Wagner-based Hydrodynamic Analysis of Wave Impact underneath the Deck of a Semi-submersible using Direct Boundary Element Method (BEM):

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1) Abstract:

The main objective of this study is to investigate the phenomenon of water impact underneath the decks of offshore structures due to propagating waves.
The decks of offshore structures may be subjected to wave induced loads, which may be not accounted for in the original design. For safe design of offshore platforms, it is important that the hydrodynamic loads and the structural response due to wave impact underneath decks of platforms are predicted accurately.
In this report, a review of the previous work on this topic with a brief introduction to slamming theory together with a proposed procedure to predict the water impact underneath the decks of floating offshore structures will be presented.
Meantime, three dimensional hydrodynamic analysis of a semi-submersible in sea waves has been performed by using the direct boundary element method.

2) Previous work:

The general problem of hydrodynamic impact has been studied extensively motivated by e.g. its importance for horizontal members in the splash zone of offshore platforms, bottom and bow-flare slamming on ships, green water impact on deck
structures of ships and wet-deck slamming on catamarans. In offshore structures Kaplan and Silbert(1976) developed a solution for the forces acting on a cylinder from the instant of impact to full immersion. Faltinsen et al.(1977) investigated the load acting on rigid horizontal circular cylinders (with end plates and length-to-diameter ratios of about one) which were forced with constant velocity through an initially calm free surface. Sarpkaya(1978) investigated forces acting on horizontal cylinders subjected to impact by a sinusoidally oscillating free surface both theoretically and experimentally. Miller(1980) presented the results of a series of

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wave-tank experiments to establish the magnitude of the wave-force slamming coefficient for a horizontal circular cylinder. Kaplan(1992) applied a method similar to the technique used in analysis of ship slamming phenomena in order to determine the time histories of impact forces on horizontal circular members and flat deck structures of offshore platforms.
The work performed on wave impact under platform decks is more limited, but it has been considered by many researchers. A more extensive theoretical analysis procedure for the assessment of the impact loading is given by Kaplan(1992). His model considers also the water exit force, i.e. the force when the wetted area reduces. Also Kaplan uses a Von Karman approach, but he includes both the slamming force and the added mass force due to the wetting of the deck. In addition, he includes a drag force by using a drag coefficient for viscous flow passed a flat plate.
In this paper, although 2-dimensional Wagner based theory for slamming analysis has been developed, but as a reasonable move regarding 3-d nature of slamming, the relative local velocity between offshore structure and sea water surface will be achieved by using 3-dimensional panel method (direct BEM).

3) Three-Dimensional Hydrodynamic Analysis of Interaction between Sea Waves and Semi-submersible:

3-1) Boundary Value Problem:

It is assumed that floating offshore structure, in this research semi-submersible, is oscillating with small amplitudes to respond to regular incident waves of small amplitudes in deep water. In linear wave theory, this general boundary value problem can be assumed to be a linear superposition of the following sub-problems:

i) the incident waves encountered by semi-submersible will be diffracted from it assuming that the semi-submersible is rigidly held in its fixed position. This is called the " **Diffraction Problem** ";

ii) as soon as the waves are diffracted, the semi-submersible is assumed to oscillate sinusoidally in previously calm water. This is known as the
" **Radiation Problem** ".

Assuming that the fluid is ideal of infinite depth and that its motion is irrotational, the total velocity potential of the flow motion can be written by a time-dependent potential, Φ:

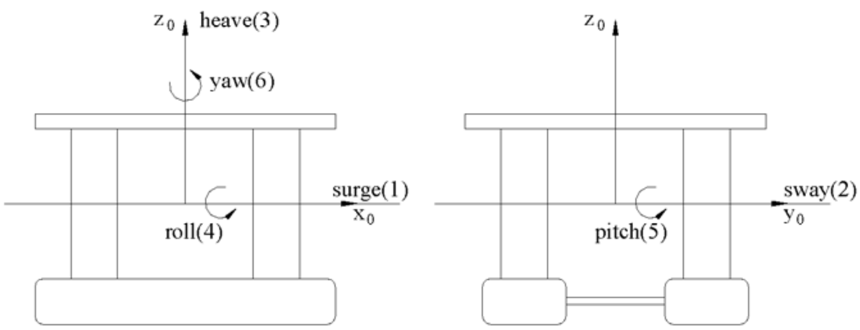
$$\Phi(x, y, z, t) = \Phi_I + \Phi_D + \Phi_R$$

Where Φ_I is the incident wave potential:

$$\Phi_I = \frac{-igA}{\omega} e^{i(kx\sin\mu + ky\cos\mu)} e^{-i\omega t}$$

where A is the wave amplitude, ω is the wave radian frequency, $k = \frac{2\pi}{\lambda}$ is the wave number, and μ is the heading angle.

