

NONLOCAL EFFECT ON VIBRATION ANALYSIS OF PIEZOELECTRIC CYLINDRICAL NANOSHELL

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ABSTRACT

Vibration analysis of piezoelectric cylindrical nanoshell (PCNS) subjected to visco-Pasternak medium with arbitrary boundary conditions is investigated using Eringen nonlocal theory and considering Donnell's shell theory and Hamilton's principle. The viscoelastic nanoshell medium is modeled as Visco-Pasternak foundation. The convergence, accuracy and reliability of the current formulation are validated by comparisons with existing experimental and numerical results. Also, the effects of nonlocality are accurately studied on frequencies of piezoelectric cylindrical nanoshell.

Keywords: Piezoelectric nanoshell, Eringen nonlocal theory, Visco-Pasternak medium, Natural frequency.

1. INTRODUCTION

Recently, with the development of nano-sized piezoelectric elements [1], some non-classical continuum theories have been introduced to develop the size-dependent continuum models [2]. With these models, Avramov presented nonlinear vibration and bifurcation behavior of single-walled carbon nanotubes using the nonlocal elasticity[3]. Nonlinear harmonic vibration of a piezoelectric-layered nanotube conveying fluid flow is investigated by Saadatnia et al. using the nonlocal theory and energy approach [4]. Recently, Ke et al. [5] extended the nonlocal theory to thermo-electro-mechanical vibration of size-dependent piezoelectric cylindrical nanoshells under various boundary conditions. Mirza and Alizadeh investigated the effects of detached base length on the natural frequencies and modal shapes of cylindrical shells [6]. Zeighampour et al. investigated wave propagation in viscoelastic single walled carbon nanotubes by accounting for the simultaneous effects of the nonlocal constant and the material length scale parameter and visco-Pasternak foundations [7]. In the present study, the vibration analysis of a piezoelectric cylindrical nanoshell subjected to Visco-Pasternak medium with arbitrary boundary conditions is investigated. The Eringen nonlocal theory is employed to derive the equation of motion of the nano-shell using Hamilton principle. A variety of new vibration results including the effects of nonlocality are accurately studied on frequencies of piezoelectric cylindrical nanoshell.

2. PROBLEM FORMULATION AND GOVERNING EQUATIONS

A cylindrical nanoshell embedded with a piezoelectric layer and visco-Pasternak medium is shown in Figure 1.

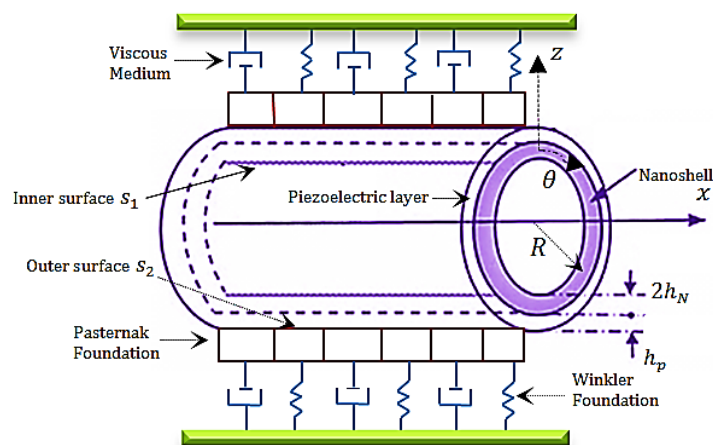


Fig. 1. A piezoelectric cylindrical nano shell with inner and outer surfaces

The nano shell has length of L , mid-surface radius R , thickness of $2h_N$, and piezoelectric thickness of h_p .

2.1. Nonlocal Eringen shell theory

According to Eringen [2], for the piezoelectric cylindrical shell, the nonlocal constitutive relations can be expressed

$$\begin{Bmatrix} \sigma_{xx(N,p)} \\ \sigma_{\theta\theta(N,p)} \\ \tau_{x\theta(N,p)} \end{Bmatrix} - (e_0 a)^2 \nabla^2 \begin{Bmatrix} \sigma_{xx(N,p)} \\ \sigma_{\theta\theta(N,p)} \\ \tau_{x\theta(N,p)} \end{Bmatrix} = \begin{bmatrix} C_{11(N,p)} & C_{12(N,p)} & 0 \\ C_{21(N,p)} & C_{22(N,p)} & 0 \\ 0 & 0 & C_{66(N,p)} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{31p} \\ 0 & 0 & e_{32p} \\ 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \bar{E}_{xp} \\ \bar{E}_{\theta p} \\ \bar{E}_{zp} \end{Bmatrix} \quad (1)$$

Also, the radial component of electric displacement D_{zp} can be presented as

$$D_{zp} - (e_0 a)^2 \nabla^2 D_{zp} = e_{31p} \varepsilon_{xx} + e_{32p} \varepsilon_{\theta\theta} + \eta_{33p} \bar{E}_{zp} \quad (2)$$

where $\nabla^2 = \partial^2 / \partial x^2 + \partial^2 / \partial (R\theta)^2$ is the Laplace operator.

