2nd International Conference on
Modern Technologies in Sciences
13th March 2019

دومین کنفرانس بین المللی
فناوری های نوین در علوم
۲۲ اسفند ماه ۱۳۹۷

SURFACE ENERGY EFFECTS ON FREQUENCY ANALYSIS OF DW PIEZOELECTRIC CYLINDRICAL NANOSHELL

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ABSTRACT

In this paper, vibration analysis of double walled piezoelectric cylindrical nanoshell (DWPECNS) subjected to Visco-Pasternak medium and van der Waals force is investigated using Gurtin–Murdoch surface/interface theory, Donnell's theory, Hamilton's and also the assumed mode method. The effects of the surface energy and length of nanoshell and piezoelectric layer, boundary condition and van der Waals (vdW) force on the natural frequencies of DWPECNS is studied. Also, the results are shown that in the nanoscale system is considered surface effects (SE) without considering the surface density (SD), we will have the maximum frequency. As a result, controlling the frequency of the system in this case is essential and it is quite clear that considering the effects of the surface energy (SE) will have a remarkably effect in the natural frequency of the piezo-viscoelastic nanoshell.

Keywords: Double walled piezoelectric nanoshell, Surface elasticity, Gurtin–Murdoch surface/interface theory, Visco-Pasternak medium, Donnell's theory, van der Waals force.

1. INTRODUCTION

Recently, investigations of vibration analysis on the electromechanical characteristics of piezoelectric structures, especially the piezoelectric nano-sized shell are attracting worldwide attention [1]. Since the classical continuum theory cannot predict the surface and the size dependent response of nano-structures, some non-classical continuum theories such as the electro-elastic surface/interface theory expanded from Gurtin-Murdoch elasticity theory has been used to analyze the surface and the size dependent vibration of piezoelectric nano-structures [2, 3]. Recently, buckling behavior of the piezoelectric functionally graded nano-shell is investigated by Zhu et al. using Electro-elastic surface/interface theory [4]. Size-dependent shear deformable shell model and molecular dynamics simulation for axial instability analysis of silicon nanoshells are investigated by Sahmani et al. with the assistance of a perturbation-based solution methodology [5]. And also, exact solution for the vibrations of cylindrical nanoshells considering surface energy effect are investigated by Rouhi et al. [6]. In the present study, Gurtin–Murdoch surface elasticity with Donnell's theory is used for vibration analysis of double walled piezoelectric cylindrical nanoshell (DWPECNS) subjected to visco-Pasternak medium. The effects of the surface energy, length and thickness of nanoshell and piezoelectric layer, boundary condition, van der Waals (vdW) force and Visco-Pasternak effects on the undamped and damped natural frequencies of piezo-viscoelastic cylindrical nanoshell is studied with arbitrary boundary conditions.

2. PROBLEM FORMULATION AND GOVERNING EQUATIONS

A double walled piezoelectric cylindrical nanoshell (DWPECNS) embedded with a piezoelectric layer in outer layer and visco-Pasternak medium shown in Figure 1. The geometrical parameters of the cylindrical shell, are the length L, the outer layer with the mid-surface radius R_2 as the second layer with thickness $2h_{N2}$ and coated by piezoelectric layer of thickness h_{p2} and also the inner layer with the mid-surface radius R_1 as the first layer and thickness $2h_{N1}$.

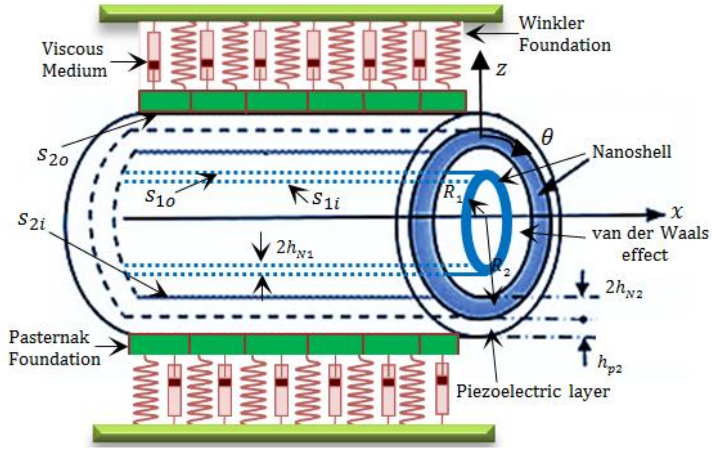

Fig. 1. *A double walled piezo-viscoelastic cylindrical nanoshell (DWPVCNS)*

2.1. Non- classical Shell theory and Governing equations

In the nano-shell and the piezoelectric layer, the constitutive relation can be expressed as [7, 8];

