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Investigation of parameter uncertainty in liquefaction probability based on meta-heuristic algorithms and Bayesian mapping function

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Abstract

Liquefaction evaluation methods in sandy soils is generally based on the deterministic analysis. In the deterministic approach, certain and non-dispersion parameters are considered. Furthermore, in these methods, establishment of exact correlation between the probability of liquefaction (P_L) and the factor of safety (F_s) is not possible. This problem is solved using the reliability analysis. In this paper, effect of the parameter uncertainty in liquefaction probability, based on the Gene Expression Programming (GEP) model for liquefaction resistance and potential evaluation based on Standard Penetration Test (SPT) is investigated. In order to verify the model, GEP results are compared with the results based on Idriss and Boulanger approach (2010). Then, First-Order Reliability Method (FORM) by using a hybrid of Particle Swarm Optimization and Genetic Algorithm (PSO-GA) in MATLAB2013a, as a robust optimization method is used to determine the reliability index (β). Bayesian mapping function is utilized to infer the relationship between probability of liquefaction and reliability index. Finally, effect of the level of parameter uncertainty on the liquefaction probability by development the Bayesian mapping functions, are investigated by using the β-P_L curves.

Keywords: Liquefaction, Gene Expression Programming, First Order Reliability Method, Hybrid algorithm (PSO-GA), Bayesian mapping function.

1. INTRODUCTION

Liquefaction occurrence is one of the most common causes of structural failures during earthquakes in saturated loose granular alluvium areas. Usually due to earthquake tensions in these regions, increasing pore-water pressure and therefore, reduction and loss of soil strength will happen and finally, soil reaches liquid consistency state. This phenomenon accompanied by remarkable settlements and cracks, eruption of mud and water, sand boiling and ground water seepage through the pore spaces between particles of un-consolidated earth materials. Due to difficulties and high costs of intact and high quality sample preparation and also existence of simple methods based on on-site tests such as Standard Penetration Test (SPT), Cone Penetration Test (CPT), Becker Penetration Test (BPT) and Shear wave velocity tests, geotechnical engineers prefer these procedures to use for evaluating soil liquefaction potential.

A new comprehensive approach for soil liquefaction assessment which considers uncertainty, is statistical analysis and specially reliability analysis. Many researches in this field has been done. Juang et al. (2004) analysis soil liquefaction based on CPT databases. They investigated the uncertainties in the Robertson and Wride model by using First-Order Reliability method (FORM) [1]. They considered parameter and model uncertainties in their probabilistic model. Hwang et al. (2004) based on statistical analysis and Chi Chi (1999) earthquake data, estimated the probability density function of cyclic shear induced-earthquake. They used First-Order Second Moment (FOSM) to obtain a correlation between probability of liquefaction (P_L) and cyclic stress ratio (CSR) [2]. Juang et al. (2006) by using the artificial neural network, presented a new relationship for soil liquefaction assessment based on 200 CPT databases [3]. Lee at el. (2010) investigated liquefaction data from one of the cities which was seen much damage in Chi Chi (1999) earthquake, Yuanlin. They considered uncertainties in dynamic soil resistance parameters in their liquefaction evaluation model. To determine the cyclic soil shear strength, they used SPT and CPT methods and illustrated a good agreement between the results of both models based on two methods [4].

In this paper, Gene Expression Programming (GEP) which is based on Genetic algorithm, is used to develop the Factor of Safety (F_s) model based on SPT databases (Cetin, 2000). In order to verify the GEP

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model, GEP results are compared with the results of Idriss and Boulanger approach (2010) [5]. The parameter uncertainty of the GEP model is calculated by using the FORM reliability analysis. A hybrid of Particle Swarm Optimization and Genetic Algorithm (PSO-GA), which are kinds of meta-heuristic optimization algorithms, are used as an optimization tool for the FORM reliability analysis. Through a rigorous reliability analysis using FORM, reliability index (β) is determined. Then Bayesian mapping function is utilized to make a relationship between the liquefaction probability and reliability index. Finally, effect of the level of parameter uncertainty is investigated by comparing the Bayesian mapping function obtained for each uncertainty level.

