Implementation of Exponential Function for Isotropic Hardening of Metallic Foams

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Abstract

The metallic foams have found wide applications in industry due to their useful properties such as low density, high hardness, thermal and acoustic insulation and high permeability to gas. These materials are used for the production of light weight structures because of the low density. Due to their high energy absorption properties of porous materials in compression stress, metalic foams can be substitute with previous structure have been used before. The major applications of porous materials are in aerospace and automotive industries. Foams are widely used in industry so prediction of their behaviour is very important. In recent years, extensive researches have been done on the behaviour of these materials and so far several constitutive equations are presented for foams. There are two types of work hardening: isotropic hardening and kinematic hardening. A new constitutive equation for isotropic hardening foam is provided in this study which includes 3 parameters. These equations are integrated using return mapping algorithm and developed in MATLAB. Then coefficients of this equation are optimized using genetic algorithm and the experimental and numerical results have been compared. Four types of aluminum foams have been studied in this investigation including Alporas, Foaminal, Alulight and Cymat that are commercially available. Comparison between numerical results and experimental data shows that this new equation is able to predict the behaviour of foams with acceptable accuracy and there is good agreement between numerical and experimental curves.

Keywords: *Aluminum foam-Isotropic hardening-Return mapping algorithm-Genetic algorithm.*

Introduction

Porous material including metal foams have desirable mechanical properties such as low weight that leads to significant weight reduction compared to other options.

Due to foam applications in various industries, it's important to predict their behaviour to avoid failure. The first efforts to offer constitutive equations were performed in 1989 by Gibson et al [1]. This equation obtained from analysis of foam cells structure. This equation recognized as GAZT yield surface. In 1994 Schreyer et al [2] offer an anisotropic model with kinematics hardening. This model have kinematic hardening parameter in yield surface equation and defined an spherical yield surface in principal stress space. In 2000, Miller [3] presented constitutive equation that in specific amount of parameter, turne to GAZT yeild surface. This equation was inspiration from Von Mises yield surface which is the simplest and most commnly offered yield surface for metals. Deshpande and Fleck [4] in 2000, proposed an isotropic yield surface for metal foams. This yield surface was a self similar model.

Aluminium foams are studied in this investigation which include Alporas, Foaminal, Alulight and Cymat foams. A micrograph cell structure of Alporas foam is shown in Figure1.

Fig.1:Cell structure of Alporas foam[5]

Theory

The most common equation for foams, is constitutive equation presented by Deshpande and Fleck. There are two types of work hardening and the work hardening used in this investigation, is isotropic hardening. In isotropic hardening, yiled surface will be expanded simillar to the original shape. Isotropic hardening is shown in figure 2.

Fig. 2: Foam yield surface in isotropic hardening [6]

Equation of yield surface is according to equation 1.

$$F = \hat{\sigma} - \sigma_{y0} \tag{1}$$

In equation 1, equivalent stress is defined according to equation 2.

$$\hat{\sigma}^2 = \frac{1}{1+\left(\frac{\alpha}{3}\right)^2}\left[\sigma_e^2 + \alpha^2 \sigma_m^2\right] \tag{2}$$

In equation 2, Von Mises and hydrostatic stress are according to equation 3 and 4.

$$\sigma_e = \sqrt{\frac{3}{2}\left(\boldsymbol{\sigma}^{dev} : \boldsymbol{\sigma}^{dev}\right)} \tag{3}$$

$$\sigma_m = \frac{1}{3} tr(\boldsymbol{\sigma}) \tag{4}$$

To illustrate the effect of work hardening in finite element software, usually a table of stress versus

1. Student

2.Assistant Professor, 03137934021, hasanpour@eng.ui.ac.ir plastic strain is used. In this research, tried to introduce a function to apply work hardening instead of this table.

Figure 3 shows a Von Mises yield surface in plane stress state. For facilitate in displaying, plane stress state has been chosen. A step forward in time takes the updated stresses outside of the yield surface. A trial stress increment is chosen which again takes the updated stresses $\boldsymbol{\sigma}_{t+\Delta t}^{tr}$ outside of the yield surface. The stress is then updated with a plastic correction bring it back on to the yield surface at time $t+\Delta t$.

This method is called return mapping algorithm. In this method, all values must be specified at the end of the step $t+\Delta t$ [7].