$$\begin{Bmatrix}\sigma_{xxN}\\ \sigma_{\theta\theta N}\\ \tau_{x\theta N}\end{Bmatrix}=\begin{bmatrix}C_{11N} & C_{12N} & 0\\ C_{21N} & C_{22N} & 0\\ 0 & 0 & C_{66N}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \gamma_{x\theta}\end{Bmatrix},\ \begin{Bmatrix}\sigma_{xxp}\\ \sigma_{\theta\theta p}\\ \tau_{x\theta p}\end{Bmatrix}=\begin{bmatrix}C_{11p} & C_{12p} & 0\\ C_{21p} & C_{22p} & 0\\ 0 & 0 & C_{66p}\end{bmatrix}\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{\theta\theta}\\ \gamma_{x\theta}\end{Bmatrix}-\begin{bmatrix}0 & 0 & e_{31p}\\ 0 & 0 & e_{32p}\\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}\bar{E}_{xp}\\ \bar{E}_{\theta p}\\ \bar{E}_{zp}\end{Bmatrix},\qquad (1)$$

With the radial component of electric field $\bar{E}_{zp}=V_p/h_{p2}$, the radial component of electric displacement D_{zp} can be presented as

$$D_{zp}=e_{31p}\varepsilon_{xx}+e_{32p}\varepsilon_{\theta\theta}+\eta_{33p}\bar{E}_{zp}\qquad (2)$$

With classical shell theory and the linear deflection and curvatures defined by Donnell's theory [7, 8], and based on the Gurtin–Murdoch surface/interface theory [2, 3], the normal stresses σ_x and σ_θ Eqs. (1) can be rewrite ten as

$$\sigma_{x(N,p)}=C_{11(N,p)}\varepsilon_x+C_{12(N,p)}\varepsilon_\theta+\frac{\upsilon_{(N,p)}\sigma_z^{S(1,2)}}{1-\upsilon_{(N,p)}},\sigma_{\theta(N,p)}=C_{21(N,p)}\varepsilon_x+C_{22(N,p)}\varepsilon_\theta+\frac{\upsilon_{(N,p)}\sigma_{zz}^{S(1,2)}}{1-\upsilon_{(N,p)}},\qquad (3)$$

$$\sigma_{x\theta(N,p)}=C_{66(N,p)}\gamma_{x\theta},$$

Where

$$\sigma_z^{S(1,2)}=\left(\frac{\left(\tau_0^{S(1,2)o}-\tau_0^{S(1,2)i}\right)}{2}+\frac{z\left(\tau_0^{S(1,2)o}+\tau_0^{S(1,2)i}\right)}{(2h_N,2h_N+h_p)}\right)\left(\frac{\partial^2 w_{(1,2)}}{\partial x^2}+\frac{1}{R_{(1,2)}^2}\frac{\partial^2 w_{(1,2)}}{\partial\theta^2}\right)$$
$$+\left(\frac{\left(\rho^{S(1,2)i}-\rho^{S(1,2)o}\right)}{2}-\frac{z\left(\rho^{S(1,2)i}+\rho^{S(1,2)o}\right)}{(2h_N,2h_N+h_p)}\right)\frac{\partial^2 w_{(1,2)}}{\partial t^2},\qquad (4)$$