2. GEP MODEL FOR F_S

To decrease the time processing, all the 160 data normalized before entering to GEP. The fitness of each model is determined by minimizing the Mean-Squared-Error (MSE) as given in Eq. (1). For development the F_s model, 110 data points are selected for training and the remaining of them as testing data. Among several models, the best one is chosen based on the rate of successful prediction as presented in Eq. (2). The successful prediction rates of liquefied and non-liquefied for the model are 80% for training and 86% for testing data. Statistical performance of the model is shown in Table1. Figure 1 illustrated the performance of the proposed *Fs* model in the classification concept. Figure 2 shows predicted F_s versus calculated F_s based on the Idriss and Boulanger (2010) approach. From the sensitivity analysis for the F_s model, $N_{1,60}$ (49.82%) is the most important parameter and the others are respectively, $CSR_{7.5}$ (43.12%), $N_{1,60}$ (29.36%), M_w (20.34%) and FC (7.18%).

$$MSE = \frac{1}{n}\left[\sum_{i=1}^{n}\left(LI - LI_p\right)^2\right] \tag{1}$$

$$\begin{aligned} FS = &\left\{\frac{1}{7.096191 \times CSR_{7.5} \times (2CSR_{7.5} + M_w) \times [(N_1)_{60} + CSR_{7.5}]} \times \sqrt[4]{M_w} \times 0.00166134\right\} \\ &+ [(N_1)_{60} \times 9.708588 + 29.22056836] \times \frac{1}{94.256681 \times CSR_{7.5}} \times \frac{(N_1)_{60}}{94.256681} \\ &\times \sqrt{9.708588} + \sin\left\{\left[(N_1)_{60} - (CSR_{7.5} \times FC \times {M_w}^2)\right]^{18}\right\} \end{aligned} \tag{2}$$

Table1- Statistical performance of the F_s model

Model	R^2 (Train)	R^2 (Test)	MSE (Train)	MSE (Test)	RMSE (Train)	RMSE (Test)
F_s	0.9952	0.9348	5E-05	7.2E-05	0.007	0.0084

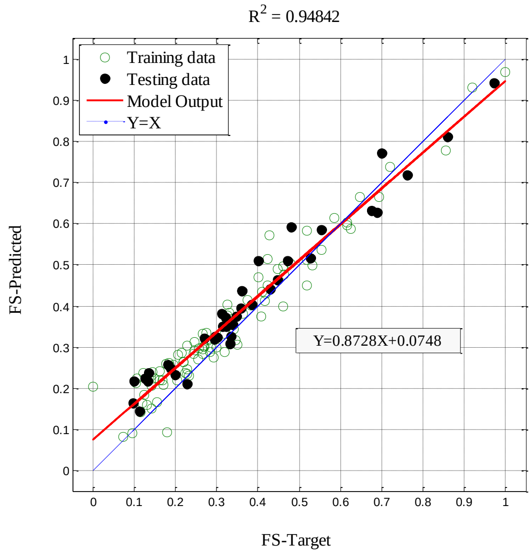

Figure 1. Performance of proposed F_s model based on classification concept

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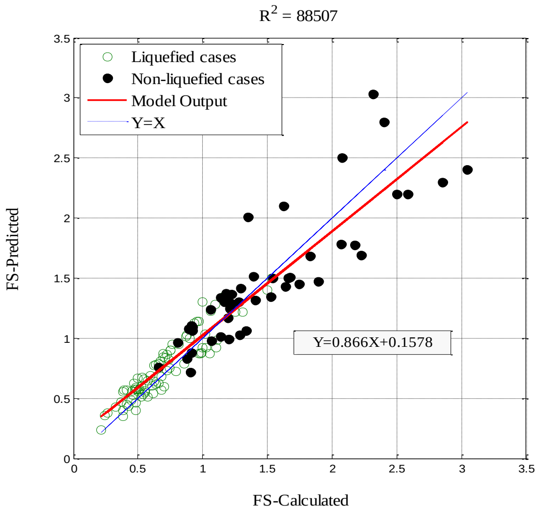

Figure 2. Comparison of the predicted model with calculated F_s based on the Idriss and Boulanger method (2010)

3. RELIABILITY ANALYSIS

Generally there is so much uncertainties in geotechnical engineering, specially in liquefaction analysis (e.g. errors due to distribution of measured data, systematic errors and human errors). So due to inability of the deterministic methods to taking into account errors in soil liquefaction potential analysis, they are not applicable to utilize. So the use of the reliability analysis is necessary[6].