As it can be seen in Figure 1, a right-handed coordinate system (x_0,y_0,z_0) fixed with respect to the mean position of the body is used, with positive z_0 vertically upwards through the centre of gravity of the platform and the origin in the plane of the undisturbed free-surface, the x_0-axis pointing towards the front and the y_0-axis pointing to port side. The x_0-z_0 and y_0-z_0 planes will be considered as planes of symmetry for the semi-submersible under consideration.



Definition of rigid body motion modes

Figure 1

Φ_D is the diffraction potential and $\Phi_R = \sum_{j=1}^{6} \phi_j(x,y,z) v_{a_j} e^{-i\omega t}$ is the radiation potential, where v_{a_j} indicates the complex amplitude of velocity in direction j, so ϕ_j is the local radiation potential due to the j[th] mode of motion as described in and only depends on the body geometry corresponding to a harmonic velocity amplitude of 1 m/s and is therefore independent of the as yet unknown body responses or velocities.

So the total velocity potential:

$$\Phi(x,y,z,t) = \phi(x,y,z)e^{-i\omega t} = (\phi_I + \phi_D + \phi_R)e^{-i\omega t} = (\phi_0 + \phi_7 + \sum_{j=1}^{6} \phi_j v_{a_j})e^{-i\omega t}$$

The solution to the above described boundary value problem should be found from a set of conditions which are to be satisfied by the flow motion potential ϕ and given as:

- Laplace equation in the fluid domain:

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0$$

- a linearized boundary condition on the free surface (S_1):

$$\frac{\partial\Phi}{\partial z}-\frac{\omega^2}{g}\Phi=0 \qquad \text{on } z=0$$

- a bottom condition on the sea floor (S_3):

$$\frac{\partial\Phi}{\partial z}=0 \qquad \text{on } z=-h \text{ (h is sea depth)}$$

- a radiation condition at a large distance from the semi-submersible (S_4):

$$\lim_{kr\to\infty}\sqrt{kr}\left(\frac{\partial\Phi}{\partial r}-ik\Phi\right)=0 \qquad \text{(Sommerfeld radiation condition)}$$

where k is the wave number and r is the radial distance from the centre of the structure in all directions.

- the kinematic boundary condition on the semi-submersible's mean wetted surface (S_2) given by:

$$\frac{\partial\Phi}{\partial n}=\left(\frac{\partial\phi_0}{\partial n}+\frac{\partial\phi_7}{\partial n}+\frac{\partial}{\partial n}\sum_{j=1}^{6}v_{a_j}\phi_j\right)e^{-i\omega t}=V_n$$

where $V_n=\sum_{j=1}^{6}\frac{\partial}{\partial t}(\eta_{a_j}e^{-i\omega t})n_j=-i\omega(\sum_{j=1}^{6}\eta_{a_j}n_j)e^{-i\omega t}$ is the normal velocity component of a point on the body surface, where η_{a_j} indicates the complex amplitude of the motion responses in 6 degrees of freedom.

In accordance with the above described linear superposition assumption, a further decomposition of the kinematic boundary condition yields the following for the radiation problem:

$$\frac{\partial\phi_j}{\partial n}=n_j \qquad j=1,2,\ldots,6 \quad \text{, on } S_2$$

and for the diffraction problem:

$$\frac{\partial\phi_0}{\partial n}+\frac{\partial\phi_7}{\partial n}=0 \qquad \text{, on } S_2$$

n_j are direction cosines on the body surface

where **n**$=in_1+jn_2+kn_3$ denotes the outward unit normal vector on wetted surface and d denotes the depth of submergence of the centre of gravity in respect of centre of coordinate system.

3-2) Forces and Moments – Equation of motion:

The forces and moments $\overline{F}$ follow from an integration of the pressure, p, over the submerged (wetted) surface, S, of the body:

$$\vec{F}=-\iint_S(p.\vec{n}).dS$$

in which $\vec{n}$ is the outward normal vector on surface dS and in the $O(x_0,y_0,z_0)$ coordinate system.
The pressure p -via the linearized Bernouli equation- is determined from the velocity potential by:

$$p = -\rho \frac{\partial \Phi}{\partial t} - \rho g z_0$$

or

$$p = -\rho \frac{\partial}{\partial t}(\Phi_I + \Phi_D + \Phi_R) - \rho g z_0$$

Having obtained the various force and moment components, it can be then constructed the equation of motion for the jth direction by applying Newton's second law of motion:

$$\sum_{k=1}^{6}\left(M_{jk}\ddot{\eta}_k\right) = \vec{F_j} \qquad \text{j=1,2,…,6}$$

where M_{jk} is the mass matrix coefficients.