Within the framework of classical shell theory, the displacement fields of the nanoshell can be written as [8]

$$u_x(x, \theta, z) = u(x, \theta) - z \frac{\partial w(x, \theta)}{\partial x}, u_\theta(x, \theta, z) = v(x, \theta) - \frac{z}{R} \frac{\partial w(x, \theta)}{\partial \theta}, u_z(x, \theta, z) = w(x, \theta), \quad (3)$$

The linear deflection and curvatures are defined by Donnell's theory as [8, 9]

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{\theta\theta} \\ \gamma_{x\theta} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_{xx}^0 \\ \varepsilon_{\theta\theta}^0 \\ \gamma_{x\theta}^0 \end{Bmatrix} + z \begin{Bmatrix} \kappa_{xx} \\ \kappa_{\theta\theta} \\ \kappa_{x\theta} \end{Bmatrix} = \begin{Bmatrix} (\partial u / \partial x) \\ (1/R)(\partial v / \partial \theta) + (1/R)w \\ (1/R)(\partial u / \partial \theta) + (\partial v / \partial x) \end{Bmatrix} - z \begin{Bmatrix} \partial^2 w / \partial x^2 \\ (1/R^2)(\partial^2 w / \partial \theta^2) \\ (2/R)(\partial^2 w / \partial x \partial \theta) \end{Bmatrix} \quad (4)$$

The total strain and kinetic energies and can be expressed as:

$$\pi = \frac{1}{2} \int_0^L \int_0^{2\pi} \{ N_{xx} \varepsilon_{xx}^0 + N_{\theta\theta} \varepsilon_{\theta\theta}^0 + N_{x\theta} \gamma_{x\theta}^0 + M_{xx} \kappa_{xx} + M_{\theta\theta} \kappa_{\theta\theta} + M_{x\theta} \kappa_{x\theta} + \eta_{33} \bar{E}_{zp} h_p \} R d\theta dx \quad (5)$$

$$T = \frac{1}{2} \iint I \left(\left(\frac{\partial u}{\partial t} \right)^2 + \left(\frac{\partial v}{\partial t} \right)^2 + \left(\frac{\partial w}{\partial t} \right)^2 \right) R d\theta dx \quad (6)$$

where the stresses and moment resultants and mass inertia are defined as:

$$(N_{ij(N,p)}, M_{ij(N,p)}) = \int_{-h_N}^{h_N} \sigma_{ijN}(1, z) dz + \int_{h_N}^{h_N+h_p} \sigma_{ijp}(1, z) dz, \quad (7)$$

$$I = \int_{-h_N}^{h_N} \rho_N dz + \int_{h_N}^{h_N+h_p} \rho_p dz + \rho^s = 2\rho_N h_N + \rho_p h_p, \quad (8)$$

From Eqs. (17)-(25) and using Eqs. (35-36), we have

$$N_{xx} - (e_0 a)^2 \nabla^2 N_{xx} = A_{11} \varepsilon_{xx}^0 + A_{12} \varepsilon_{\theta\theta}^0 + B_{11} \kappa_{xx} + B_{12} \kappa_{\theta\theta} - N_{xp} \quad (9a)$$

$$N_{\theta\theta} - (e_0 a)^2 \nabla^2 N_{\theta\theta} = A_{21} \varepsilon_{xx}^0 + A_{22} \varepsilon_{\theta\theta}^0 + B_{21} \kappa_{xx} + B_{22} \kappa_{\theta\theta} - N_{\theta p} \quad (9b)$$

$$N_{x\theta} - (e_0 a)^2 \nabla^2 N_{x\theta} = A_{66} \gamma_{x\theta}^0 + B_{66} \kappa_{x\theta}, \quad (9c)$$

