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Multiplicity of steady solutions in lid-driven cavity flows using an Incompressible Generalized Lattice Boltzmann Method

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Abstract

This work is concerned with the computation of two-sided lid-driven square cavity flows by the Lattice Boltzmann Method (LBM) to obtain multiple stable solutions. The velocity field is solved by an incompressible generalized lattice Boltzmann method. In the two-sided square cavity two of the walls move with equal velocity move in such a way that parallel walls move in opposite directions with the same velocity. Conventional numerical solutions show that the symmetric solutions exist for all Reynolds numbers for all the geometries, whereas multiplicity of stable states exist only above certain critical Reynolds numbers. Here we demonstrate that Lattice Boltzmann method can be effectively used to capture multiple steady solutions for all the aforesaid geometries. The strategy employed to obtain these solutions is also described. At low Reynolds numbers, the resulting flow field is symmetric with respect to one of the cavity diagonals for the two-sided driven cavity, while it is symmetric with respect to both cavity diagonals for the four-sided driven cavity. It is found that for parallel motion of the walls, there appears a pair of counter-rotating secondary vortices of equal size near the center of a wall. Because of symmetry, this pair of counter-rotating vortices has similar shapes and their detailed study as to how they grow with increasing Reynolds number has not yet been made by lattice Boltzmann Method.

Keywords: *Lattice Boltzmann Method - D2Q9 model - Two-sided square cavity - parallel and antiparallel.*

Introduction

In recent years, lattice Boltzmann methods have become a powerful numerical method for simulating fluid flow in different types of flow fields as porous media, multiphase flows, microfluidics and nanofluids [1-5].

The lid-driven cavity flow is not only technically important but also of great scientific interest because it displays almost all fluid mechanical phenomena in the simplest of geometrical settings. The classical cavity problem has attracted considerable attention because its flow configuration is relevant to many industrial applications and academic research [6–8]. It is known that cavity flows arise in applications such as short-dwellcoating, drug-reducing riblets in aerodynamics, removal of species from structured surfaces, mixing and flow in drying devices.

A number of experimental and numerical studies have been conducted to investigate the flow field of a single-sided lid-driven cavity flow in the last several decades. The features of the single-sided lid-driven cavity flow consist of a large primary eddy and secondary corner eddies. Several flow characteristics like flow instability, corner eddies and transition to turbulence can be observed in this system. Conventional numerical solutions reveal that in a single-sided cavity flow beyond the critical Reynolds number, Hopf bifurcation takes place with the steady-flow solution becoming unstable.

The single-sided lid-driven cavity flow problem was extended to two-sided lid-driven cavity by Kuhlmann and other investigators [9–14] and they have done several experiments on the two-sided lid-driven cavity with various spanwise aspect ratios. They numerically simulated the rectangular cavity flow for parallel and antiparallel motion of two of the walls and showed that a plethora of vortex patterns can be generated with different aspect ratios and directions of motion of the walls. Kuhlmann et al. [9,10] extended the one-sided lid driven cavity problem to a two-sided problem, where the flow is driven by the parallel or antiparallel motion of two facing walls. The facing walls could be either the left and right walls or the upper and lower walls. At low Reynolds number, the flow consists of separate co- or counter-rotating primary vortices that form adjacent to each moving wall. At higher Reynolds numbers, instabilities arise in the flow due to the interaction between the two primary vortices. Moreover, their results showed that multiple flow solutions may exist, depending on the cavity aspect ratio and the value of the Reynolds number.

Albensoeder et al. [11] were among the first to investigate the nonlinear regime and find multiple two-dimensional steady states in rectangular two-sided lid-driven cavities. They have found five and seven flow states in parallel and antiparallel motion respectively. Luo and Yang [15] numerically investigated flow bifurcation with and without heat transfer in a two-sided lid-driven rectangular cavity. More recently, the multiplicity of flow states induced by the motion of two sided non-facing lid-driven square cavity flow and four-sided lid-driven cavity flow have been investigated by Wabha [16]. He found the critical Reynolds numbers of 1073 for the two-sided non-facing lid-driven square cavity and 129 for the four-sided lid-driven square cavity, beyond which it is possible for multiple steady states to exist.