Fig.3:Integration by return mapping algorithm [7]

In return mapping algorithm, difference between trial stress and stress on the yield surface in each step is small. For this reason, the following equations must be satisfied simultaneously. These equations are known as the residual equations.

$$\boldsymbol{r_\sigma} = \boldsymbol{\sigma} - \boldsymbol{\sigma}_{n+1} \tag{5}$$

$$r_p = p - p_{n+1} \tag{6}$$

$$r_F = F(\boldsymbol{\sigma}, p) \tag{7}$$

In these equations, 'p' is the work hardening variable. For solving the equations by return mapping algorithm, Newton-Raphson method is used. By putting unknown parameters in the vector $\boldsymbol{v}$, and also putting residual functions in vector function $\boldsymbol{m}(\boldsymbol{v})$, equations 8 and 9 are obtained. So, final state of stress and work hardening is the root of the vector function '$\boldsymbol{m}$', according to equation 10.

$$\boldsymbol{v} = \begin{Bmatrix} \boldsymbol{\sigma} \\ p \\ d\lambda \end{Bmatrix} \tag{8}$$

$$\boldsymbol{m}(\boldsymbol{v}) = \begin{Bmatrix} \boldsymbol{r_\sigma}(\boldsymbol{v}) \\ r_p(\boldsymbol{v}) \\ r_F(\boldsymbol{v}) \end{Bmatrix} \tag{9}$$

$$\boldsymbol{v}^{(i+1)} = \boldsymbol{v}^{(i)} - \left[\frac{\partial \boldsymbol{m}}{\partial \boldsymbol{v}} \boldsymbol{v}(i)\right]^{-1} \boldsymbol{m}(\boldsymbol{v}(i)) \tag{10}$$

Jacobian matrix $\frac{\partial \boldsymbol{m}}{\partial \boldsymbol{v}}$ is achieved by partial derivative of residual functions in the form of equation 11.

$$\frac{\partial \boldsymbol{m}}{\partial \boldsymbol{v}} = \begin{bmatrix} \frac{\partial \boldsymbol{r_\sigma}}{\partial \boldsymbol{\sigma}} & \frac{\partial \boldsymbol{r_\sigma}}{\partial p} & \frac{\partial \boldsymbol{r_\sigma}}{\partial d\lambda} \\ \frac{\partial r_p}{\partial \boldsymbol{\sigma}} & \frac{\partial r_p}{\partial p} & \frac{\partial r_p}{\partial d\lambda} \\ \frac{\partial r_F}{\partial \boldsymbol{\sigma}} & \frac{d r_F}{\partial p} & \frac{\partial r_F}{\partial d\lambda} \end{bmatrix} \tag{11}$$

In Newton-Raphson method, iteration continues until the set of residual equations become less than a predetermined amount.

A typical stress–strain curve for metal foams shows three distinct regions, illustrated in Figure 4. There is (1) an initial linear elastic region. The region of (2) is known as the plateau region, in which the stress is almost constant for a large range of strain. Compression of the collapsed cell walls occurs in the densification region (3) [8].

Fig.4: Typical stress strain curve of a metal foam[8]

To insert the effect of work hardening, various functions can be used. The parameters of these functions are determined depending on the shape of the stress-strain curve. With considering work hardening function, yield function is according to equation 12.

$$F = \hat{\sigma} - \sigma_{y0} - hp \tag{12}$$

'h' which determining the function of work hardening, must be specified in such a way that the points obtained by equations solving of return mapping algorithm, placed on the stress–strain experimental diagram with a good agreement. Here, by assuming an uniaxial compression load and also work hardening function as an exponential function, the relevant parameters are determined. In the equation 13 the proposed function for work hardening has been shown.

$$h = h_0 + \alpha_1 . e^{(\alpha_2 . \varepsilon)} \tag{13}$$

Adaptation of experimental and numerical stress-strain curves, indicates that offered function can predict foam behaviour as well. Genetic algorithm is one of the efficient methods for extracting the coefficients. In this algorithm, coefficients of equation are determined in such way that difference between stress in experimental and numerical state, be least amount. In this issue, the proposed objective function is the sum of the least square. In other words, squared differences between experimental diagram points and Points obtained from genetic algorithm, should be minimal.

$$S = \sum_{i=1}^{n} \left(\sigma_{\exp} - \sigma_{num}\right)^2 \tag{14}$$

To do this ,an uniaxial compression test specimen is considered that its experimental stress-strain curve is available. Since this test is uniaxial, all values are along axis 1.