If a symmetrical distribution of masses with respect to the plane y_0=0 is assumed, and with respect to further simplification for double symmetrical bodies like semi-submersible, the mass matrix is expanded as follows:

$$M = \begin{bmatrix} m & 0 & 0 & 0 & mz_{0_G} & 0 \\ 0 & m & 0 & -mz_{0_G} & 0 & 0 \\ 0 & 0 & m & 0 & 0 & 0 \\ 0 & -mz_{0_G} & 0 & m_{44} & 0 & 0 \\ mz_{0_G} & 0 & 0 & 0 & m_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & m_{66} \end{bmatrix}$$

where m is the mass of the body, which in a floating structure is equal to that of the water displaced, and m_{kj} (j=k) correspond to the mass moments of inertia and m_{0_G} is the vertical position of centre of gravity.

By substitution above mentioned definitions of forces, equation of motion will be in the form of: (Faltinsen(1990))

$$\sum_{k=1}^{6}\left[(M_{jk} + A_{jk})\ddot{\eta}_k + B_{jk}\dot{\eta}_k + C_{jk}\eta_k\right] = F_j e^{-i\omega t} \qquad \text{j=1,2,...,6}$$

where A_{jk} and B_{jk} are the added mass and damping coefficients that can be formally written due to harmonic motion mode η_j as:

$$\rho \iint_S \frac{\partial \Phi_R}{\partial t} n_k dS = \rho \iint_s \left(\frac{\partial}{\partial t}\sum_{j=1}^{6}\phi_j v_j\right) n_k dS = -A_{kj}\frac{d^2\eta_j}{dt^2} - B_{kj}\frac{d\eta_j}{dt}$$

If the structure has zero forward speed and there is no current by using Green's second theorem, it can be shown that $A_{jk} = A_{kj}$ and $B_{jk} = B_{kj}$ (Sarpkaya and Isaacson(1981)).

The matrix of hydrodynamic added masses, A, for double symmetrical bodies is obtained as:

$$A = \begin{bmatrix} A_{11} & 0 & A_{13} & 0 & A_{15} & 0 \\ 0 & A_{22} & 0 & A_{24} & 0 & 0 \\ A_{31} & 0 & A_{33} & 0 & 0 & 0 \\ 0 & A_{42} & 0 & A_{44} & 0 & 0 \\ A_{51} & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix}$$

The matrix of potential damping, B, likewise has the form above.
Finally, C is the matrix of restoring forces or hydrostatic stiffness matrix and for double symmetrical bodies is written as: (Faltinsen(1990))

$$C = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

where:

$$C_{33} = \rho g A_{WP}$$

$$C_{44} = \rho g V \overline{GM}_T$$

$$C_{55} = \rho g V \overline{GM}_L$$

Here A_{WP} is the waterplane area, V is the displaced volume of water, $\overline{GM}_T$ and $\overline{GM}_L$ are transverse and longitudinal metacentric heights, respectively.
For a moored structure, additional restoring forces have to be added. However, the effect of a spread mooring system on the linear wave-induced motion is generally quite small. In special cases, in particular for long wavelengths, the mooring system will have an influence.
$F_j e^{-i\omega t}$ (j=1,2,…,6) in equation of motion are wave exciting forces and moments consisting of incident and diffraction components:

$$F_j e^{-i\omega t} = \rho \iint_S \left(\frac{\partial \Phi_I}{\partial t} + \frac{\partial \phi_D}{\partial t}\right) n_j dS = -i\omega\rho e^{-i\omega t} \iint_S (\phi_I + \phi_D) n_j dS$$

From the kinematic boundary condition on the semi-submersible's mean wetted surface that described before:

$$\frac{\partial \phi_J}{\partial n} = n_j$$

$$\frac{\partial \phi_I}{\partial n} + \frac{\partial \phi_D}{\partial n} = 0$$

So:

$$F_j = -i\omega\rho \iint_S (\phi_I + \phi_D) \frac{\partial \phi_j}{\partial n} dS \qquad \text{for j=1,2,…,6}$$

in which ϕ_j is the radiation potential in direction j.

The potential of the incident waves, ϕ_I, is known and described before, but the diffraction potential, ϕ_D, has to be determined. Green's second theorem provides a relation between this diffraction potential, ϕ_D, and a radiation potential, ϕ_j:

$$\iint_S \phi_D \frac{\partial \phi_j}{\partial n} dS = \iint_S \phi_j \frac{\partial \phi_D}{\partial n} dS$$

but $\frac{\partial \phi_I}{\partial n} = -\frac{\partial \phi_D}{\partial n}$ so:

$$\iint_S \phi_D \frac{\partial \phi_j}{\partial n} dS = -\iint_S \phi_j \frac{\partial \phi_I}{\partial n} dS$$

This elimination of the diffraction potential results into the so-called **Haskind** relations:

$$F_j e^{-i\omega t} = -i\omega\rho e^{-i\omega t} \iint_S \left(\phi_I \frac{\partial \phi_j}{\partial n} - \phi_j \frac{\partial \phi_I}{\partial n} \right) dS \qquad \text{for j=1,2,…,6}$$

So the expression for wave exciting forces and moments (diffraction problem) depends only on the incident wave potential, ϕ_I, and the radiation potential, ϕ_D.