Lattice Boltzmann Method

An incompressible generalized lattice Boltzmann method, in that the collision takes place in the moment

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space $\Re$ spanned by the vectors while the streaming step takes place in velocity space spanned by vectors f has been used for simulation of fluid flow. This method can be written as in the following two steps

$$\overline{f}_i(x,t) = f_i(x,t) - M^{-1}S[m_i(x,t) - m_i^{eq}(x,t)] + F_i \quad (1)$$

$$f_i(x+\xi_i\delta t, t+\delta t) = \overline{f}_i(x,t) \quad (2)$$

Where $f_i(x,t)$ and $\overline{f}_i(x,t)$ signify the pre-collision and post-collision states of the particle distribution functions, respectively. M is the one-to-one and linear transformation, by which the vectors in the velocity space are mapped to the vectors in the moment space, and vice versa as

$$|m\rangle = M|f\rangle \quad (3)$$

$$|f\rangle = M^{-1}|m\rangle \quad (4)$$

This transformation matrix M is easily determined using the Gram–Schmidt orthogonalization procedure to monomials of Cartesian components of the discrete velocities, i.e. $\left\{C_{\alpha x}^{m}\ C_{\alpha y}^{n}\ \mid m,n \geq 0;\ m,n \ \text{integer}\right\}$ in two dimensional symmetrical domains and is defined as follows,

$$M = \begin{pmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & -1 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 0 & -1 & 1 & 1 & -1 & -1 \\ -4 & -1 & -1 & -1 & -1 & 2 & 2 & 2 & 2 \\ 0 & 1 & -1 & 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & -1 \\ 0 & -2 & 0 & 2 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & -2 & 0 & 2 & 1 & 1 & -1 & -1 \\ 4 & -2 & -2 & -2 & -2 & 1 & 1 & 1 & 1 \end{pmatrix} \quad (5)$$

and the diagonal relaxation matrix S, is given as,

$$S = diag\,(0 \quad 0 \quad 0 \quad s_3 \quad s_4 \quad s_5 \quad s_6 \quad s_7 \quad s_8) \quad (6)$$

where parameters $s_i (i = 0,1,\ldots,7,8)$ are easily obtained using the Chapman–Enskog expansion procedure and the linear stability analysis. To satisfy the stability condition, the relaxation rates must satisfy this inequality, $1 < s_i < 2$ The relaxation rates pertaining to the seventh and eighth directions have to be equal and can be calculated in the following form,

$$s_7 = s_8 = \frac{1}{3\upsilon + 0.5} \quad (7)$$

Where υ is the kinematic viscosity.

The quasiequilibria, to which the moments are relaxed, can be easily obtained through the optimizations of the isotropy and Galilean invariance of the method. For the D2Q9 model, these equilibria are defined as,

$$\begin{aligned} m_3^{eq} &= 6p + 3(u_x^2 + u_y^2) \\ m_4^{eq} &= -9p - 3(u_x^2 + u_y^2) \\ m_5^{eq} &= -u_x \\ m_6^{eq} &= -u_y \\ m_7^{eq} &= u_x^2 - u_y^2 \\ m_8^{eq} &= u_x u_y, \end{aligned} \quad (8)$$

Where p is pressure, u_x and u_y are the horizontal and vertical velocity components in the lattice domain, respectively. These quantities can be calculated as

$$p = \frac{c_s^2}{1-\omega_0}\left[\sum_{i=1}^{8} f_i - \frac{3}{2}\omega_0 |\mathbf{u}|^2\right] \quad (9)$$

$$\mathbf{u} = \sum_{i=1}^{8} f_i e_i \quad (10)$$

Where ω_0 is equal to 4/9.

Code validation and grid independence

It is shown on table 1 that with increasing the grid density from (101×101) to (201×201), the relative change in the value of the stream function at the center of the primary vortex is less than 1%, confirming that computed results on the (101×101) grid are indeed grid independent. Also to validate the present numerical method, the LBM code is used to compute the single lid-driven flow in a square cavity on a (101×101) lattice. A lid velocity of U = 0.1 is considered in this work. Fig. 1-(a) depicts the streamline pattern at Re = 1000 obtained through LBM, which closely resembles those given by Ghia et al. [7]. Fig. 1-(b) show the comparison of steady-state u-velocity profile along a vertical line and v velocity profile along a horizontal line passing through the geometric center of the cavity at Re = 1000. It is observed that the agreement between the present LBM results and Ghia's work used for comparison is excellent. Thus the present LBM code stands validated.