The first step in the reliability analysis is to define a performance function. In the liquefaction potential evaluation the cyclic stress ratio (CSR) and the cyclic resistance ratio (CRR) are defined by Q and R respectively. The margin of safety, Z is defined as the difference between the resistance and the load. So the performance function of liquefaction potential assessment is presented by Eq. (3).

$$Z = R - Q \tag{3}$$

If Z<0, it indicates the failure mode and the occurrence of liquefaction. The dark area of the probability density function (PDF) of Z as shown in Figure 3 depicts the probability of liquefaction (P_L).

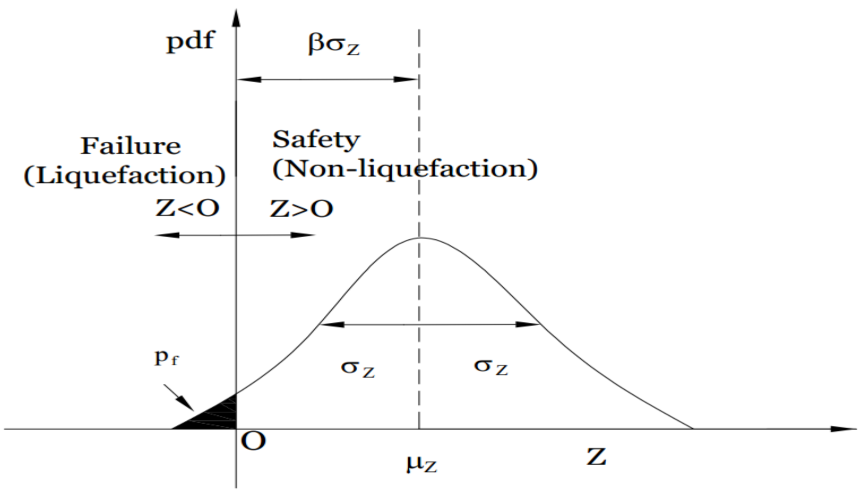

Figure 3. Probability density function of liquefaction performance function (modified from Baecher and Christian 2003)

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Due to some uncertainties in CRR and CSR, *R* and *Q* are treated as random variables and the reliability index (β) is defined as the inverse of the standard deviation as given in Eq. (4).

$$\beta = \frac{\mu_z}{\sigma_z} = \frac{\mu_R - \mu_Q}{\sqrt{\sigma_R^2 + \sigma_Q^2 - 2\rho_{RQ}\sigma_R\sigma_Q}} \tag{4}$$

where μ_R, μ_Q are respectively the mean values of *R* and *Q* and σ_R, σ_Q are respectively the standard deviations of *R* and *Q*, $\sigma^2{}_R$, $\sigma^2{}_Q$ are respectively the variances of *R* and *Q* and ρ_{RQ} is the correlation coefficient between *R* and *Q*.

If *R* and *Q* are random variables with normal distribution, then the performance function is also normally distributed. The probability of liquefaction (P_L) can be obtained through the notional failure probability concept, presented as Eq. (5).

$$p_f = P_L = P[Z \leq 0] == \Phi\left(-\frac{\mu_z}{\sigma_z}\right) = \Phi(\beta) = 1 - \Phi(\beta) \tag{5}$$

where Φ is the cumulative distribution function (CDF) for a standard normal variable.

Despite this method simplicity, it has significant drawbacks. For example when the failure function linearized around the average values of the random variables, this method estimate the different reliability index (β) by choosing the different functions for the same problem. For solving this problem, Hasofer and Lind (2000) defined a geometric interpretation of the reliability index (β), which is defined as the shortest distance between the limit state function and the origin in the standard variable space and they suggested to use a linear approximation of the failure surface at the design point.