2.2. Governing equations

In this section, the governing equations of the piezoelectric cylindrical nanoshell are obtained by applying the assumed mode method. With considering of the surface stress effect, the total strain and kinetic energies and can be expressed as:

$$\pi=\frac{1}{2}\int_0^L\int_0^{2\pi}\int_{-h_{N1}}^{h_{N1}}\left(\sigma_{ijN}\varepsilon_{ij}\right)R_1dzd\theta dx+\frac{1}{2}\int_0^L\int_0^{2\pi}\left(\sigma_{ij}^{S_{1i}}\varepsilon_{ij}\right)_{(z=-h_{N1})}R_1d\theta dx$$
$$+\frac{1}{2}\int_0^L\int_0^{2\pi}\left(\sigma_{ij}^{S_{1o}}\varepsilon_{ij}\right)_{(z=h_N)}R_1d\theta dx+\frac{1}{2}\int_0^L\int_0^{2\pi}\int_{-h_{N2}}^{h_{N2}}\left(\sigma_{ijN}\varepsilon_{ij}\right)R_2dzd\theta dx$$
$$+\frac{1}{2}\int_0^L\int_0^{2\pi}\left(\sigma_{ij}^{S_{2i}}\varepsilon_{ij}\right)_{(z=-h_{N2})}R_2d\theta dx+\frac{1}{2}\int_0^L\int_0^{2\pi}\int_{h_{N2}}^{h_{N2}+h_{p2}}\left(\sigma_{ijp}\varepsilon_{ij}-\bar{E}_{zp}D_{zp}\right)R_2dzd\theta dx\qquad (5)$$
$$+\frac{1}{2}\int_0^L\int_0^{2\pi}\left(\sigma_{ij}^{S_{2o}}\varepsilon_{ij}-\bar{E}_{zp}D_i^{S_{2o}}\right)_{(z=h_{N2}+h_{p2})}R_2d\theta dx$$

$$T_n=\frac{1}{2}\iint I_n\left(\left(\frac{\partial u_n}{\partial t}\right)^2+\left(\frac{\partial v_n}{\partial t}\right)^2+\left(\frac{\partial w_n}{\partial t}\right)^2\right)R_nd\theta dx\qquad (6)$$

Where the stresses and moment resultants are defined in [4].

The work done by the visco-pasternak medium and vdW forces can be expressed as [9]

$$W_c = \frac{1}{2}\int_0^L C_w \left(\frac{\partial w_2}{\partial t}\right)^2 dx, \tag{7}$$

$$W_{wp} = -\int_0^L \left(K_w w_2 - K_p \nabla^2 w_2\right) w_2\, dx \tag{8}$$

$$W_{vdw} = \int_0^L C_{vdw}(w_2 - w_1) w_1\, dx - \int_0^L C_{vdw}\frac{R_1}{R_2}(w_2 - w_1) w_2\, dx \tag{9}$$

for discretizing equations of motion, the assumed mode method combined with Lagrange–Euler's is used. using following dimensionless parameters for Eqs. (5-9)

$$\begin{gathered}(\bar{u},\bar{v},\bar{w})_n = \frac{(u,v,w)_n}{h_{Nn}}, \xi = \frac{x}{L}, (\bar{A}_{ijn},\bar{B}_{ijn},\bar{D}_{ijn}) = \frac{(A_{ijn},B_{ijn},D_{ijn})}{A_{11Nn}(1,h_{Nn},h_{Nn}^2)}, \bar{\tau}_0^{s_{n(i,o)}} = \frac{\tau_0^{s_{n(i,o)}}}{A_{11Nn}},\\ (\bar{A}_{ijn}^*,\bar{B}_{ijn}^*,\bar{D}_{ijn}^*) = \frac{(A_{ijn}^*,B_{ijn}^*,D_{ijn}^*)}{A_{11Nn}(1,h_{Nn},h_{Nn}^2)}, \bar{F}_{11n}^* = \frac{F_{11n}^*}{A_{11Nn}h_{Nn}}, \bar{E}_{11n}^* = \frac{E_{11n}^*}{A_{11Nn}h_{Nn}^2}, \bar{N}_{(x,\theta)p2}^* = \frac{N_{(x,\theta)p2}^*}{A_{11N2}},\\ \bar{M}_{(x,\theta)p2}^* = \frac{M_{(x,\theta)p2}^*}{A_{11N2}h_{N2}}, m_{0n} = \frac{L}{R_n}, m_1 = \frac{L}{h_N}, m_{2n} = \frac{h_{Nn}}{R_n}, m_{3n} = \frac{I_n}{2\rho_{Nn}h_{Nn}}, \tau = t\sqrt{\frac{A_{11Nn}}{2\rho_{Nn}h_{Nn}L^2}} = \Omega t,\\ \bar{\Omega} = \frac{\omega}{\Omega}, \bar{K}_{wn} = \frac{K_{wn}L^2}{m_{3n}A_{11Nn}}, \bar{K}_{pn} = \frac{K_{pn}}{m_{3n}A_{11Nn}}, \bar{C}_{w2} = \frac{C_w \Omega L^2}{m_{3n}A_{11Nn}}, \bar{C}_{vdwn} = \frac{C_{vdw}L^2}{m_{3n}A_{11Nn}},\end{gathered} \tag{10}$$