As it described before, the space dependent of incident wave potential in deep water:

$$\phi_I = \frac{-igA}{\omega} e^{i(kx \sin\mu + ky \cos\mu)}$$

So:

$$\frac{\partial \phi_I}{\partial n} = \frac{-igA}{\omega}(ik)\left\{ \frac{\partial x}{\partial n} \sin\mu + \frac{\partial y}{\partial n} \cos\mu \right\} e^{i(kx \sin\mu + ky \cos\mu)} = ik\phi_I \{ n_1 \sin\mu + n_2 \cos\mu \}$$

hence:

$$F_j e^{-i\omega t} = -i\omega\rho e^{-i\omega t} \iint_S \phi_I n_j dS + k\omega\rho e^{-i\omega t} \iint_S \phi_I \phi_j \{ n_1 \sin\mu + n_2 \cos\mu \} dS$$

The first term in the expression for the wave loads is the so-called **Froude-Krilov** force or moment, which is the wave load caused by the undisturbed incident wave. The second term is caused by the wave disturbance due to the presence of the (fixed) body, the so-called **diffraction force**.

3-3) Formulation of Hydrodynamic Analysis of Interaction between a Semi-submersible and Sea Waves using Direct Boundary Element Method:

By introducing the approximate solution into above mentioned formulas for ϕ_j and its flux functions, the errors will be produced. For minimizing these errors, three dimensional fundamental solution of Laplace's equation as the weighting function is applied as follows:

$$\int_{\Omega} \left(\nabla^2 \phi_j\right) u^* d\Omega = \int_{S_1} \left(\frac{\partial \phi_j}{\partial z} - \frac{\omega^2}{g} \phi_j \right) u^* ds + \int_{S_2} \left(\frac{\partial \phi_j}{\partial n} - n_j \right) u^* ds + $$

$$+ \int_{S_3} \left(\frac{\partial \phi_j}{\partial z} \right) u^* ds + \int_{S_4} \left(\frac{\partial \phi_j}{\partial n} - ik\phi_j \right) u^* ds \qquad , j=1,2,\ldots,6$$

where: $u^* = \frac{1}{4\pi r}$

$$r = \sqrt{(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2}$$

in which:

X (x,y,z): Field Point

X_i (x_i ,y_i ,z_i) : Source Point

By suitable change and by applying Green's second identity, it can be shown that:

$$\phi_j^{\,i} + \int_{S_1} \left(\phi_j \frac{\partial u_i^*}{\partial n} - \frac{\omega^2}{g} \phi_j u_i^* \right) ds + \int_{S_2+S_3} \left(\phi_j \frac{\partial u_i^*}{\partial n} \right) ds + \int_{S_4} \left(\phi_j \frac{\partial u_i^*}{\partial n} - ik\phi_j u_i^* \right) ds =$$

$$= \int_{S_2} n_j u_i^* ds$$

This equation relates the value of u^* at the point 'i' (in the interior domain Ω) with the values of ϕ_j over the boundary S.

By moving the point 'i' to the boundary and dealing with the problem of singularity of integrands at this point (source point), finally it can be shown that: (Brebbia(1980))

$$C_i \phi_j^{\,i} + \int_{S_1} \left(\phi_j \frac{\partial u_i^*}{\partial n} - \frac{\omega^2}{g} \phi_j u_i^* \right) ds + \int_{S_2+S_3} \left(\phi_j \frac{\partial u_i^*}{\partial n} \right) ds + \int_{S_4} \left(\phi_j \frac{\partial u_i^*}{\partial n} - ik\phi_j u_i^* \right) ds =$$

$$= \int_{S_2} n_j u_i^* ds$$

where:

$C_i = 1$ if source point is inside the domain Ω

$C_i = 0$ if source point is outside of the domain Ω

$C_i = 1/2$ if source point is on the smooth boundary S

Hence, in order to formulate the problem in terms of boundary element method, source points will choose on the boundary of the domain under consideration, and by discretizing the boundary into n elements and choosing the unknown values, ϕ_j, as the nodes of these elements, above formulation will be solved by applying numerical integration process, in this research, Gussian quadrature integration method.

3-4) Numerical Analysis of KHAZAR Semi-Submersible by using Direct Boundary Element Method (Case study):

Khazar Semi-Submersible Drilling Unit (KSSDU) is the largest semi-submersible drilling platform in the Caspian Sea, north of Iran, which is under construction in the

Caspian Sea Complex Yard located at suburb of city of Neka on the coast of caspian Sea in Iran. The project was awarded to a consortium of the local Iran Marine Industrial Company (SADRA) and GVA consultants of Sweden by National Iranian Oil Company. The unit will be operating in the southern waters of the Caspian Sea with a depth of around 1,000 meters. General particulars of this platform (shown in Table 1) which is used as the case study in this research are as follow:

The rigid body motion response amplitude operator (RAO) in 6 degrees of freedom of KHAZAR semi-submersible in response of heading waves with time periods from 3 s to 30 s have been calculated by using direct boundary element method. The water depth was assumed 500 m (deep water) and all calculations has been made for 19.5 m draught (operational draught).