Table 1: Grid study: Properties of primary vortex for onesided lid driven cavity flow (Re = 1000)

Reference	Grid	x	y
Present work	51×51	0.539	0.573
Present work	101×101	0.535	0.569
Present work	201×201	0.533	0.567
Ghia et al. [7]	129×129	0.531	0.562

Results and discussions

Problem statement for two-sided parallel square cavity flow

The boundary conditions for the parallel wall motion are shown in Figure 2. In the parallel motion we consider, both the upper and lower plates move from left to right in the *x* direction with the same velocity.

Figure 3 shows the streamline patterns for various Reynolds numbers on a 101×101 lattice structure. Expectedly, the streamlines are found to be symmetrical with respect to the horizontal centerline. Figure 3(a) shows the streamline pattern for *Re* = 100. Two counter-rotating primary vortices symmetrical to each other are seen to form with a `free' shear layer in between. At this Reynolds number the primary vortex cores are seen to be somewhat away from the centers of the top and the bottom halves of the cavity towards the right hand top and right hand bottom corners respectively. Figure 3(b) shows the streamline pattern for *Re* = 400. At this Reynolds number is seen also a pair of counter-rotating secondary vortices symmetrically placed about the horizontal centerline near the centers of the right wall. Figures 3(c) and 3(d) show the streamline patterns for *Re* = 1000 and *Re* = 2000 respectively. It is seen that with the increase in Reynolds number the primary vortex cores move towards the centers of the top and bottom halves of the cavity and the secondary vortex pair grow in size. At all the Reynolds numbers the counter-rotating pairs of primary and secondary vortices maintain their symmetry about the horizontal centerline

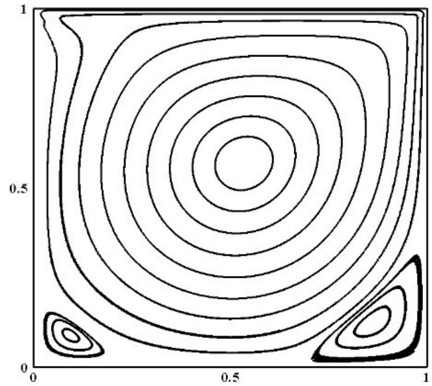
Figure 4 shows the comparison for horizontal velocity profiles along vertical lines passing through different points of the cavity for various Reynolds numbers.

Table 1: locations of the vortices for parallel wallmotion

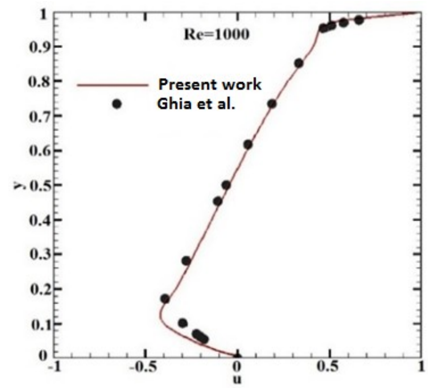
Re	TPV		BPV		TSV		BSV	
	x	y	x	y	x	y	x	y
100	0.61	0.79	0.61	0.20	-	-	-	-
400	0.58	0.75	0.58	0.23	0.98	0.52	0.98	0.465
1000	0.53	0.75	0.53	0.24	0.95	0.53	0.95	0.469
2000	0.51	0.75	0.51	0.24	0.93	0.53	0.93	0.458

Conclusion

In this work a relatively unexplored flow configuration in a two-sided lid-driven square cavity is computed with the LBM. The velocity field is solved by an incompressible generalized lattice Boltzmann method. In the case of parallel wall motion, besides two primary vortices, there also appears a pair of counter-rotating secondary vortices symmetrically placed about the centerline parallel to the motion of the walls. About this centerline also appears a 'free' shear layer with the increase in Reynolds number.



(a)



(b)

Fig. 1: Code validation: (a) Streamline pattern for the single-sided lid-driven cavity flow, (b) u-velocity along vertical center line (Re = 1000).