and displacement in terms of generalized coordinate and mode function reference [8], and substituting into Eqs. (5-9) and applying the Euler–Lagrange method results in the following reduced-order model of the system:

$$[(M)_u^u]_n\{\ddot{\bar{u}}_n\} + [(M)_u^w]_n\{\ddot{\bar{w}}_n\} + [(K)_u^u]_n\{\bar{u}_n\} + [(K)_u^v]_n\{\bar{v}_n\} + [(K)_u^w]_n\{\bar{w}_n\} = \bar{F}_{un}, \tag{11}$$

$$[(M)_v^v]_n\{\ddot{\bar{v}}_n\} + [(M)_v^w]_n\{\ddot{\bar{w}}\} + [(K)_v^v]_n\{\bar{v}_n\} + [(K)_v^u]_n\{\bar{u}_n\} + [(K)_v^w]_n\{\bar{w}_n\} = \bar{F}_{vn}, \tag{12}$$

$$\begin{gathered}[(M)_w^w]_n\{\ddot{\bar{w}}_n\} + [(C)_w^w]_p\{\dot{\bar{w}}_n\} + \left[[(K)_w^w]_n + (-1)^n \bar{C}_{vdw}\left(\frac{\bar{R}_1}{\bar{R}_2}\right)^m [(K)_w^{vdw}]_n\right]\{\bar{w}_n\}\\ +(-1)^{n+1}\bar{C}_{vdw}\left(\frac{\bar{R}_1}{\bar{R}_2}\right)^m [(K)_w^{vdw}]_n\{\bar{w}_k\} + [(K)_w^u]_n\{\bar{u}_n\} + [(K)_w^v]_n\{\bar{v}_n\} = \bar{F}_{wn},\end{gathered} \tag{13}$$

where (M) , (C) and (K) are mass, damping and stiffness matrixes, respectively, in directions of u_n, v_n and w_n. Also, $\bar{F}_{un}$, $\bar{F}_{vn}$ and $\bar{F}_{wn}$ are applied loads by piezoelectric voltage and surface stress and for free vibration of piezoelectric nanoshell $\bar{F}_n$ are zero, as a result ($\bar{F}_{un} = \bar{F}_{vn} = \bar{F}_{wn} = 0$). $[(K)_w^{vdw}]_n$ is stiffness matrix for van der Walls effect and also for $n = 1: m = 0$, $k = 2, p = 0$ and for $n = 2: m = 1, k = 1, p = 1$. All coefficients of mass and stiffness matrixes Eqs. (35)-(37) are presented in Appendix 3. Also, the eigenvalue equation can be obtained in the following format:

$$\left[[K] - \omega_{ij}^2[M]\right]\{u_{ij}\ \ v_{ij}\ \ w_{ij}\}^T = 0, \tag{14}$$

3. NUMERICAL RESULTS AND DISCUSSIONS

In this section, the surface energy effect on the free vibration analysis of a piezoelectric cylindrical nano-shell with arbitrary boundary conditions is investigated. In order to simplify the presentation, C-C, S-S, C-S and C-F represent clamped edge, simply supported edge, clamped-simply supported edge and clamped-free edge, respectively. The bulk and surface material properties of DWPVCNS and ZnO layers are listed in Table 1 [9, 10].