Table 1- General Particulars (KHAZAR Semi-Submersible):

Length over all (approx.)	98.60 m
Beam over all (approx.)	78.84 m
Width outside Pontoons	73.4 m
Pontoon length (moulded)	80.56 m
Pontoon width (moulded)	18.68 m
Pontoon height (moulded)	7.5 m
Column Diameter moulded)	12.9 m
Column spacing, longitudinal	54.72 m
Column spacing, transverse	54.72 m
Height to Box Bottom	28.5 m
Height to Lower Deck	29.7 m
Height to Tween Deck	33.0 m
Height to Upper Deck	36.5 m
Displacement at Transit Draught (7.2 m , spec.grav.=1.01 tonnes/m^3)	20665 tonnes
Displacement at Survival Draught (16.2 m , spec.grav.=1.01 tonnes/m^3)	26525 tonnes
Displacement at Operational Draught (19.5 m , spec.grav.=1.01 tonnes/m^3)	28621 tonnes
Air-gap at Still Water (at draught=19.5m)	9.0 m

Some results of calculations -which have been produced by the program written based on direct BEM described before- have been presented in following charts. For comparison, the results produced by designer (GVA consultants) have also been shown.

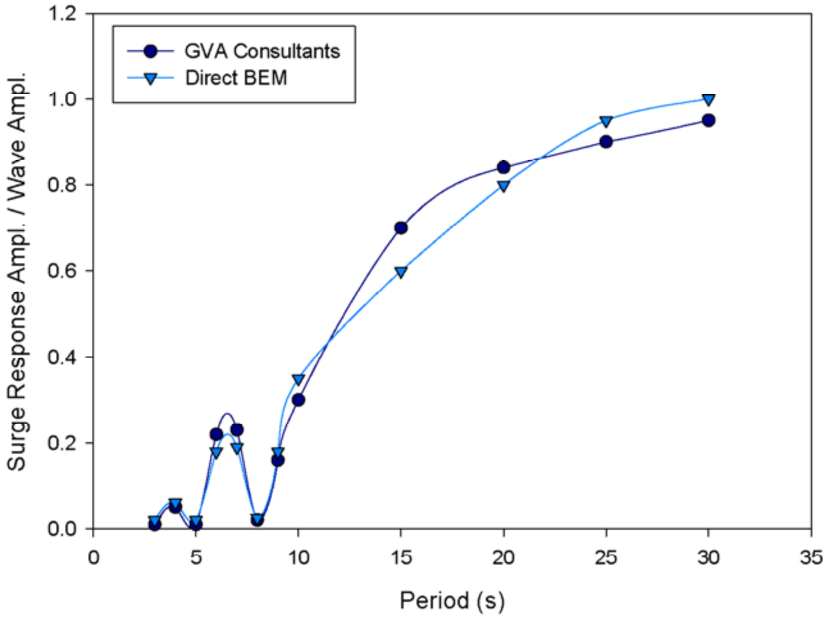


Figure 2, RAO for Surge

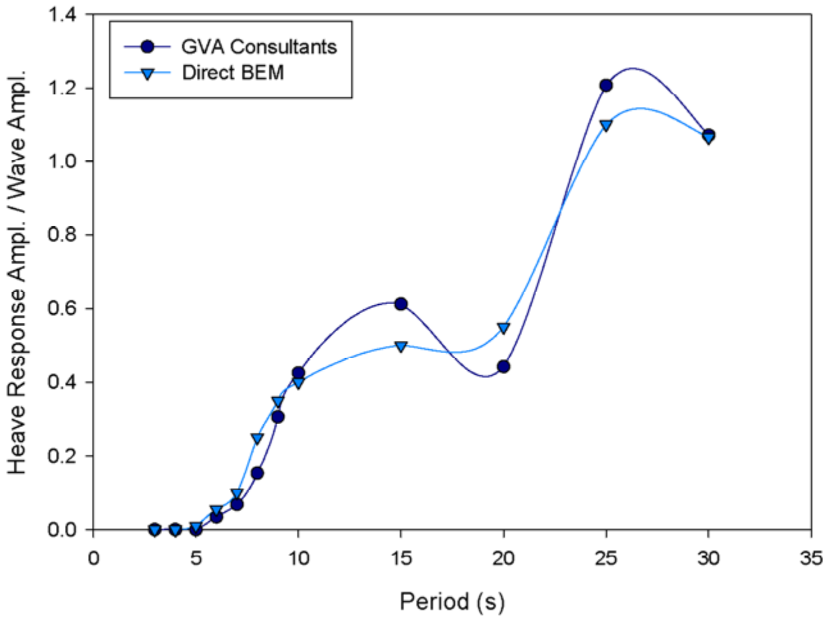


Figure 3, RAO for Heave

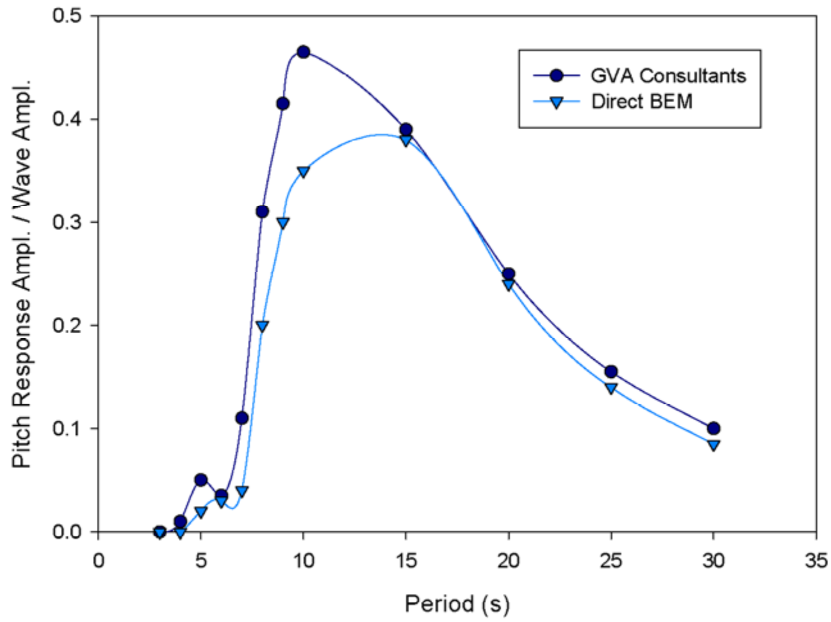


Figure 4, RAO for Pitch

As it can be seen from these charts, there is a good agreement between direct boundary element method and the most common method –indirect BEM- which has been used by GVA Consultants Company.
Next step will be imposing the slamming effect –derivation will be presented in the next chapter- into equation of motion and studying on its influences on hydrodynamic behaviour of platform and its structural responses, as well.

4) Wave Slamming Underneath the Deck:

4-1) Introduction:

Impulse loads with high pressure peaks occur during impact between a body and water. This is often called "slamming". The duration of slamming pressure measured at one place on the structure is of the order of milliseconds (Faltinsen(1990)). It is very localized in space. The position where high slamming pressure occur changes with time. Slamming pressures are sensitive to how the water hits the structure.