$$M_{xx} - (e_0 a)^2 \nabla^2 M_{xx} = B_{11} \varepsilon_{xx}^0 + B_{12} \varepsilon_{\theta\theta}^0 + D_{11} \kappa_{xx} + D_{12} \kappa_{\theta\theta} - M_{xp}, \quad (9d)$$

$$M_{\theta\theta} - (e_0 a)^2 \nabla^2 M_{\theta\theta} = B_{21} \varepsilon_{xx}^0 + B_{22} \varepsilon_{\theta\theta}^0 + D_{21} \kappa_{xx} + D_{22} \kappa_{\theta\theta} - M_{\theta p}, \quad (9e)$$

$$M_{x\theta} - (e_0 a)^2 \nabla^2 M_{x\theta} = B_{66} \gamma_{x\theta}^0 + D_{66} \kappa_{x\theta}, \quad (9f)$$

The work done by the surrounded viscoelastic medium and the harmonic excitation can be written as [10]

$$W_{vm} = - \int_0^L \int_0^{2\pi} \left\{ \int_0^w \left(K_w w - K_p \nabla^2 w + C_w \frac{\partial w}{\partial t} \right) dw \right\} R d\theta dx \quad (10)$$

By taking the variations of displacements u, v and w , integrating by parts, and equating the coefficients of δu

δv and δw to zero, the governing equations of motion can be derived from Hamilton's principle as:

$$\delta u: \frac{\partial N_{xx}}{\partial x} + \frac{1}{R} \frac{\partial N_{x\theta}}{\partial \theta} = (1 - (e_0 a)^2 \nabla^2) I \frac{\partial^2 u}{\partial t^2}, \quad \delta v: \frac{\partial N_{x\theta}}{\partial x} + \frac{1}{R} \frac{\partial N_{\theta\theta}}{\partial \theta} = (1 - (e_0 a)^2 \nabla^2) I \frac{\partial^2 v}{\partial t^2},$$

$$\delta w: \frac{\partial^2 M_{xx}}{\partial x^2} + \frac{2}{R} \frac{\partial^2 M_{x\theta}}{\partial x \partial \theta} + \frac{1}{R^2} \frac{\partial^2 M_{\theta\theta}}{\partial \theta^2} - \frac{N_{\theta\theta}}{R} \quad (11)$$

$$= (1 - (e_0 a)^2 \nabla^2) \left(I \frac{\partial^2 w}{\partial t^2} + C_w \frac{\partial w}{\partial t} + K_w w - K_p \left(\frac{\partial^2 w}{\partial x^2} + \frac{1}{R^2} \frac{\partial^2 w}{\partial \theta^2} \right) \right),$$

and boundary conditions are obtained as follows:

$$N_{xx} n_x + \frac{1}{R} N_{x\theta} n_\theta = 0, \quad N_{x\theta} n_x + \frac{1}{R} N_{\theta\theta} n_\theta = 0, \quad (12)$$

$$\left(\frac{\partial M_{xx}}{\partial x} + \frac{1}{R} \frac{\partial M_{x\theta}}{\partial \theta} \right) n_x + \left(\frac{1}{R} \frac{\partial M_{x\theta}}{\partial x} + \frac{1}{R^2} \frac{\partial M_{\theta\theta}}{\partial \theta} \right) n_\theta = 0,$$

$$M_{xx}n_x + \frac{1}{R}M_{x\theta}n_\theta = 0, \quad \frac{1}{R}M_{x\theta}n_x + \frac{1}{R^2}M_{\theta\theta}n_\theta = 0,$$

2.2. Solution procedure

In the assumed mode method, displacement and shear deformation are written in terms of generalized coordinate and mode function as follows [9]:

$$\begin{bmatrix} u(x, \theta, t) \\ v(x, \theta, t) \\ w(x, \theta, t) \end{bmatrix} = \sum_{m=1}^{M_1} \sum_{j=1}^N \begin{bmatrix} [u_{m,j,c}(\tau) \cos(j\theta) + u_{m,j,s}(\tau) \sin(j\theta)] \chi_{mj}(\xi) \\ [v_{m,j,c}(\tau) \sin(j\theta) + v_{m,j,s}(\tau) \cos(j\theta)] \phi_{mj}(\xi) \\ [w_{m,j,c}(\tau) \cos(j\theta) + w_{m,j,s}(\tau) \sin(j\theta)] \beta_{mj}(\xi) \end{bmatrix} + \sum_{m=1}^{M_2} \begin{bmatrix} u_{m,0}(\tau) \chi_{m0}(\xi) \\ v_{m,0}(\tau) \phi_{m0}(\xi) \\ w_{m,0}(\tau) \beta_{m0}(\xi) \end{bmatrix} \quad (13)$$

