f: [a,b] \to \mathbb{R}$; $k_i(i = 1,2): [a,b] \times [a,b] \to \mathbb{R}$ and u: $[a,b] \to \mathbb{R}$, with f and $k_i(i = 1,2)$
known function satisfying (1).
When modeling real problems, one has frequently to deal with Eq. (1) which has the theoretical and computational features
of both Volterra and Fredholm equations.

Suitable conditions on $k_i(i = 1,2)$and λ are posed in order to obtain a unique solution of (1) in C[a,b], under the assumption that f∈C[a,b] (for proofs in particular Banach spaces, with different conditions on $k_i(i = 1,2)$, refer, for example, to [6,1]).
However, in this paper, to state some results about the convergence of the proposed methods, we will assume the following
conditions:

(i) $k_1, k_2 \in C^1([a,b] \times [a,b])$,
(ii) $|\lambda| > 2\,(b-a)M$ where $M = \max(\| k_1\|_\infty, \|k_2\|_\infty)$

From a numerical point of view it is well known that Volterra equations are generally solved through iterative methods.
On the contrary direct methods are more suitable to the solution of Fredholm equations (see for example [2] and [3]). We tried to afford the solution of (1) both through an iterative as well as through a direct numerical method. The aim of this
paper is to analyze and compare these two different numerical methods.
We can reduce (1) to two equivalent compact forms:

$$[\lambda I + \mathcal{K}_1 + \mathcal{K}_2]u = f,$$
$$u = \frac{1}{\lambda}[-\mathcal{K}_1 u - \mathcal{K}_2 u + If], \qquad (2),(3)$$

where operators κ_1 and κ_2 are defined as follows:

$$\mathcal{K}_1 g = -\int_a^x k_1(x,y)g(y)dy \quad x \in [a,b],$$
$$\mathcal{K}_2 g = -\int_a^b k_2(x,y)g(y)dy \quad x \in [a,b], \qquad (4),(5)$$

and I is the identity operator in C[a,b].
The structure of (2) and (3) suggests the idea of constructing and comparing two different classes of methods: collocation direct methods and iterative fixed point methods (normally referred in particular to non linear equations).
Both kinds of methods are proposed for a particular class of approximating functions.
Namely, the first method (in the following called CSp) is a collocation method based on a linear spline class approximation;
the second one (in the following called FPSc) is a fixed point method built on Schauder linear bases.
We compare the two methods in terms of both efficiency and adaptability to the peculiarities of the integral model under

consideration. Convergence analysis is carried out in both cases.some preliminaries about the particular linear splines class and to the description and analysis of
the numerical collocation method. describe the Schauder basis correlated to the fixed point method and to study the related convergence problems. numerical results for both methods are given. some final remarks on the comparison between the two methods and some open problems.
we present a numerical model suitable to (1) based on a collocation method using approximating splines,
in particular the so called variation-diminishing Schoenberg (VDS) splines [12].
Firstly, let us recall some background on linear VDS splines (see for example [8]).

If we define

$$r_n = [\lambda I + \tilde{\kappa}]S_n u - f, \tag{6}$$

the values$\bar{u}_i$, approximating the function u in $t_{i+1}(i = 1,2, \dots, n)$are determined imposing that rn, defined in (6), is zero in a set of collocation points $\tau_k(k = 1,2, \dots, n)$ chosen in [a,b]:

$$r_n(\tau_k) = 0 \tag{7}$$

and solving the linear system(7).

we present a numerical model suitable to (1) based on a fixed point method with Schauder bases.
Firstly we recall some necessary background on linear Schauder bases.

we define the operator $\mathcal{T}: C[a,b] \rightarrow C[a,b]$ defined as it follows:

$$\mathcal{T}(u) = \frac{1}{\lambda}[-\mathcal{K}_1 u - \mathcal{K}_2 u + If]. \tag{8}$$

Under the hypotheses of existence and uniqueness of the solution of (1) it is well known that the fixed point theorem assures
a unique fixed point for the operator $\mathcal{T}$.

For the collocation method we consider:
_ the mesh points as equidistant points in [0,1] with double nodes in 0 and 1,
_ the mesh points all simple in (0, 1) as the approximation points,
_ the collocation points as a set of distinct points of the same number of the approximation points.
For the fixed point method we consider the subset of [0,1] defined as

$$H = \left\{0, 1, \frac{1}{2}, \frac{1}{4}, \frac{3}{4}, \ldots, \frac{1}{2^j}, \frac{3}{2^j}, \ldots, \frac{2^j - 1}{2^j} \ldots \right\} \quad j = 0, 1, \ldots$$

supporting the Schauder basis in C[0, 1].

$$\{t_1, \ldots t_n\} = \left\{\frac{i-1}{2^{j_0}}\right\}, \quad (i = 1, \ldots, 2^{j_0} + 1).$$

We denote R1 and R2 respectively the max(jerrj) in the fixed point and in the collocation methods, being err the set of
errors related to the approximation of the unknown function in a suitable subset of points in [0, 1].
Numerical results:

Example. $\lambda = 15, u(x) = x \text{and} f(x) = 15x - e^x(xe^x - e^x + 2)$

	$n = 17, m = 1$	$n = 33, m = 6$
$R1$	$1.1\ 10^{-16}$	$1.1\ 10^{-16}$
$R2$	$9.9\ 10^{-16}$	$5.9\ 10^{-16}$

For the fixed point method an algorithmic instability is experienced, while in the collocation case the equations are
more sensitive to the lack of stability of the collocation system. This last effect is currently under study.

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