Table 1: *Material property of DWCNS and piezoelectric ZnO [9, 10]*

$E_N(TPa)$	υ_N	$\rho_N(kg\ m^{-3})$	$E_p(Gpa)$	υ_p
1	0.27	2300	140	0.5686
$\rho_p(kg\ m^{-3})$	$e_{31p}(C\ m^{-2})$	$e_{32p}(C\ m^{-2})$	$\eta_{33p}(F\ m^{-1})$	
5610	-0.51	-0.51	8.91×10^{-8}	

The accuracy of the present study was verified in the previous section. Here, some numerical results are presented to explore the effects of involved parameters on the vibration behavior of cylindrical piezoelectric nano-shell. The geometrical parameters used in these numerical results are as;
$R_1 = 0.5\,nm$, $R_2 = 1$ nm, $L = 10R_2$, $h_{N(1,2)} = 0.02R_1$, $h_{p2} = 0.02R_1$, $K_w = 1\times10^{15}\ (N/m^3)$, $K_p = 2$ N, $C_w = 5\times10^{-7}\ (N.S/m)$, $C_{vdw} = 10\times10^{15}$ Pa, $\mu^{s_1} = \mu^{s_2} = 11.7$ N/m, $\lambda^{s_1} = \lambda^{s_2} = 14.2$ N/m, $\tau_o^{s_1} = \tau_o^{s_2} = 1\,N/m$, $\rho_{s_{1i}} = \rho_{s_{1o}} = \rho_{s_{2i}} = 3.17\times10^{-7}\ kg/\ m^3$, $\rho_{s_{2o}} = 5.61\times10^{-6}\ kg/\ m^3$.

We first want to study the effect of surface density (SD) (i.e. effects of $\rho_{s_{1i}}, \rho_{s_{1o}}, \rho_{s_{2i}}$ and $\rho_{s_{2o}}$) and other surface effects (SE) (effects of $\mu^{s_1}, \mu^{s_2}, \lambda^{s_1}, \lambda^{s_2}, \tau_o^{s_1}, \tau_o^{s_2}$) on dimensionless undamped and damped natural frequency of the multi walls nano-structure, here DW piezoelectric nano-shell, versus length-to-small radius ratio (L/R_2) in figure (2).

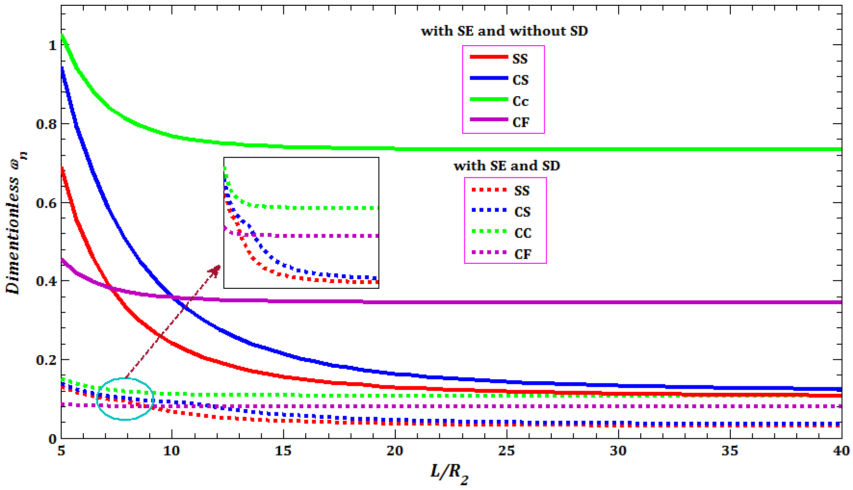

Fig. 2. *The effect of surface density (SD) on ω_n versus L/R_2 for different boundary conditions*

As seen in Figure 2, the natural frequency of the system for two cases, with and without surface density, with the presence of other surface effects and for different ratios of the length-to-small radius ratio (L/R_2) of the second layer is considered. It is clear that for all boundary conditions, considering the surface density will result in a significant reduction in the natural frequency of the system. And the reason is that considering the surface density leads to increased stiffness of the system and therefore reduces the natural frequency of DW piezoelectric nano-shell. In Fig. 3, which shows the natural frequency of the system in terms of different length-to-small radius of outer layer ratio L/R_2, we want to consider the previous subject in detail for four different cases: i) with (SE) and without (SD), ii) without (SE) and (SD), iii) with (SE) and (SD), iv) without (SE) and with (SD). As can be seen, for all boundary conditions, the maximum frequency occurs when the effects of the surface are considered without the presence of surface density and also, the lowest frequency occurs when the surface density effect is considered without regard to other surface parameters related to the internal and external layers of the system.