In uniaxial loading, equivalent stress, Von Mises stress and hydrostatic stress are simplified according to equation 15-17.

$$\sigma_e = \sigma \tag{15}$$

$$\sigma_m = \frac{1}{3}\sigma \tag{16}$$

$$\hat{\sigma} = \sigma \tag{17}$$

According to mentioned equation, residual equation of return mapping algorithm, are expressed according to equation 18-20.

$$r_\sigma = \sigma - E\varepsilon + d\lambda E\sqrt{1+\frac{\beta^2}{9}} \tag{18}$$

$$r_p = p - p_n - d\lambda\left(\frac{2}{3}\left(1+\frac{\beta^2}{9}\right)\right)^{1/2} \tag{19}$$

$$r_F = \sigma - hp - \sigma_{y0} \tag{20}$$

Solving equations 18-20, and obtaining the coefficient of work hardening function in one dimension, can predict the behaviour of various metallic foams in uniaxial compression and other loading conditions. Properties of some metallic foams, are listed in table 1.

Table 1: Foams properties

Foam	$E(MPa)$	$\sigma_{y0}(MPa)$
Foaminal	206.4	5.1219
Alulight	235.61	13.43
Alporas	48.87	2.59
Cymat	74.33	4.46

Modulus of elasticity for foam obtained from the slope of elastic part of stress-strain curve as follow.

$$E = \frac{\sigma_{y0}}{\varepsilon_{y0}} \tag{21}$$

Results and Discussion

After solving of genetic algorithm, the optimized values for parameters are listed in table 2. The curve of stress-strain in experimental and numerical state, are compared in figures 5-8.

Table 2: Optimized values of foam hardening coefficients

Foam	S	α_1	α_2	h_0
Foaminal	0.04	0.228	3.662	1863.8
Alulight	0.0033	0.250	4.053	7668.3
Alporas	0.15	0.002	8.497	5185.5
Cymat	0.07	0.111	2.956	5883.2

Fig.5: Comparison of experimental and numerical stress-strain curve for Cymat foam [9]

Fig.6: Comparison of experimental and numerical stress-strain curve for Foaminal foam [10]

Fig.7: Comparison of experimental and numerical stress-strain curve for Alulight foam [11]

Fig.8: Comparison of experimental and numerical stress-strain curve for Alporas foam [12]

Conclusions

Region 1, is linear elastic region, and stress is in a linear relation with strain.

Comparison of stress-strain curves show that the exponential function can properly predict work hardening behavior of foam. Results show that work hardening function follows the region 2 of the stress-strain curve and there is good agreement between experimental and numerical results. It was also observed that this function predict the stress raise in region 3 of diagram. Check the value of error determined that difference between numerical and experimental diagram is negligible and these two diagram are well coincide. Another point is the dependence of the work hardening to stress.This show that after arrival to plastic phase, work hardening is function of both plastic and elastic parts of strain.

Nomenclatures

E	Modulus of elasticity
F	Yield function
$\boldsymbol{m}(\boldsymbol{v})$	Vector function
p	Hardening variable
r_σ	Stress residual
r_p	Hardening variable residual
r_F	Yield function residual
S	Objective function
$\boldsymbol{v}$	Unknown vector variable

Greek symbols

α	Shape of the yield surface
β	Shape of the potential surface
ε	Strain
σ_e	Von Mises stress
σ_m	Hydrostatic stress
$\sigma_{\exp}$	Experimental stress
σ_{num}	Numerical stress
$\boldsymbol{\sigma}^{dev}$	Deviatoric stress
σ_{y0}	Yield stress
$\hat{\sigma}$	Equivalent stress

σ^{tr} Trial stress

$d\lambda$ Plastic multiplier

Figures:

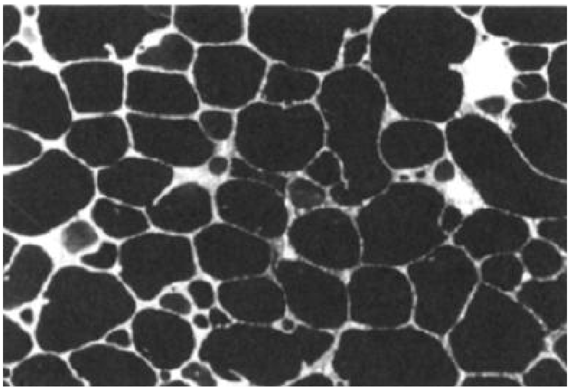

Figure1.Cell structure of Alporasfoam[5]