where $\chi_i(\xi)$, $\phi_r(\xi)$ and $\beta_s(\xi)$ are modal functions which satisfy the required geometric boundary conditions. By substituting Eqs. (13) into Eqs. (11), and using following dimensionless parameters

$$(\bar{u}, \bar{v}, \bar{w}) = \frac{(u, v, w)_n}{h_N}, \xi = \frac{x}{L}, (\bar{A}_{ij}, \bar{B}_{ij}, \bar{D}_{ij}) = \frac{(A_{ij}, B_{ij}, D_{ij})}{A_{11N}(1, h_N, h_N^2)}, (\bar{M}, \bar{N})_{(x,\theta)p}^* = \frac{(M, N)_{(x,\theta)p}^*}{A_{11N}},$$

$$m_0 = \frac{L}{R}, m_1 = \frac{L}{h_N}, m_2 = \frac{h_N}{R}, m_n = \frac{I_n}{2\rho_N h_N}, \mu = \frac{e_0 a}{L}, \tau = t \sqrt{\frac{A_{11N}}{2\rho_N h_N L^2}} = \Omega t, \bar{\Omega} = \frac{\omega}{\Omega} \quad (14)$$

$$\bar{K}_w = \frac{K_w L^2}{m_3 A_{11N}}, \bar{K}_p = \frac{K_p}{m_3 A_{11N}}, \bar{C}_w = \frac{C_w \Omega L^2}{m_3 A_{11N}}$$

And using Hamilton principle, the following reduced-order model of the system:

$$[(M)_{uH}^u + (M_{bc})_{uH}^u] \{\ddot{\bar{u}}\} + [(K)_{uH}^u + (K_{bc})_{uH}^u] \{\bar{u}\} + [(K)_{uH}^v + (K_{bc})_{uH}^v] \{\bar{v}\} + [(K)_{uH}^w + (K_{bc})_{uH}^w] \{\bar{w}\} + \bar{F}_{upH} = 0, \quad (15a)$$

$$[(M)_{vH}^v] \{\ddot{\bar{v}}\} + [(K)_{vH}^u + (K_{bc})_{vH}^u] \{\bar{u}\} + [(K)_{vH}^v + (K_{bc})_{vH}^v] \{\bar{v}\} + [(K)_{vH}^w] \{\bar{w}\} + \bar{F}_{vpH} = 0, \quad (15b)$$

$$[(M)_{wH}^w] \{\ddot{\bar{w}}\} + [(C)_{wH}^w] \{\dot{\bar{w}}\} + [(K)_{wH}^u + (K_{bc})_{wH}^u] \{\bar{u}\} + [(K)_{wH}^v + (K_{bc})_{wH}^v] \{\bar{v}\} + [(K)_{wH}^w + (K_{bc})_{wH}^w] \{\bar{w}\} + \bar{F}_{wpH} = 0, \quad (15c)$$

Natural frequencies and mode shapes can be obtained from solving following eigenvalue equation:

$$[[K] - \omega_{mn}^2 [M]] \{u_{mn} \ v_{mn} \ w_{mn}\}^T = 0, \quad (16)$$

3. NUMERICAL RESULTS AND DISCUSSIONS

The non-homogeneous nano-shell considered in the following examples is composed of stainless steel and nickel. The material properties for nanoshell (stainless steel and nickel) and also the piezoelectric layer (PZT-4 material) are shown in Table 1 and Table 2, respectively [10, 11].

Table 1: Properties of stainless steel and nickel [10, 11]

| Stainless steel | | | Nichel | | |
|-----------------|---------|------------------------|-------------|---------|------------------------|
| $E_B (GPa)$ | ν_B | $\rho_B (kg \ m^{-3})$ | $E_T (GPa)$ | ν_T | $\rho_T (kg \ m^{-3})$ |
| 208 | 0.381 | 8166 | 205 | 0.31 | 8900 |

Table 2: Properties of PZT-4 [10, 11]

| $E_p (GPa)$ | ν_p | $e_{31p} (C/m^2)$ | $e_{32p} (C/m^2)$ | $\eta_{33p} (10^{-11} F/m)$ | $\rho_p (kg \ m^{-3})$ |
|-------------|---------|-------------------|-------------------|-----------------------------|------------------------|
| 95 | 0.3 | -5.2 | -5.2 | 560 | 7500 |

Also, the material property of surface effects and geometrical parameters used in all following results are shown in Table 3.