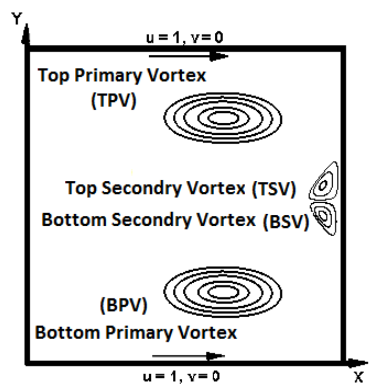


Fig. 2: Geometry and boundary conditions of the Two-SidedLid-Driven Cavity for parallel wall motion

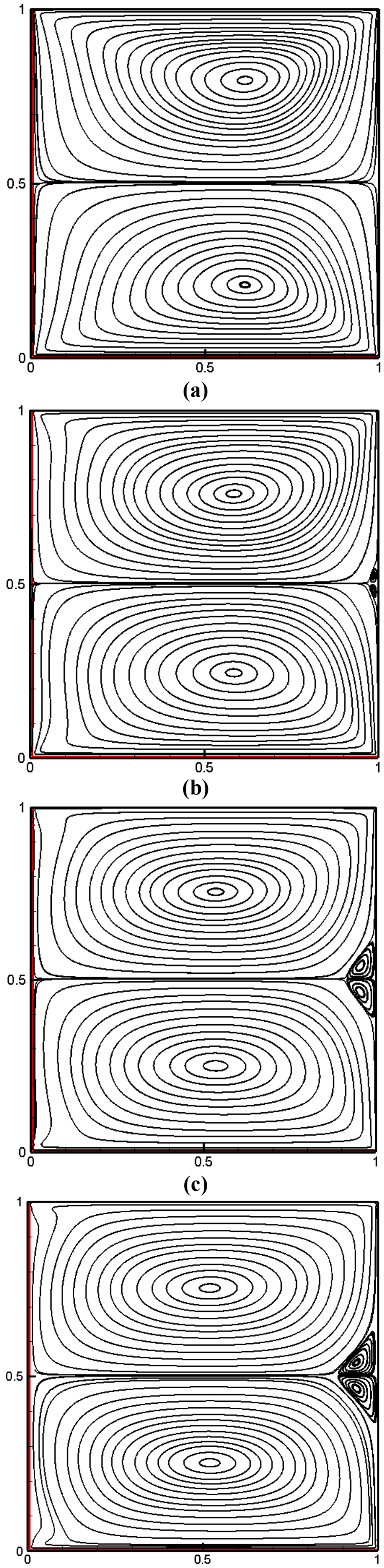
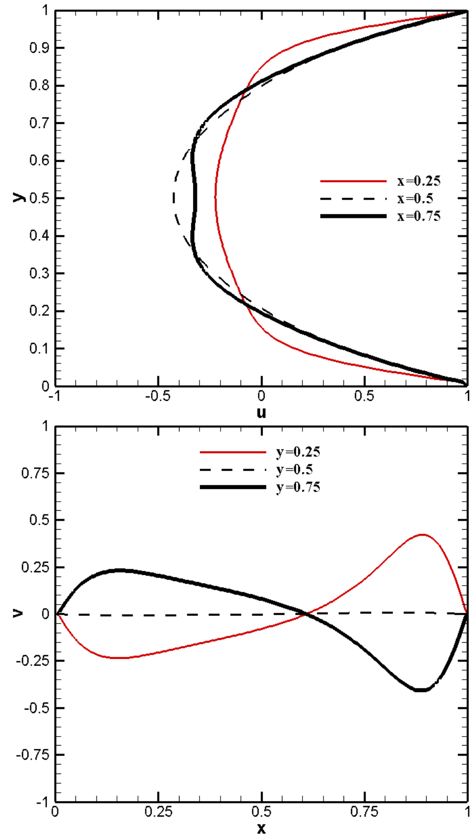
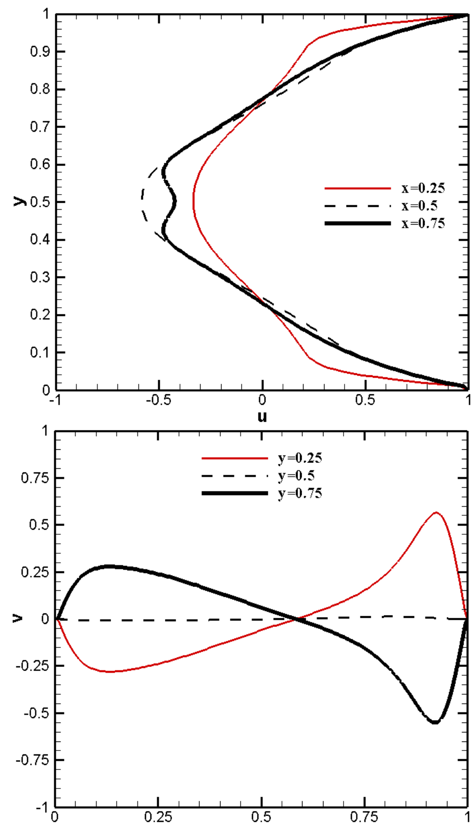