A fully satisfactory theoretical treatment on slamming has been prevented so far by the complexity of the problem:

- Slamming is a strongly non-linear phenomenon which is very sensitive to relative motion and contact angle between body and free surface.
- Predictions in natural seaway are inherently stochastic; slamming is a random process in reality.
- Since the duration of wave impact loads is very short, hydro-elastic effects are large.
- Air trapping may lead to compressible, partially supersonic flows where the flow in the water interacts with the flow in the air.

Classical theories approximate the fluid as inviscid, irrotational, incompressible and this allows a (predominantly) analytical treatment of the problem in the framework of potential theory.

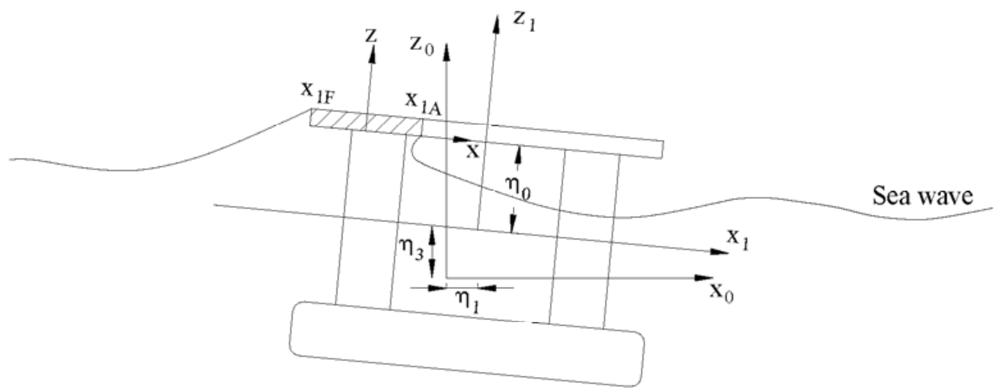
Von Karman(1929) was the first to study theoretically water impact (slamming). He idealized the impact as a 2-D wedge entry problem on the calm-water surface to estimate the water impact load on a seaplane during landing

Wagner(1932) derived a more realistical water impact theory. He used similar arguments to Von Karman but considered the effect of the spray roots.

Wagner's theory can be applied to arbitrarily shaped bodies as long as the deadrise angle is small enough not to trap air, but not so small that air trapping plays a significant role. Wagner's theory is simple and useful, even if the linearization is sometimes criticized. Many experimental studies have checked the accuracy of Wagner's theory (Bertram(2000)). Measured peak impact pressures are typically a little lower than estimated. This suggested that Wagner's theory gives conservative estimates for practical use.