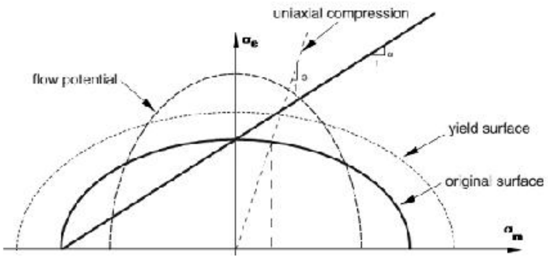

Fig. 2: Foam yield surface in isotropic hardening [6]

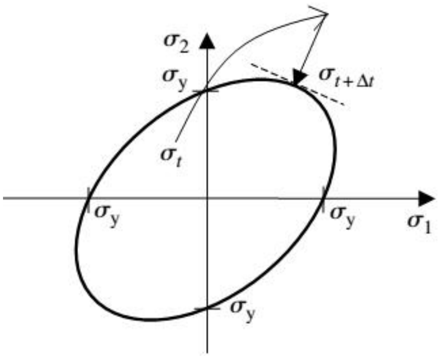

Fig.3:Integration by return mapping algorithm [7]

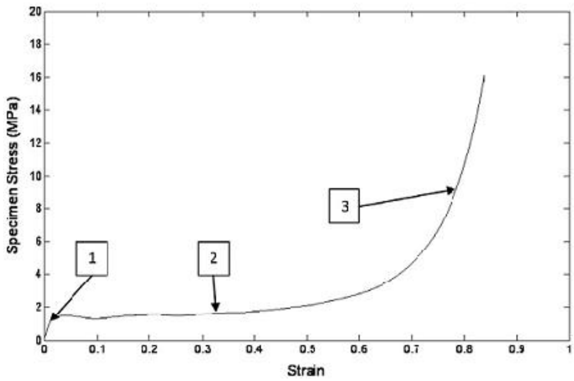

Fig.4: Typical stress strain curve of a metal foam[8]

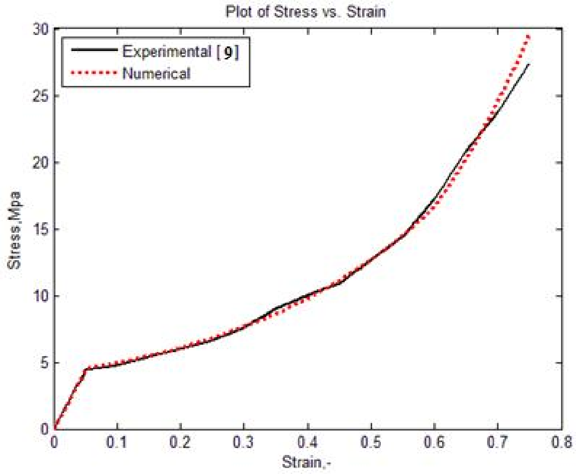

Fig.5: Comparison of experimental and numerical stress-strain curve for Cymat foam [9]

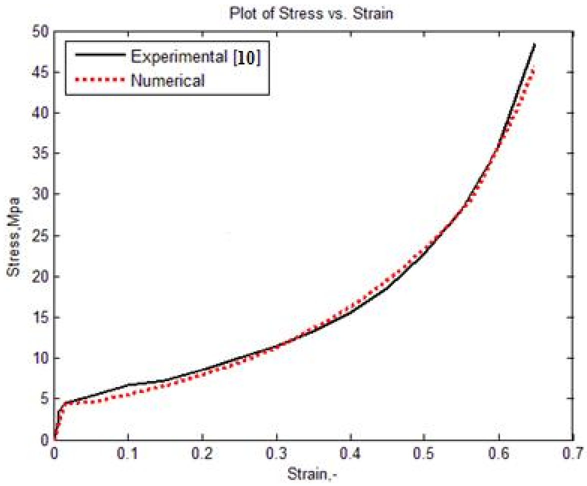

Fig.6: Comparison of experimental and numerical stress-strain curve for Foaminal foam [10]