Table 3: The material property of surface effects and geometrical parameters

| $R(m)$ | L/R | h_N/R | h_p/R | $C_w (N.S/m)$ | $K_w (N/m^3)$ | $K_p (N)$ | $V_p (V)$ |
|--------------------|-------|---------|---------|--------------------|--------------------|--------------------|--------------------|
| 1×10^{-9} | 10 | 0.05 | 0.03 | 3×10^{-7} | 1×10^{20} | 1×10^{-2} | 1×10^{-5} |

3.1. Convergence studies

A convergence study of the natural frequency ω_n of the SS piezoelectric nanoshells for Eringen nonlocal theory is presented in Table 4 with varying total numbers of nodes N and circumferential wave numbers n .

Table 4: Convergence of dimensionless natural frequencies $\omega_n = \bar{\Omega}_n R/L$ of the Eringen nonlocal theory

| n | $\mu = 0$ | | $\mu = 0.02$ | | $\mu = 0.05$ | | $\mu = 0.1$ | |
|-----|-----------|----------|--------------|----------|--------------|----------|-------------|----------|
| | $N = 1$ | $N = 2$ | $N = 1$ | $N = 2$ | $N = 1$ | $N = 2$ | $N = 1$ | $N = 2$ |
| 1 | 0.567012 | 0.543459 | 0.554995 | 0.531641 | 0.502426 | 0.480069 | 0.391778 | 0.372328 |
| 2 | 1.064967 | 1.060514 | 0.987180 | 0.982913 | 0.748597 | 0.744961 | 0.471781 | 0.469163 |
| 3 | 1.577035 | 1.575583 | 1.350387 | 1.349085 | 0.871548 | 0.870564 | 0.496306 | 0.495676 |

As can be seen from Table 4, for all nonlocal parameter μ , the natural frequencies decrease with increase of the node number N and increase with increase of the circumferential wave number n . As a result, the convergence mode number for the paper results (Table 4), is $n = 2$ and $m = 2$.

3.2. Parametric study

In this subsection the natural frequency analysis of SS piezoelectric cylindrical nanoshell is presented using the Eringen nonlocal theory.

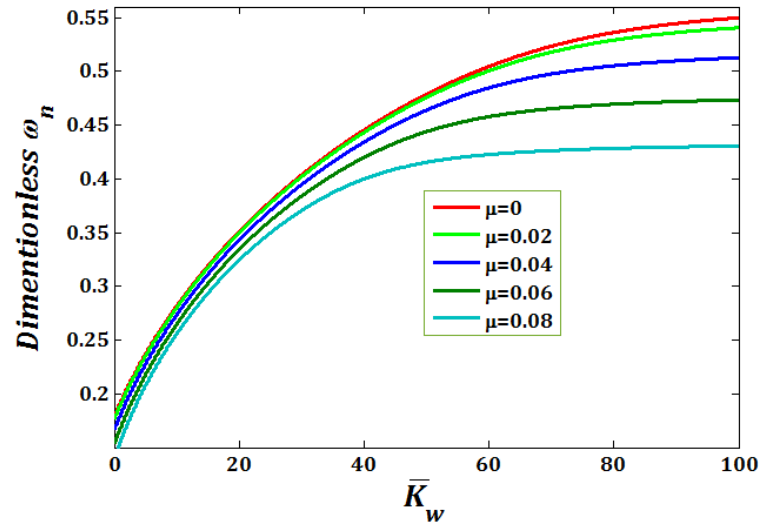


Fig. 2. The effect of \bar{K}_w on ω_n with different values of nonlocal parameter μ

Figure 2 illustrates the effect of dimensionless stiffness coefficient of Winkler foundation \bar{K}_w on dimensionless natural frequencies ω_n of the piezoelectric nano-shell. It can be seen that the dimensionless natural frequency increase with the increase of the \bar{K}_w . Also, the natural frequency decreases with increase of nonlocal parameter μ . The reason is that a higher nonlocal parameter μ leads to a decrease in the nanoshell stiffness, and cause to lower natural frequencies of nanoshell. Figure 3 discusses the effect of the length-to-small radius ratio (L/R) on the dimensionless natural frequency ω_n of SS piezoelectric nanoshells for different values of nonlocal parameter μ using Eringen nonlocal theory.