4-2) The impact's hydrodynamic formulation for the Wagner based method:

Propagating incident waves are assumed. The rigid body motion is described by six degrees of freedom with respect to a global coordinate system, (x_0,y_0,z_0). The (x_0,y_0,z_0)-coordinate system is earth fixed and attached to the mean body position, i.e. no drift forces are assumed and thus the body has zero mean velocity. The coordinate system is right-handed, with positive z-axis vertically upwards through the body's centre of gravity. The x_0y_0-plane is located on the undisturbed free surface. Let the oscillatory translatory displacements parallel to the x_0-, y_0-, and z_0-axis be referred to as surge, sway, and heave respectively, and denoted as η_1, η_2, and η_3. The angular oscillatory displacements of the rotational motions about the same axes are denoted as η_4, η_5, and η_6, i.e. roll, pitch and yaw respectively. See the illustration in Figure 1. A two-dimensional problem is assumed. Accordingly, the y-axis can be omitted. This is the situation shown in Figure 5, where the waves propagate along the positive x_0-axis. Small body motions are assumed so that $\sin\eta_5 \sim \eta_5$ and that $\cos\eta_5 \sim 1$.



Body motion during impact

Figure 5

Three coordinate systems are shown in the figure. The (x_1,z_1)-coordinate system is right-handed and body fixed. Its axes and origin are located so that the coordinate system coincides with the global earth-fixed coordinate system when the body is in its mean position. The relationship between the body fixed and the earth fixed coordinate systems may be written as:

$x_0 = x_1 + \eta_1$ and $z_0 = z_1 + \eta_3 - x_1\eta_5$

The third coordinate system has its origin in the centre of the instantaneous wetted part of the deck. The front end, or upstream end, of the wetted length is defined as x_{1F} , while the aft end, or downstream end, of the wetted length is denoted as x_{1A}, when measured in the body-fixed x_1z_1-reference frame. Similarly to the notation used in Faltinsen(1990) for impact problems, half of the wetted deck length is denoted as c, so that the total length of the wetted part of the deck is equal to 2c. This gives the following relations between the xz- and the x_1z_1-coordinate systems:

$x = x_1 - x_{1F} - c$ and $z = z_1 - \eta_0$

Whereη_0 is the deck clearance (air gap) in still water. The different coordinate systems are shown in Figure 5.

As assumptions of study on the hydrodynamic boundary value problem of the impact, two-dimensional, irrotational flow and an incompressible fluid are taken into account. The approximation of two dimensional flow requires head or beam sea and that the incident waves are long relative to the diameter of the platform legs (semi-submersible's columns) so diffraction due to these members is neglected. The fluid flow can be described by the total velocity potential $\Phi = \phi_I + \phi$, where ϕ is the velocity potential due to the impact and ϕ_I is the known undisturbed incident wave potential. A boundary value problem (BVP) for ϕ can be set up for each time instant and for a given wetted body area. The two-dimensional Laplace equation becomes the governing equation in the fluid domain:

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial z^2} = 0$$

which must be satisfied in the entire fluid domain. To solve above equation, boundary conditions on the free surface and the wetted body surface are also required.

The dynamic free surface condition is obtained from Bernoulli's equation by imposing atmospheric pressure on the free surface. It is assumed that the impact occurs over a small time instant, meaning that the gravitational acceleration g is negligible relative to the impact induced fluid accelerations, and that the rate of change of ϕ with time is generally larger than the rate of change ϕ with respect to the spatial coordinates. This gives $\frac{\partial\phi}{\partial t} = 0$ on the free surface. Since the initial value of ϕ is zero, this gives $\phi = 0$ on the free surface.

When solving the boundary value problem this condition is applied on the horizontal line z=0, i.e. the dynamic free surface condition becomes:

$$\phi = 0 \qquad \text{on } z=0$$

This condition implies that no waves will be generated. This dynamic free surface condition is often used in impact studies. Wagner used this condition in the outer domain (i.e. outside the spray root).

In addition, the kinematic free surface condition states that a fluid particle on the free surface remains on the free surface. Also the kinematic condition is satisfied on z=0.

The body boundary condition is defined as:

$$\frac{\partial\Phi}{\partial n} = U \cdot n \qquad \text{on the wetted body surface}$$

where $\frac{\partial}{\partial n}$ denotes differentiation along the normal direction to the body surface, U is the velocity of the body, and **n=**(n_1,n_3) is the unit normal vector of the body surface. **n**
is positive into the fluid domain.