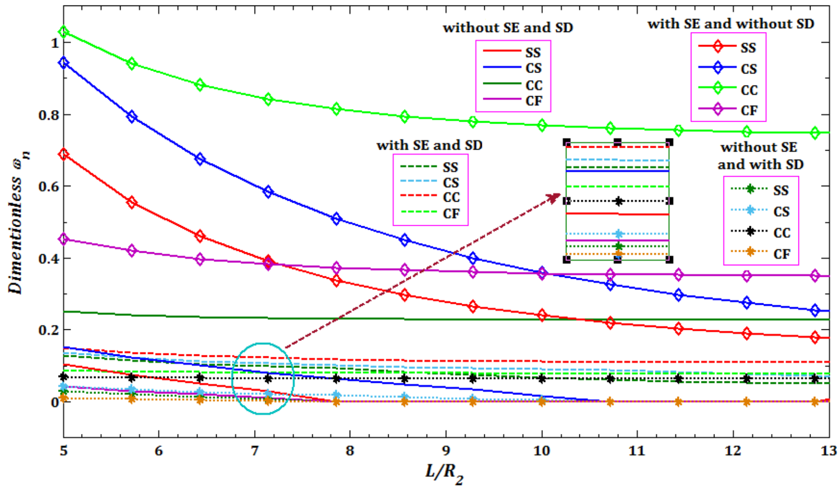

Fig. 3. *The effect of surface density (SD) and surface effects (SE) on ω_n versus L/R_2*

With regard to the above-mentioned explanations about the importance of surface effects (SE) in the absence of surface density (SD), the effect of van der Waals force C_{vdw} on dimensionless ω_n in terms of L/R_2 ratio for various boundary conditions illustrate in Figure 4. It can be seen that for all boundary conditions with surface energy effects (SE), considering the effect of van der Waals force, it increases the natural frequency of the system. Also, the frequency decreases with increasing length to radius ratio L/R_2 and for a less than 10, this decrease is more significant, and for almost $L/R_2 > 20$, except for SS and CS boundary conditions in $C_{vdw} = 0\ pa$, and increasing of L/R_2 ratio will not have much effect on the frequency.

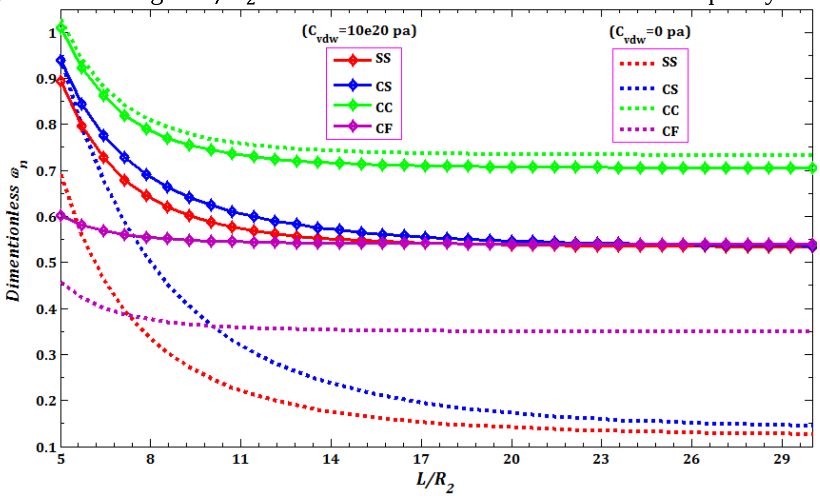

Fig. 4. *The effect of van der Waals force* C_{vdw} *on* ω_n *versus* L/R_2 *with surface effects (SE)*

4. CONCLUSION

Vibration analysis of DWPECNS subjected to visco-Pasternak medium is investigated by Gurtin–Murdoch surface elasticity and Donnell's theory and Hamilton's principle combined with the assumed mode method. A variety of new vibration results including natural frequencies with and without surface energy effects for piezoelectric cylindrical nano-shell with non-classical constraints as well as different material parameters are presented. The result are shown that in the nanoscale system is considered surface effects (SE) without considering the surface density (SD), we will have the maximum frequency and this case will be considered as the critical state of the system. As a result, controlling the frequency of the system in this case is essential and it is quite clear that considering the effects of the surface energy give to a significant effect in the natural frequency of the piezo- viscoelastic nanoshell.

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