Based on the Hasofer-Lind approach all the normal variables are transformed to their reduced form in standard normal space with zero mean and unit standard deviation. So *R* and *Q* can be shown as Eq. (6)-(7) as standard normal variables. The distance between the origin and the limit state line is presented as Eq. (8).

$$R' = \frac{R - \mu_R}{\sigma_R} \tag{6}$$

$$Q' = \frac{Q - \mu_Q}{\sigma_Q} \tag{7}$$

$$d = \frac{\mu_R - \mu_Q}{\sigma_R^2 + \sigma_Q^2} \tag{8}$$

Liquefaction performance function, *Z* depends on multiple variables such as $N_{1,60}$, FC, σ_v, σ_v', a_{max}, M_w. So the Eq. (3) can be presented as Eq. (9).

$$Z = R - Q = g(z) \tag{9}$$

where *z* is a vector of uncorrelated random variables $z=\{N_{1,60}, FC, \sigma_v, \sigma_v', a_{max}, M_w\}$.

Eq. (8) can be extended for six random variables, which are first converted to standard normal variables (z_i') as obtained in Eq. (10). So the reliability index (β) based on First-Order reliability method can be calculated by Eq. (11), which $g(z)=0$ is a constraint function.

$$d = \sqrt{z_1'^2 + z_2'^2 + \cdots + z_6'^2} = \sqrt{z'^T z'} \tag{10}$$

$$\beta = min\left(z'^T z'\right)^2 \qquad g(z) = 0 \tag{11}$$

Because of the lognormal distribution good fitting to the many geotechnical parameters, In this paper, each variable is assumed to follow lognormal distribution and the mean and standard deviation of equivalent normal variables can be calculated respectively by Eq. (12)-(13) [7].

$$\xi_i = \sqrt{\ln(1 + \delta_{zi}^2)} \tag{12}$$

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$$\lambda_i = \ln\mu_{zi} - 0.5\xi_i^2 \tag{13}$$

where ξ_i is the standard deviation of the equivalent normal variable, λ_i is the mean of the equivalent normal variable, μ_{zi} is the mean of the random variable z_i and δ_{zi} is the coefficient of variation of z_i.

Several numerical techniques can be used to minimize the reliability index (β). One of the new optimization methods is a hybrid of Particle Swarm Optimization and Genetic Algorithm (PSO-GA). This algorithm is depends only on the quality of the response variables, while most optimization methods need knowledge of derivatives of the objective function, which may not be easy to obtain derivatives in different optimization problems such as reliability problems. The benefits of using this approach is its easy mechanism to use by computer and in this study, simulation is performed by MATLAB2013a software.

4. PARAMETER UNCERTAINTY

The coefficients of correlation among six input variables are given in Table 2. Due to the lack of the consideration of the model uncertainty in the FORM analysis, the calculated β and the relating P_L through the notional concept are underestimated or overestimated. Then, Bayesian mapping function are utilized to make a relationship between F_s and P_L to interpret the probability of liquefaction occurrence. Figure 4 shows a difference between the notional concept and the Bayesian mapping function in β-P_L curves. The second curve is calibrated empirically with the field manifestations of databases, so it can be a good approximation of the true probability of liquefaction [8, 9, 10, 11, 12, 13].

Table 2- Coefficients of correlation of the input variables

Input Parameters	Input Parameters					
	$N_{1,60}$	FC	σ'_v	σ_v	a_{max}	M_w
$N_{1,60}$	1	0	0.3	0.3	0	0
FC	0	1	0	0	0	0
σ'_v	0.3	0	1	0.9	0	0
σ_v	0.3	0	0.9	1	0	0
a_{max}	0	0	0	0	1	0.9
M_w	0	0	0	0	0.9	1