Solving for $\frac{\partial\phi}{\partial n}$, the body boundary condition becomes:

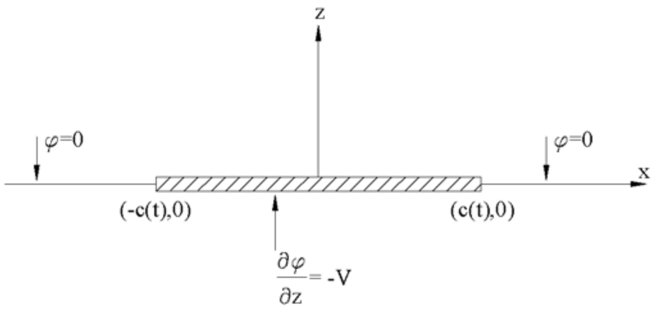
$$\frac{\partial\phi}{\partial n} = U \cdot n - \frac{\partial\phi_I}{\partial n} \qquad \text{on the wetted length of the deck}$$

For impact with small local dead-rise angle, i.e. small angle between the body surface and the free surface at their intersections, above equation may be approximated as, Faltinsen(1990);

$$\frac{\partial\phi}{\partial z} = -V_r \qquad \text{on } z=0 \quad \text{and} \quad |x| < c(t)$$

where V_r is the relative normal velocity between the body and the fluid. The impact velocity gets contributions from both the fluid particle velocities in the incident wave (ϕ_I) and from the rigid body velocity of the deck. 2c(t) is an

approximation of the wetted length of the deck. The boundary value problem for ϕ is illustrated in Figure 6. The shaded rectangle symbolizes the instantaneous wetted area, and as it mentioned before, the xz-coordinate system has its origin at the centre of this area (see Figure 5).
Making the assumption that the normal velocity, V_r, along the wetted length is constant, a solution to this problem can be shown in this form: (Newman(1977), by using conformal mapping technique)



Boundary value problem in simplified analysis of impact between a two-dimensional body and water

Figure 6, Faltinsen(1990)

$\phi = -V_r\left(c^2 - x^2\right)^{\frac{1}{2}}$ on z=0 and $|x| < c(t)$

$\phi = 0$ on z=0 and $|x| > c(t)$

Therefore, the hydrodynamic pressure on the body will be:

$$P = -\rho\frac{\partial\phi}{\partial t} = \rho V_r \frac{c}{\left(c^2 - x^2\right)^{\frac{1}{2}}}\frac{dc}{dt} + \frac{dV_r}{dt}\left(c^2 - x^2\right)^{\frac{1}{2}}$$

So, the corresponding vertical force on the body will be:

$$F_z = \int_{-c}^{c} P dx = \rho V_r c \frac{dc}{dt}\int_{-c}^{c}\frac{dx}{\left(c^2 - x^2\right)^{\frac{1}{2}}} + \frac{dV_r}{dt}\int_{-c}^{c}\left(c^2 - x^2\right)^{\frac{1}{2}}dx = V_r\frac{d}{dt}\left(\rho\frac{\pi}{2}c^2\right) + \left(\rho\frac{\pi}{2}c^2\right)\frac{dV_r}{dt}$$

where $m_z = \rho\frac{\pi}{2}c^2$ is the added mass in heave for the flat plate in infinite fluid.

So:

$$F_z = V_r\frac{dm_z}{dt} + m_z\frac{dV_r}{dt} = \frac{d}{dt}\left(m_z V_r\right)$$

Therefore, the impact force on the body is equal to time rate of change of vertical fluid momentum component.
It can be shown that:

$$F_z = \rho\pi c\left(V_r\frac{dc}{dt} + \frac{c}{2}\frac{dV_r}{dt}\right)$$

The quantities c and $\frac{dc}{dt}$ are determined from the relative degree of wetting of the deck underside, which are found in terms of the wave elevation and the vertical motion of the offshore structure as the incident wave travels along the deck from its initial contact location. The first term on the right in the above formula continually varies up to the time when the wetted length c reaches the end of the deck plate, after which $\frac{dc}{dt}=0$ and that term is then zero throughout the remaining time that the particular wave elevation is contacting the deck.

5) Conclusion and Further Research:

In this paper, the direct boundary element method- as a new and powerful method in ocean engineering- for 3-d hydrodynamic analysis of the semi-submersible in sea waves has been developed and as a case study, some RAOs for KHAZAR semi-submersible has been produced and compared with results presented by designer which shown a good agreement.

The derivation of slamming force underneath the deck of semi-submersible has been done by using Wagner's theory. Wagner's theory can be applied to arbitrarily shaped bodies as long as the deadrise angle is small enough not to trap air. Many experimental studies have checked the accuracy of Wagner's theory (Bertram(2000)).

In this research, although 2-dimensional Wagner based theory for slamming analysis has been developed, but as a reasonable move regarding 3-d nature of slamming, the relative local velocity between offshore structure and sea water surface will be achieved by using 3-dimensional panel method (direct BEM).

As the further research, next step will be composition of hydrodynamic behaviour of the semi-submersible in sea waves and slamming force due to impact wave underneath of deck, with each other and study on this combination's effects from different point of views.

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