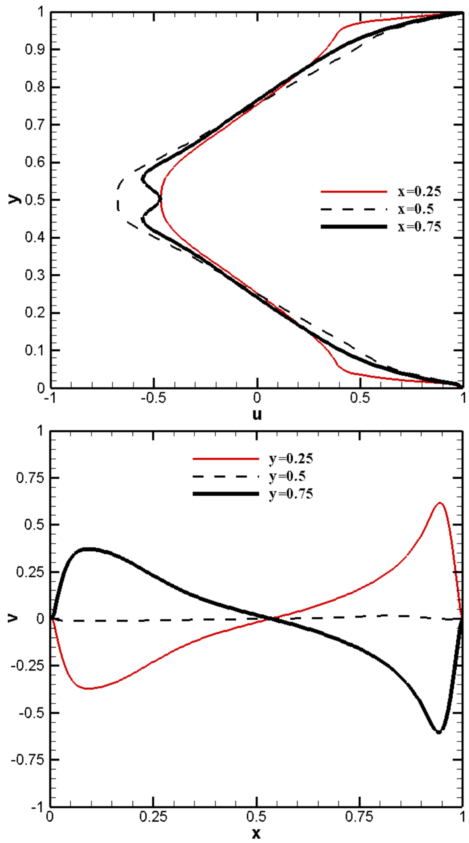
Fig. 3: Streamline pattern for parallel wall motion at (a) *Re* = 100 (b) *Re* = 400(c) *Re* = 1000 and (d) *Re* = 2000



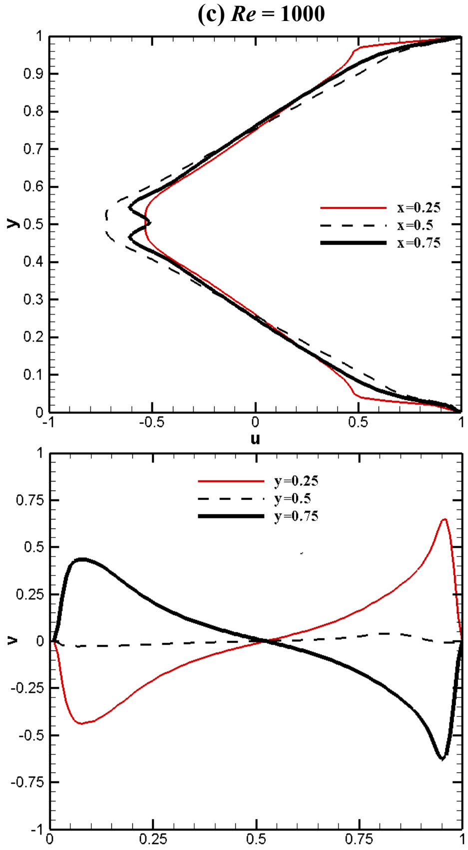
(a) *Re* = 100



(b) *Re* = 400



(c) *Re* = 1000



(d) *Re* = 2000

Fig. 4:horizontal velocity *u* along vertical lines (*x*=0.25, 0.50, 0.75) and vertical velocity *v* along horizontal lines (*y*=0.25, 0.50, 0.75) for different Reynolds numbers

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