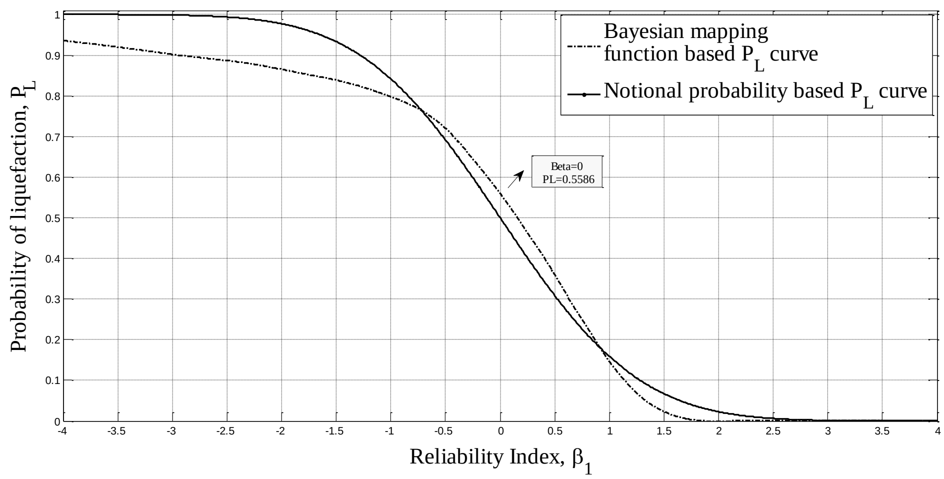

Figure 4. Estimated β-P_L curves obtained from the reliability analysis

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By comparing the Bayesian mapping functions obtained for each level of uncertainty, the effect of the parameter uncertainty levels are investigated. Figure 5 shows the effect of five levels of uncertainty (COV=0.10, 0.25, 0.40, 0.55, 0.70) of the $N_{1,60}$ on β-P_L curves. The results show that the cases between COV=0.1 and COV=0.25 the level of $N_{1,60}$ has a little effect on the β-P_L curve and the effect of the uncertainty becomes significant at a greater COV values.

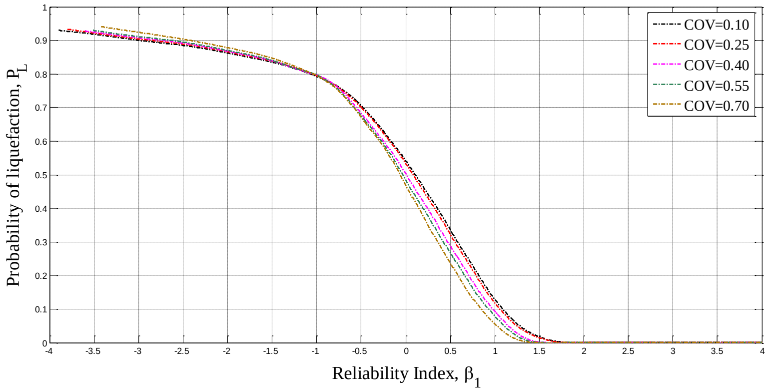

Figure 5. Effect of COV of $N_{1,60}$ on the Bayesian mapping function

Similarly, the effect of the uncertainty of *FC* for four different COV values (COV= 0.10, 0.75, 1.25, 2.00) are considered and as expected from the *Fs*-GEP model sensitivity analysis, the *FC* has not a great effect on the Bayesian curve as illustrated in figure 6.

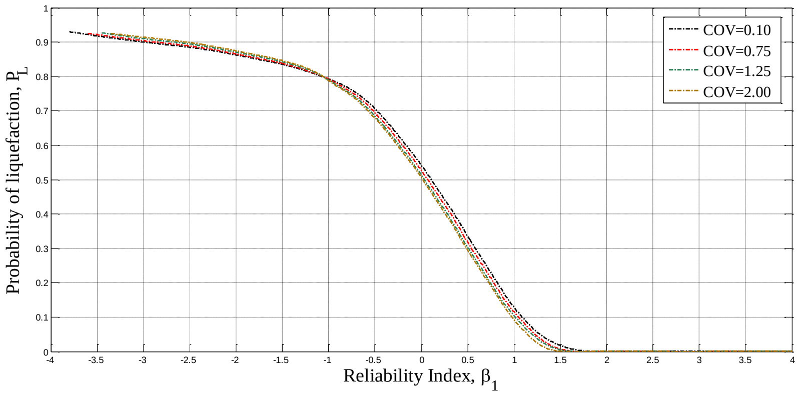

Figure 6. Effect of COV of *FC* on the Bayesian mapping function

To investigate the effect of the level of uncertainty of σ_v', four different scenarios of sensitivity analysis of COVs (0.05, 0.15, 0.25, 0.35) are performed. The results show that from COV=0.05 to 0.15 and COV=0.25-0.35 the level of σ_v' uncertainty has little effect on the β-P_L relationship but a little more difference is observed from COV=0.15 to COV=0.25, as shown in Figure 7.