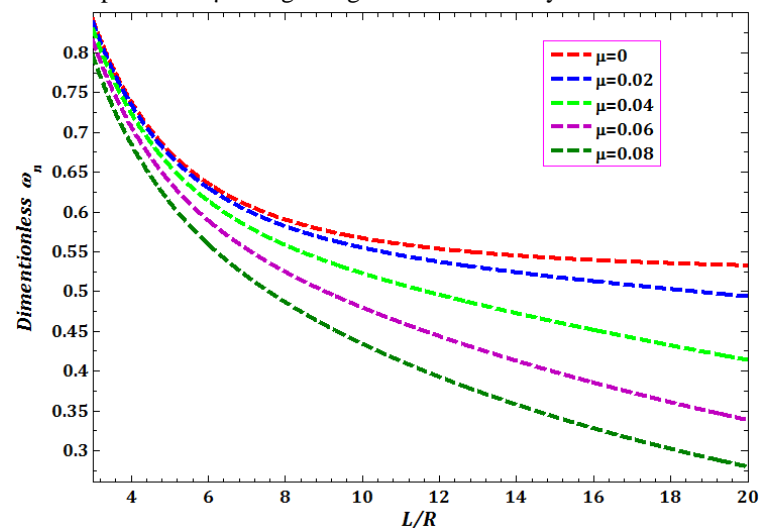


Fig. 3. The effect of L/R ratio on ω_n with different values of nonlocal parameter μ

For SS piezoelectric nanoshells, the dimensionless natural frequency decreases with the increase of the ratio (L/R). As shown in Figure 3, the natural frequency decreases with increase of nonlocal parameter μ . The reason is that a higher nonlocal parameter μ leads to a decrease in the nanoshell stiffness, and cause to lower natural frequencies of nanoshell.

And finally, the effect of the small radius ratio h_N/R on dimensionless natural frequencies ($\omega_n = \bar{\Omega}_n R/L$) of the SS piezoelectric nanoshell for different values of nonlocal parameter μ are shown in Figure 4 using Eringen nonlocal theory. As shown in Figure 9, the dimensionless natural frequency increase with the increase of the ratio h_N/R . Also, the natural frequency decreases with increase of nonlocal parameter μ . The reason is that a higher nonlocal parameter μ leads to a decrease in the nanoshell stiffness, and cause to lower natural frequencies of nanoshell.

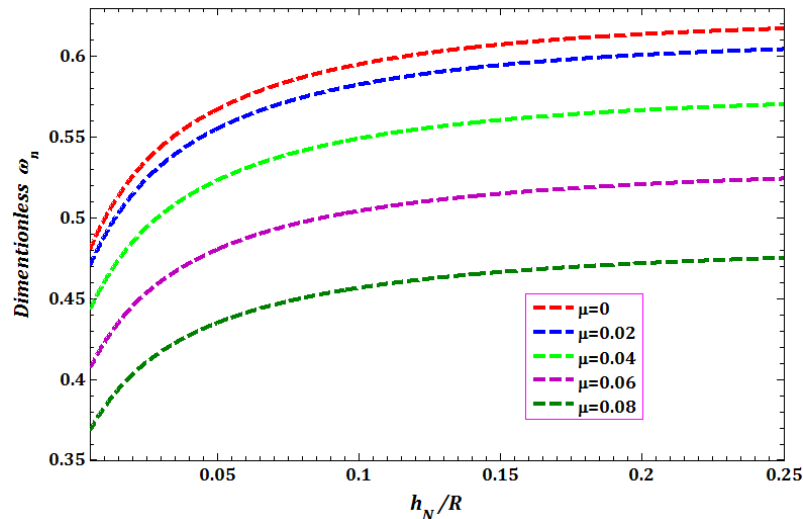


Fig. 4. Dimensionless natural frequency ω_n versus h_N/R for different nonlocal parameter μ

4. CONCLUSION

Vibration analysis of piezoelectric cylindrical nanoshell subjected to visco-Pasternak medium with arbitrary boundary conditions is investigated by Eringen nonlocal theory and using Donnell's theory and Hamilton's principle. A variety of new vibration results including natural frequencies with and without nonlocal for piezoelectric cylindrical nano-shell with non-classical restraints. The convergence is validated with excellent agreements achieved. Also, the effects of nonlocal parameter μ are accurately studied on frequencies of piezoelectric cylindrical nanoshell versus \bar{K}_w , L/R ratio and h_N/R ratio.

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