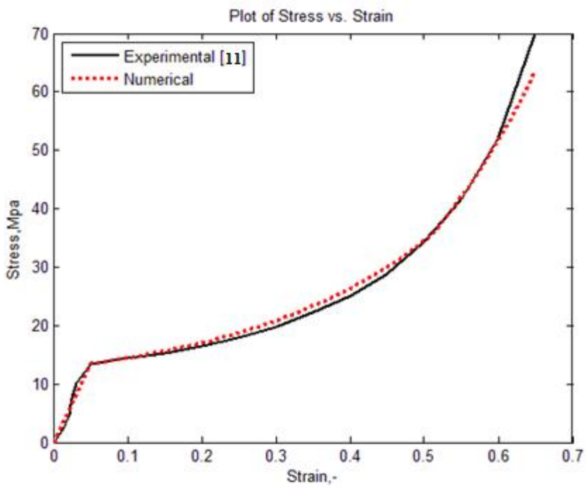

Fig.7: Comparison of experimental and numerical stress-strain curve for Alulight foam [11]

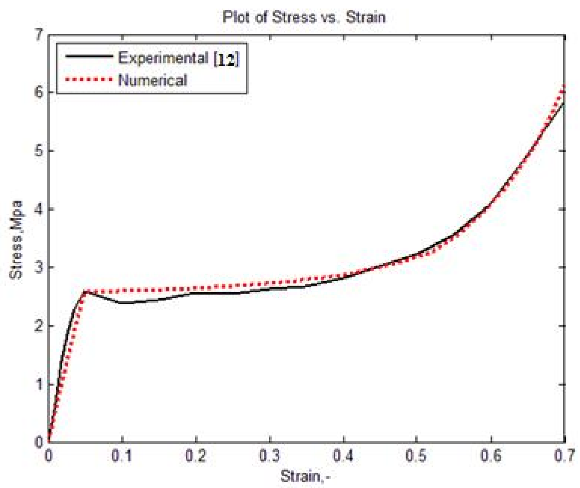

Fig.8: Comparison of experimental and numerical stress-strain curve for Alporas foam [12]

References

[1] Gibson L. J. Ashby M. F. Zhang J. and Triantafillou T. C., Failure surfaces for cellular materials under multiaxial loads—I. Modelling,*International Journal of Mechanical Sciences,* Vol. 31, No. 9,1989, pp. 635-663.

[2] Schreyer H. L. Zuo Q. H. and Maji A. K.,Anisotropic plasticity model for foams and honeycombs,*Journal of engineering mechanics,* Vol.120, No. 9, 1994, pp. 1913-1930.

[3] Miller R. E., A continuum plasticity model for the constitutive and indentation behaviour of foamed metals, *International Journal of Mechanical Sciences,*Vol. 42, No. 4, 2000, pp. 729–754.

[4] Deshpande V. S. Fleck N. A., Isotropic constitutive models for metallic foams,*Journal of the Mechanics and Physics of Solids*, Vol. 48, No. 6, 2000, pp. 1253-1283.

[5] Ashby M. F. Evans A. Fleck N. A. Gibson L. J. Hutchinson J. W. and Wadley H. N. G., *Metal foams: a design guide*, Elsevier, 2000.

[6] Systèmes D., Abaqus 6.12 Theory Manual. *Dassault Systèmes Simulia Corp, Providence, Rhode Island,* 2012.

[7] Dunne F.Petrinic N., *Introduction to computational plasticity*, andOxford University Press New York, 2005.

[8] Merrett R. P. Langdon G. S. and Theobald M. D,The blast and impact loading of aluminium foam, *Materials & Design*, Vol. 44,2013, pp. 311-319.

[9] Liu H.Cao Z. K. Luo H. J. Shi J. C. and Yao G. C., Performance of closed-cell aluminum foams subjected to impact loading, *Materials Science and Engineering: A*, 570, 2013, pp. 27-31.

[10] Peroni L. Avalle M. and Peroni M., The mechanical behaviour of aluminium foam structures in different loading conditions, *International journal of impact engineering*, Vol. 35, No. 7, 2008, pp. 644-658.

[11] McCullough K. Y. Fleck N. A. and Ashby M. F., Uniaxial stress–strain behaviour of aluminium alloy foams, *Acta Materialia*, Vol. 47, No. 8, 1999, pp. 2323-2330.

[12] Kadkhodapour J. and Raeisi S., Micro–macro investigation of deformation and failure in closed-cell aluminum foams, *Computational Materials Science 83*, 2014, pp. 137-148.