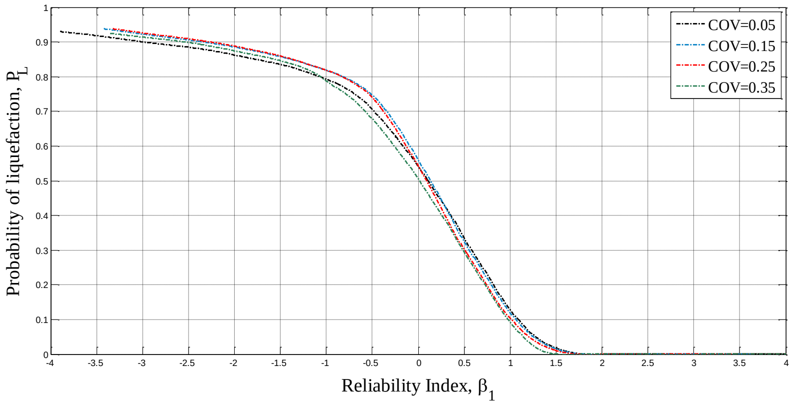

Figure 7. Effect of COV of σ_v' on the Bayesian mapping function

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Six different scenarios of sensitivity analyses for assessing the levels of uncertainty of a_{max} and M_w are studied. (COV a_{max}=0.1 & COV M_w=0.01, COV a_{max}=0.1 & COV M_w=0.02, COV a_{max}=0.2 & COV M_w=0.01, COV a_{max}=0.2 & COV M_w=0.02, COV a_{max}=0.3 & COV M_w=0.01, COV a_{max}=0.3 & COV M_w=0.02). By changing the COV of a_{max}, β-P_L curve has seen a significant effect but the effect of M_w uncertainty is not remarkable as indicated by Figure 8.

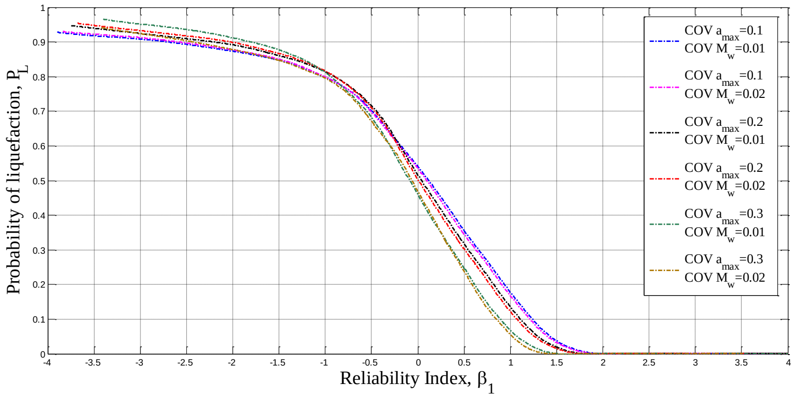

Figure 8. Effect of COVs of a_{max} *and* M_w on the Bayesian mapping function

5. CONCLUSIONS

In this paper, Gene Expression Programming (GEP) has been used as a tool to develop the factor of safety (F_s) model. The efficiency of the GEP model is indicated by comparing the results of the GEP model with the results calculated based on Idriss and Boulanger approach (2010). F_s model is utilized to form the constrained function.

First-Order Reliability Method (FORM) requires an optimization algorithm to locate the design point and to determine the reliability index (β). For this purpose, in this paper a hybrid of Particle Swarm Optimization and Genetic Algorithm (PSO-GA) in MATLAB2013a is utilized to minimize the objective function, then the notional probability of liquefaction (P_L) can be obtained through the FORM analysis. Also the Bayesian mapping function on the basis of Bayesian theory of conditional probability is used to establish a relationship between β and P_L to interpret the probability of liquefaction occurrence. It can be found that, due to lack of consideration of model uncertainty, calculated β values are not accurate and thus, the notional probability curve is not exactly correct. It can also be observed from the Bayesian mapping function that, where the β is equal to 0, P_L is equal to 0.5586, which is approximately near to P_L=0.5. The results indicate the robustness of this method. So Bayesian mapping function can be used for primary estimation of P_L of the databases in the absence of the parameter uncertainty. Finally, effect of the level of parameter uncertainty (levels of $N_{1,60}$, FC, σ_v', a_{max} and M_w uncertainty) is investigated. According to the β-P_L curves, can be concluded that, where the current parameter uncertainty differs remarkably from those assumed in the base analysis, the β-P_L mapping function are not applicable to use.

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